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ENGINEERING SERIES

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AN INTRODUCTORY TEXTBOOK OF ELECTRICAL ENGINEERING

BY

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IN THE UNIVERSITY OF FLORIDA

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PREFACE

This book has been written to serve as an elementary textbook of electrical engineering, for the use of students who have some knowledge of college physics and mathematics through elementary calculus. It is intended to give a brief general acquaintance with the more important principles and facts of electrical engineering. For students who are specializing in electrical engineering it may serve as a preliminary to more advanced courses; for other students it should be a suitable text for the short courses in electrical engineering sometimes given to students who are specializing in other branches of engineering or in general science.

The length of the text makes it suitable for use in courses lasting one semester. Before its publication in book form, the text has been used in mimeographed form at the University of Florida in a course arranged for two recitations per week for one semester, with one two-hour laboratory period per week. In this course the classes have covered all the text (including about two thirds of the problems) excepting Chapters I and XXII, which were covered in other courses.

The book is so arranged that certain omissions can be made without loss of continuity. All the portions in fine print may be omitted. The last three chapters treat subjects which are somewhat distinct from those of the other chapters, and can therefore be omitted without affecting the completeness of the rest of the book. The first chapter can be omitted by students who are already sufficiently well grounded in the fundamental principles of electricity and magnetism.

The problems which follow each chapter are not intended to be difficult. Their purpose is to illustrate the principles discussed in the text and to give the student practice in applying them. Except in Chapter I, the problems in each set are intended to be arranged in the order of their importance, so that if omissions are made the later problems in any set should be omitted rather than the earlier ones.

In issuing this text the author takes pleasure in acknowledging his indebtedness to Dean A. A. Potter of Purdue University and to Professor Joseph Weil of the University of Florida for many valuable suggestions and criticisms; to Messrs. J. W. Graff, J. G. Keck, and R. D. Ross, former students of the University of Florida, for assistance in preparing the line drawings; and to numerous electrical manufacturers for permission to use illustrations taken from their publications. Individual acknowledgments are made to the manufacturers in the legends of the illustrations they have supplied.

JOHN ROBERT BENTON

UNIVERSITY OF FLORIDA

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AN INTRODUCTORY TEXTBOOK OF ELECTRICAL ENGINEERING

INTRODUCTION

It is not much more than a hundred years since experimenters first began to acquire systematic knowledge of electricity, and much less than a hundred years since mankind began to apply it to useful purposes. Yet today the use of electricity has become so widespread and varied that there is hardly any branch of human activity in which it is not turned to account. It is used not only in nearly all the industries but also (in this country) in almost every household, on almost every farm, and even on almost every vehicle and boat.

The usefulness of electricity arises principally from the fact that it offers convenient means for transmitting energy and transforming it into such forms as may be desired. For example, by means of electricity the supply of energy at some distant point — perhaps a waterfall a hundred miles away — can be drawn upon in mechanical form to drive a machine (by means of an electric motor); or in thermal form to heat a furnace; or in the form of visible radiation to light a room; and the energy in each of these forms is instantly available to the operator upon the mere turning of a handle or switch, placed to suit the convenience of the operator and not necessarily very close to the appliance it controls.

The subject of electrical engineering includes the principles and practice involved in the generation and distribution of electrical energy and in all its useful applications, most of which fall under the following headings: (1) mechanical power; (2) lighting; (3) heating; (4) chemical applications; (5) transmission of intelligence, by telephone or telegraph, with or without wires.

Since electricity is useful primarily as a means for conveying energy, those parts of the subject that deal with the generation

and distribution of electrical energy are of fundamental and general importance; while those that deal with the applications are of more limited or specialized importance. Accordingly, most general treatises on electrical engineering are devoted primarily to the subjects of the generation, distribution, and transformation of electrical energy (especially its transformation to mechanical energy); while the special applications, if included at all, receive treatment in subsidiary chapters. Special applications, although belonging to electrical engineering, are not fundamental to it, and are often treated as distinct specialized branches of engineering, under such names as illuminating engineering, electrochemistry, telegraph engineering, telephone engineering, and radio engineering.

This book is devoted almost entirely to the fundamental parts of electrical engineering; and the subjects of illuminating engineering, electrochemistry, and electrical transmission of intelligence are entirely excluded from its scope.

CHAPTER I

RÉSUMÉ OF ELECTROMAGNETIC PRINCIPLES AND DEFINITIONS

1. Magnetic poles. The most easily observed property of a magnet is that of attracting iron, and it is a familiar fact that every magnet shows this property most prominently at two particular regions of its surface, known as "poles." If the magnet has the form of a long thin bar, the poles are usually located at the extreme ends, and the middle parts of the bar do not show the magnetic property of attracting iron, to any appreciable extent. In such cases the polar regions may be very small in comparison with the dimensions of the whole magnet, and each pole may then be looked upon as a center of attraction and regarded as concentrated at a single point. To be sure, it is never strictly true that a pole is located at a single mathematical point; but often a polar region is so small, in comparison with the distances across which its action is being considered, that only an insignificant error is committed in considering the whole polar region to be concentrated at one point.

Every magnet has two poles; so that if an attempt is made to study the forces exerted by one of the poles on some piece of iron, the results will be complicated by forces coming from the other pole at the same time. Although it is not possible to construct a magnet having only one pole, it is possible, by using a very long bar, to construct one in which the poles are very far apart. Then, if a piece of iron is placed near to one pole, the other pole is so much farther away that any force due to it will be insignificant in comparison with that due to the nearer pole.

By experimenting with the concentrated poles at the ends of long bars Coulomb was able to show that the force with which any two poles act upon each other varies inversely as the square of the distance between them, this force being one

of repulsion if the two poles are of the same kind and one of attraction if they are of opposite kinds.

2. Unit pole. A magnet pole is said to have a strength of one unit pole if it exerts a force of one dyne upon an equal pole placed one centimeter away from it in air. A magnet pole which exerts a force of m dynes upon a unit pole one centimeter away in air is said to have a strength of m units.

From the definition just given, and the results of Coulomb's experiments, it follows that the force F in dynes between any two concentrated magnetic poles of strengths m_1 and m_2 placed r centimeters apart in air is given by

$$F = \frac{m_1 m_2}{r^2}. \tag{1}$$

This statement is often called Coulomb's law.

The force between magnet poles depends not only upon the strengths of the poles and their distance apart but also upon the medium surrounding them. When this is taken into account, the expression for the force becomes

$$F = \frac{m_1 m_2}{\mu r^2}, \tag{2}$$

where μ is a constant of the medium surrounding the poles, and is called the *permeability* of the medium.

The permeability of air is taken as unity, so that for air equation (2) becomes the same as equation (1). For iron and its alloys the permeability is very large; for a few other substances the permeability differs appreciably from that of air; but for the great majority of ordinary substances (including all common gases and liquids) the permeability does not differ appreciably from that of air. Accordingly equation (1) can be used pretty generally, although it is really a special case of the more general equation (2). Nevertheless the special case is the one that nearly always occurs.

3. Magnetic field. A portion of space throughout which magnetic forces are present is called a *magnetic field*. To observe the existence of such forces a magnetic pole may be introduced into the space, and the force which is found to act upon it at any point determines the direction and strength of the magnetic field at that point.

If a magnet pole of strength m units is found to be acted upon by a force of F dynes when placed at a certain point in a magnetic field, then the strength, or intensity, H, of the magnetic field at that point is defined as the force per unit pole, or

$$H = \frac{F}{m}. \qquad (3)$$

The unit of field strength is accordingly a field of such strength that a unit pole receives a force of one dyne. This unit is called the *gauss*.

Besides noting the formal definition of the gauss, it is desirable to form a direct conception of this unit in terms of magnetic fields with which the reader may be already familiar. Thus the magnetic field of the earth (total) has a strength of about half a gauss, and its horizontal component is from a sixth to a quarter of a gauss (in the United States). The gauss may then be thought of as a rather weak field, only a few times stronger than the earth's field. The field strengths used in the magnetic fields of ordinary dynamos range from about 5000 gausses to about 14,000 gausses.

4. Lines of force. A line which is so drawn that its direction at every point is the same as the direction of the force that would act upon a north magnetic pole * placed at the same point is called a *line of force*. The arrangement of the magnetic forces in a magnetic field can be represented very conveniently by drawing the lines of force. Thus, the field due to an isolated north pole is shown by Fig. 1; that due to a north pole and a south pole (at the ends of a bar magnet) is shown in Fig. 2.

After having started from the surface of a north pole, a line of force must continue without limit unless it reaches the surface of a south pole. It is not possible for the line to stop, or reach a "dead end," within the magnetic field in the same medium, since this would correspond to a field of zero intensity; if the field had any intensity at all, it would be in some direction, and that direction would determine the continuance of the line of force. The line (within the same medium) must always be a smooth curve without angles, since an angle in a line of force would indicate a point at which the force on a magnet pole

* The term *north* magnetic pole refers to a pole that is attracted toward the north when acted upon by the earth's field.

would have two different directions at the same time, which is impossible. For the same reason it is not possible for two lines of force to cross each other.

In the fields shown in Figs. 1 and 2 it is evident that the regions where the lines of force are closest together are the same as those where the field is most intense. This suggests a method by which the lines of force may be so drawn as to indicate the intensity of the field as well as its direction. Consider a surface drawn so as to be everywhere normal to the direction of the magnetic field, and let us agree that the *number* of lines of force drawn through each square centimeter of this surface shall be made the same as the number of gausses of field intensity for that square centimeter. Then, at all parts of the chosen surface, the field intensity will be represented by the number of lines of force per square centimeter. Now if any other surface normal to the lines (in the same medium) is considered, it will be found that the field intensity there is also given by the number of lines per square centimeter. This may be accepted here as an experimental fact; but it can also be deduced by mathematical reasoning that will not be given here, as a consequence of the inverse-square law which governs the forces of attraction or repulsion between magnetic poles.

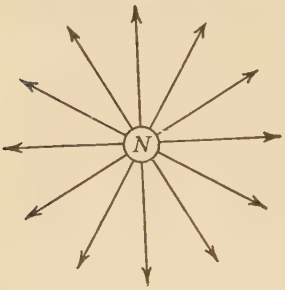


FIG. 1. Field due to an isolated north pole

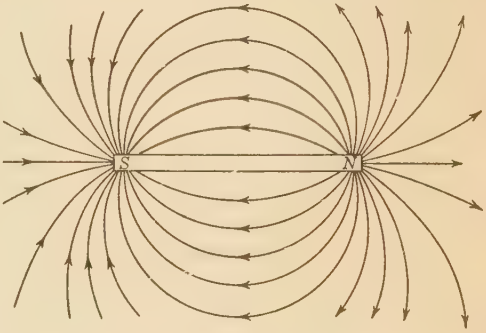


FIG. 2. Magnetic field due to a north pole and a south pole

If a surface which is not normal to the lines of force is considered, then it is easy to see that the number of lines per square centimeter of such a surface will represent the component of the intensity of the magnetic field normal to the surface.

5. Field due to an isolated magnetic pole. An isolated magnetic pole of strength m units will exert upon a unit pole, at a distance r in air, a force F equal to $\frac{m \times 1}{r^2}$. Accordingly the field intensity H due to an isolated magnetic pole in air is equal to $\frac{m}{r^2}$, where r is the distance from the pole.

When r equals unity, the field intensity is numerically equal to m, and accordingly should be represented by drawing m lines of force per square centimeter of a surface normal to the lines. But the surface normal to the lines of force will be a sphere of unit radius having its center at the pole. The total area of the sphere is 4π, so that the total number of lines to be drawn is $4\pi m$. Thus the field due to an isolated pole of strength m units in air is represented by $4\pi m$ equally spaced radial lines.

If the pole is not in air but in a medium whose permeability is μ, then the field intensity at a distance r is given by $\frac{m}{\mu r^2}$, or

$$H = \frac{m}{\mu r^2}. \tag{4}$$

6. Magnetic flux. Consider a surface of area A at right angles to a uniform magnetic field of intensity H in air. The intensity of the field (in air), multiplied by the area of the surface, is called the *magnetic flux* through the surface, or

$$\Phi = HA, \tag{5}$$

where Φ stands for magnetic flux. In the C. G. S. system, the unit for expressing magnetic flux is evidently that flux which passes through an area of one square centimeter placed at right angles to a uniform magnetic field whose intensity is one gauss. This unit is called the *maxwell*.

If lines of force are drawn to represent a magnetic field, one line per square centimeter being drawn for each gauss of intensity as described in section 4, then it is evident that the number of maxwells of magnetic flux through any area at right angles to the field will be the same as the total number of lines of force through that area. For this reason magnetic flux is often expressed in *lines*.

If the field is not of uniform intensity, then the flux through any surface normal to the field may be found by dividing the surface into

a number of small elements, each so small that the field is substantially uniform over any one element. Then the flux $d\phi_1$ through any element of area dA_1 will be, by definition,

$$d\phi_1 = H_1 \cdot dA_1,$$

and the flux through the whole surface will be

$$\Phi = H_1 \cdot dA_1 + H_2 \cdot dA_2 + H_3 \cdot dA_3 + \text{etc.}$$

By using the notation of integral calculus, the same thing is expressed as

$$\Phi = \int H \cdot dA,$$

where the integration is to be carried out over the surface A.

This is also the same as saying that the flux is equal to the whole area multiplied by the average of the field intensity throughout the area.

If a surface is not normal to the direction of the magnetic field, then the flux through it is equal to the area multiplied by the component of the field intensity normal to the surface (or by the average value of this component, if the field is not uniform). In this case also the flux will equal the number of lines of force that cross the surface. For, assuming a uniform field and a plane surface, and letting θ represent the angle between the direction of the field and the perpendicular to the surface, we have the flux given by

$$\Phi = H \cos \theta \cdot A.$$

But $\cos \theta \cdot A$ equals the projection of the surface at right angles to the field, and the number of lines per unit area of this projection is H. So the total number of lines through this projection (and therefore also through the oblique surface A) is equal to $H \cdot (\cos \theta \cdot A)$, which is the same as the flux through A.

If the magnetic field is not in air, but in some other medium whose permeability is μ, then the magnetic flux through a surface of area A is

$$\Phi = \mu H \cdot A, \qquad (6)$$

if the field is uniform and perpendicular to A. This should be taken as the general definition of magnetic flux. (If the field is not uniform or not perpendicular to A, the expression (6) is modified in the manner just discussed above, but using μH in place of H.)

By equation (4) the field intensity at distance r from an isolated pole of strength m in a medium of permeability μ is $H = \frac{m}{\mu r^2}$. The total flux from the pole will be that passing through a sphere whose center is at the pole; if the sphere is taken with radius r, then its area is $4\pi r^2$. Then, according to equation (6), the flux is

$$\Phi = \mu \cdot \frac{m}{\mu r^2} \cdot 4\pi r^2 = 4\pi m. \tag{7}$$

This is independent of the radius chosen for the sphere, as would be expected; and it is also independent of the medium in which the pole is placed. Accordingly the total flux from a unit pole is 4π maxwells (or lines), regardless of the medium surrounding the pole.

7. Flux density. The magnetic flux per unit of area of a surface at right angles to the flux is called the *flux density** and is usually denoted by B. If the field is in air, B is evidently equal to H; otherwise it is equal to μH. B is expressed in gausses, just like H; but it is a different physical quantity from H, and should not be confused with H.

It is evident that lines can be drawn to represent flux density, in just the same manner as those to represent magnetic force. Such lines serve just as well to represent the distribution of the field as do the lines of force; in any actual field both the quantities B and H are present, and the field at any point can be specified just as well by the one as by the other. In air the same lines represent both B and H; in other media the lines of flux density are μ times as numerous as those of magnetic force, but have the same direction as the lines of force (except under certain conditions in crystalline media).

8. Fundamental character of the notion of a magnetic field. The foregoing discussion began by developing the notion of a (concentrated) magnetic pole; later, the notion of a magnetic field was developed. The notion of the magnetic pole is most convenient for some of the fundamental definitions of magnetic quantities; but the notion of the magnetic field is more convenient in studying a great many magnetic phenomena.

* Also called the *magnetic induction.*

The magnetic field should be regarded not merely as an imaginary thing which is convenient for picturing the distribution of magnetic conditions, but rather as a physical reality existing in that portion of space in which the field is located. From this point of view the magnetic field may be considered the fundamental thing, and the existence of the poles (or polar regions) merely a consequence of the existence of the field; a north polar region being any region of the field from which magnetic lines diverge, and a south polar region any region toward which the lines converge.

For example, when a previously unmagnetized bar of iron is placed in a magnetic field, it develops two polar regions and exhibits all the properties of a magnet. If the field is examined before and after the introduction of the iron, it will be found to have been altered, somewhat as shown in Fig. 3, which represents the effect on a previously uniform field of the introduction of a previously unmagnetized bar of iron. The iron may be thought of as merely the means for altering the field, and the appearance of the poles as a consequence of such alteration.

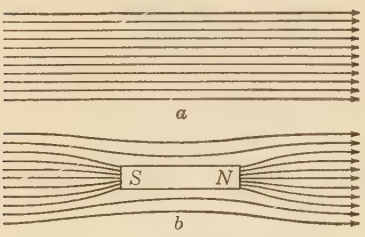


FIG. 3. Magnetic field

a, before introduction of a bar of iron; *b*, after introduction of a bar of iron

By proper experimental methods* it can be shown that in this case (and also in the case of a permanent magnet) magnetic flux passes through the cross section of the iron, as well as through any surface across the field outside the iron; and that the amount of the flux within the iron is the same as if the lines of flux density passed from the air into the interior of the iron, continued through the iron, and passed out again into the air without interruption. Accordingly the lines of flux density must not be regarded as ending at a polar region, but as continuing through the iron; and not simply as incapable of having a "dead end" within any one medium, but as incapable of having an end, whatever medium they may traverse.

In the case of a permanent magnet each line of flux density must then form a closed curve, and the magnet itself may be

* Based on the electromotive force induced in a circuit round the magnet.

looked upon as a sort of girdle holding together a bundle of closed lines of flux. In fact, the presence of the magnetic flux may be considered to be the fundamental thing about the magnet, and its poles to be merely the regions where the lines of flux density pass from iron to air or from air to iron.

9. Magnetic properties of electric currents. It was discovered by Oersted in 1819 that a magnetic needle was deflected when an electric current flowed in a conductor near it. This is the same thing as saying that the region about an electric current is a magnetic field. By examining the direction of the force on a north magnetic pole near a current it can be shown that the lines of force always form closed curves that link with the path of the current, although the exact form and distribution of these curves varies with the shape of the path followed by the current. The force on the magnetic pole is accordingly never one of attraction to the path of the current or repulsion from it; but the force acts sideways to the direction of the current. In the particular case that the current flows in a long straight conductor (with the return path too distant to have appreciable effect), the direction of the force on a magnetic pole is at right angles to the plane containing the current and the pole; and the lines of force are accordingly circles in planes at right angles to the current, having their centers on the path of the current.

The strength of the field at any point depends upon the location of the point, the strength of the current, and the shape and position of the path of the current.

If the path of the current is divided into short elements of length dl, each element may be considered to make a contribution of its own to the field at the point P; and the total field intensity at the point P will be the resultant of the intensities contributed by all the separate elements. The field intensity dH contributed by one element of current is given by

$$dH = I \cdot \frac{dl}{r^2} \cdot \sin (r, dl), \tag{8}$$

where I stands for the current strength, dl for the length of the element, r for the distance from the element to the point P, and (r, dl) for the angle between r and dl; and the direction of dH is perpendicular both to r and to the direction of dl.

The positive direction of the field (direction of force on a north magnetic pole) is related to the positive direction of the

current in the same manner as the direction of motion of a point on the circumference of a right-hand screw is related to the direction of motion of the axis of the screw, as shown by Fig. 4.

10. Unit of current strength. The magnetic field accompanying an electric current is just as fundamental a characteristic of the current as are the phenomena which take place within the conductor that carries the current. In fact, the magnetic field due to a current can be used for defining the strength or intensity of the current and for defining the unit of current.

The C. G. S. unit * of current is defined as a current of such strength that if one centimeter of length of the current is bent into a circular arc with a radius of one centimeter, a magnetic field of one gauss is produced at the center of the arc. If, under the same circumstances, the current produces a field of N gausses, it is considered to have a strength of N units. The C. G. S. unit of current is called the *abampere*; and the *ampere* is defined as one tenth of an abampere.

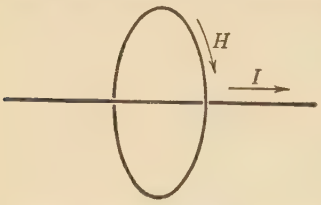


FIG. 4. Directions of a current and the magnetic field that accompanies it

11. Force on a current in a magnetic field. Since a magnet pole receives a force from an electric current, it follows from Newton's law of action and reaction that the electric current must receive from the magnet pole a force of equal strength but in opposite direction. This force acts on the material that carries the current, and its direction at any point is at right angles to the current and to the direction of the magnetic field at that point. Such a force acts upon a current placed in any magnetic field, as well as in the field associated with the pole of a magnet. If the field is uniform and at right angles to the current, the force on l centimeters of the current is given by the relation

$$F = lBI, \tag{9}$$

where F is the force in dynes, B the flux density of the magnetic field in gausses, and I the current in abamperes.

The directions of B, I, and F are related to one another in the same way as the outward directions of the thumb and the first

* Strictly, the C. G. S. *electromagnetic* unit. There is also a C. G. S. electrostatic unit of current strength, which, however, is not of practical importance.

two fingers of the *left* hand, as shown in Fig. 5. It may help to memorize this rule to note that when the thumb and fingers are taken in order, the corresponding quantities B, current, and F come in alphabetical order (but not the symbols B, I, and F — wherefore the *left* hand must be used).

Equation (9) holds true, using the unit of current already defined. It is also possible to define the C.G.S. unit of current by using equation (9), as the current of such strength that when placed at right angles to a uniform magnetic field of one gauss of flux density it receives a force of one dyne per centimeter of length of the current. The unit so defined is evidently identical with that defined in section 10.

If the field is not at right angles with the current, the force is given by lB_cI, where B_c stands for the component of flux density at right angles to the current.

Equation (9) is the fundamental equation for electric motors, since their action depends on the force exerted by a magnetic field upon current-carrying conductors.

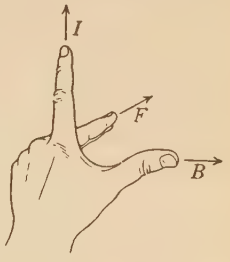


FIG. 5. Relative directions of F, B, and I

12. Practical system of units for electrical quantities. The abampere, or C. G. S. electromagnetic unit for current strength, was defined in section 10; and it is possible to define C. G. S. electromagnetic units for electromotive force, resistance, and all other electrical quantities. Such units have the advantage of being directly related to the fundamental units of length, mass, and time (centimeter, gram, and second) and to all other units of the C. G. S. system, without the use of reduction factors such as would be required for units defined by independent arbitrary standards.

On the other hand, the C. G. S. units have some disadvantages. For one thing, their sizes are not always convenient for expressing the magnitudes of the quantities most commonly met with. For another thing, the experimental work of calibrating instruments in terms of C.G.S. units cannot always be carried out as accurately or easily as in terms of units defined in some other way. Again, it is desirable to have a unit for any kind of quantity defined in terms of some physical standard simply related to the unit, which can be constructed once for

all (or else easily reproduced from specifications); and this is not practicable for the C. G. S. electromagnetic units.

For these and other reasons a different set of units, called the practical system of units, has been agreed upon. These units have been defined by various international electrical congresses, and the definitions so reached have been enacted into law in this country and in most other countries.

In defining the practical units the aim has been to make each as nearly as possible equal to the corresponding C. G. S. electromagnetic unit multiplied by some power of ten. This is accomplished accurately enough for all practical purposes by the definitions agreed on; but when these definitions are based on different principles from those for the C. G. S. units, the practical units are not *identical* with the corresponding C. G. S. units multiplied by the proper powers of ten. For most purposes the differences can be neglected, but in precise scientific work they may be appreciable.

The *ampere* is defined as one tenth of the C. G. S. electromagnetic unit; but it is also defined as "the practical equivalent of the unvarying current which when passed through a solution of silver nitrate in water in accordance with standard specifications deposits silver at the rate of 0.001118 gram per second." This latter definition serves for all practical purposes, although the former definition is the fundamental one.

The practical unit of quantity of electricity is the *coulomb*, and is defined as the quantity transferred by one ampere in one second.

All the other practical units require for their definitions the ohm (or units defined in terms of the ohm), which is defined in terms of a physical standard.

13. Electrical resistance and the ohm. When a current flows through any conductor, heat appears in the conductor at a rate proportional to the square of the current. The heat represents energy given to the conductor, and the rate at which the energy is being given, or power, is proportional to the square of the current, or

$$P = RI^2, \tag{10}$$

where P stands for the power, I for the current, and R is a proportionality factor depending on the material and dimensions of the conductor. This factor is the *resistance* of the conductor,

which is accordingly defined as the energy converted into heat in the conductor per second per unit of current squared. The C. G. S. unit of resistance is the resistance of a conductor such that heat is produced in it at a rate equivalent to one erg per second when traversed by a current of one abampere. This unit is called the *abohm*.

The *ohm* is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of the length of 106.3 centimeters. It is substantially equal to 10^9 abohms.

If, in equation (10), R is expressed in ohms and I in amperes, P will be in watts. The rate of production of heat, in calories per second, can be obtained by dividing by the mechanical equivalent of heat, which gives

$$\text{Calories per second} = 0.24\, RI^2. \tag{11}$$

14. Potential; electromotive force; the volt. When a current is flowing through some conductor, the power required to maintain it may also be written

$$P = VI, \tag{12}$$

where P is the power, I the current, and V a factor of proportionality which evidently represents the quantity of power per unit of current. The factor V is the *potential difference* between the ends of the conductor, potential difference between any two points being defined in general as the work done per unit quantity of electricity transferred from one of the points to the other.

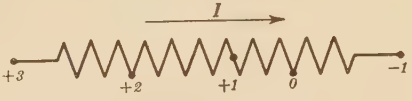


FIG. 6. Distribution of potential along a current-carrying conductor

Evidently there is a potential difference not only between the ends of the conductor but also between other points along it. If some one point O on the conductor is chosen as a reference point or zero point for potential, the potential difference between any point A and the point O is called the *potential* of the point A. The potential is counted as decreasing in the direction in which the current flows. Its arrangement along a conductor is illustrated by Fig. 6.

Since $P = RI^2$, it follows that $VI = RI^2$; whence

$$V = RI. \tag{13}$$

This is Ohm's law for the case of a conductor not containing any electromotive force.

In much of the previous discussion it has been found convenient to fix attention on the current in some limited conductor, although in reality every (unvarying) current flows round a closed circuit, so that the current does not require that electricity be supplied continuously to the circuit, but merely that the same electricity be kept continuously in motion around the circuit. But this requires that energy be supplied continuously to the circuit. The necessary energy may be supplied by a battery or a generator connected in the circuit; then if the power (or rate of supply of energy) delivered by the battery or generator is P when the current is I, we may write

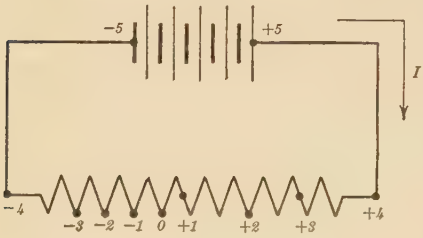


FIG. 7. Distribution of potential in a circuit containing an electromotive force

$$P = EI, \tag{14}$$

where E is a factor of proportionality which evidently represents the power supplied per unit of current maintained. E is called the *electromotive force* of the battery or generator. Although its name suggests some kind of force, it is not really of the nature of a force acting upon the electricity in the circuit, but rather of the nature of *work per unit quantity* of electricity passing through the battery or generator. The work per unit quantity is the same as the difference of potential; so the electromotive force of a battery or generator may be thought of as a difference of potential which the battery or generator maintains between its own terminals, and by reason of which it is able to supply the energy for maintaining a current in the circuit to which it is connected. The electromotive force of the battery or generator which maintains a current corresponds to an *increase* of potential in the direction in which the current flows, as shown in Fig. 7.

Since the potential at any given point has but one value, the electromotive force E (or increase of potential in passing through the battery or generator) must equal the decrease of potential RI throughout the circuit within which the battery or generator is connected, or,

$$E = RI. \tag{15}$$

This is Ohm's law for the case of a complete circuit.

If the circuit contains more than one electromotive force, then E in (15) must be taken as the algebraic sum of all the electromotive forces in the circuit, and R must be taken to include the internal resistance of all batteries or generators. Those electromotive forces that act in opposition to the flow of current absorb power, of course, from the current; those that act in the same direction with the current supply the power necessary to maintain the current.

The practical unit of electromotive force and of potential, the *volt*, is defined as the electromotive force which, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere. This unit is substantially equal to 10^8 times the C. G. S. electromagnetic unit of electromotive force, to be defined later.

When the ampere and the volt are used for units in equations (12) and (14), the power comes out in *watts*.

The term *voltage* is often used for electromotive force or for potential difference.

15. Magnetic field at the center of a circular coil. Consider a current flowing in a circuit having the form of a circle of radius r. Imagine a unit magnetic pole at the center of the circle. The field due to this pole (in air) will have a strength of $\frac{1}{r^2}$ gausses at all points of the circle carrying the current, and will be perpendicular to the current. Accordingly the conductor carrying the current will receive a force $F = lBI = 2\pi r \cdot \frac{1}{r^2} \cdot I$, or $\frac{2\pi I}{r}$, in a direction at right angles to the plane of the circle.* This is the force exerted by the unit pole upon the conductor, and must

* If the medium between conductor and pole is different from air, the same result is obtained. In this case the field strength due to the pole is $\frac{1}{\mu r^2}$; but $B = \mu H$, so that $B = \frac{1}{r^2}$, just as above.

be equal and opposite to the force exerted by the conductor upon the pole. Accordingly the pole receives a force of $\frac{2\pi I}{r}$ dynes. Therefore the force per unit pole, or field strength, at the center of the circle, is

$$H = \frac{2\pi I}{r}. \tag{16}$$

If the circuit consists of a coil of N turns wound in the form of a circle of radius r, then it is evident that the field strength at the center is

$$H = \frac{2\pi NI}{r}. \tag{17}$$

(Of course it is not possible to wind a coil of many turns so that all of them will coincide exactly with a single circle, and accordingly this result becomes inexact if the cross section of the coil becomes appreciable in comparison with its radius.)

16. Magnetic field within a long solenoid. A coil wound helically about a long cylindrical or tubular surface is called a *solenoid.* If the solenoid is very long in comparison with its diameter, the magnetic field due to a current flowing in its winding is uniform and parallel to the axis of the solenoid at all points within the solenoid and distant from its ends; and the intensity of the field in gausses is

$$H = 4\pi nI, \tag{18}$$

where n stands for the number of turns per centimeter of length of the solenoid, and I is the current in abamperes.

Proof for a point on the axis of a solenoid wound uniformly on a circular cylinder. Consider a point P on the axis of the solenoid, and let r be the radius of the solenoid. Consider one turn of the winding at a distance x from P. Strictly, this turn is a portion of a helix; but as a close approximation it may be treated as a circle. If a unit pole is placed at P, it produces a field of strength $\frac{1}{r^2 + x^2}$ at all points of the circle whose center is at the distance x from P. The component of this field perpendicular to the axis of the solenoid is $\frac{1}{r^2 + x^2} \cdot \frac{r}{\sqrt{r^2 + x^2}}$; and accordingly the force parallel to the axis on one turn of the solenoid is $F = lBI = 2\pi r \cdot \frac{r}{(r^2 + x^2)^{\frac{3}{2}}} \cdot I$. (The component of field parallel to the axis produces forces perpendicular to the axis and differing in direction at different parts of the circle,

and canceling out for the whole circle.) By the law of action and reaction the force on the unit pole is equal but opposite to the force just derived, and accordingly the contribution to the field due to the turn at the distance x from P is given by

$$dH = \frac{2\pi r^2 I}{(r^2 + x^2)^{\frac{3}{2}}}.$$

If n stands for the number of turns per unit length, the contribution from all the turns within the length dx will be

$$dH = \frac{2\pi r^2 I}{(r^2 + x^2)^{\frac{3}{2}}} \cdot n \cdot dx.$$

If this expression is integrated between the limits $-\infty$ and $+\infty$, the expression (18) is obtained.

Equation (18) is independent of the medium within or about the solenoid, as can be deduced by reasoning similar to that of the footnote to section 15.

17. Electric effects of moving magnetic fields. When a magnet is in motion, of course the magnetic field belonging to it moves with it. Similarly, when a current-carrying conductor moves, its magnetic field moves with it. When a current is started in a previously inactive circuit, a magnetic field is brought into existence in the region about it, and this is equivalent to moving a magnetic field into that region.

Faraday discovered in 1831 that when a magnetic field is in motion an electromotive force is produced along any path across which the magnetic field cuts. If the path lies in a conductor that forms part of a closed circuit, then the electromotive force causes a current to flow. An electromotive force is produced also in a conductor that moves across a stationary magnetic field, the effect being determined entirely by the relative motion between the field and the conducting path, regardless of any "absolute" motion that either may possess.

The intensity of the electromotive force along any path is proportional to the rate at which magnetic flux is being cut by that path. This principle can be used to define a unit of electromotive force. Accordingly the C. G. S. unit of electromotive force is defined as that electromotive force which is produced in a conductor by cutting magnetic flux at the rate of one maxwell

per second. The name *abvolt* has been given to this unit. The volt is substantially equal to 10^8 abvolts.*

If a straight conductor is at right angles to a uniform magnetic field and moves in a direction perpendicular to itself and to the field, then the flux that it cuts per second equals the flux density B multiplied by the area A which it sweeps over per second. But $A = lv$, where l is the length of the conductor, and v is its velocity. Accordingly the rate of cutting of the magnetic flux is $B \cdot lv$, and we have

$$E = lBv, \tag{19}$$

where E is the electromotive force, which will be in abvolts if l is in centimeters, B in gausses, and v in centimeters per second.

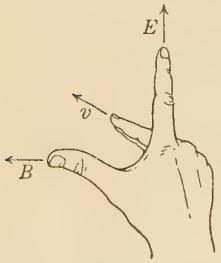


FIG. 8. Relative directions of E, B, and v

The directions of B, E, and v are related to one another in the same way as the outward directions of the thumb and first two fingers of the right hand, as shown in Fig. 8. It may help to memorize this rule to note that when the thumb and fingers are taken in order, the corresponding quantities B, E, and v come in alphabetical order.

If the conductor, field, and velocity are not mutually perpendicular, then the electromotive force will be equal to lB_cv_c, where v_c means the component of velocity at right angles to the length of the conductor, and B_c the component of the flux density at right angles both to the conductor and to the direction of its velocity.

Equation (19) is the fundamental equation of electric generators, since these act on the principle of producing electromotive force by moving conductors across magnetic fields.

When a conductor forms a closed circuit, the different parts of the circuit may cut magnetic flux at different rates or even in different directions; but the electromotive force round the whole circuit will be determined by the rate of cutting of magnetic flux by the whole circuit, or the rate at which flux passes from the outside of the circuit to the inside. This is the same

* This, of course, is not intended for a definition of the volt but merely as a statement of fact. The definition of the volt was given in section 14.

as the rate of change of the total flux linking with the circuit. Accordingly, for a closed circuit,

$$E = -\frac{d\Phi}{dt}, \tag{20}$$

where E is the electromotive force round the circuit, in abvolts; Φ is the magnetic flux linking with the circuit, in maxwells; and $\frac{d\Phi}{dt}$ means the rate of change of Φ with respect to the time (expressed in the usual notation of calculus), in maxwells per second. A minus sign is used to indicate that the positive direction of the electromotive force is related to the positive direction of the rate of change of the flux, in the same way as the forward direction along the screw thread is related to the forward direction along the axis in a *left*-hand screw, as shown by Fig. 9.

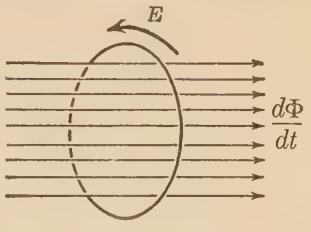


FIG. 9. Directions of a change of flux and the electromotive force that accompanies it

Equation (20) is more convenient to use for some applications than equation (19); but both are statements of the same physical principle for different particular conditions.

If the circuit has the form of a coil of N turns, all linking with the same magnetic flux, then the electromotive force $-\frac{d\Phi}{dt}$ will be induced in each turn, and the electromotive force for the whole coil will be

$$E = -N \cdot \frac{d\Phi}{dt}. \tag{21}$$

Since an electric current is always accompanied by a magnetic field that links with the path of the current, it follows that when the current starts or stops or changes in value its magnetic field must start or stop or change in value. This is equivalent to the motion of a magnetic field in the region surrounding the current, as previously stated. The process takes place in the following manner:

The lines of force of the magnetic field are closed curves that link with the path of the current. When the current starts or increases in value, these lines spread outward, and additional lines are emitted from the conductor; the lines thus behave somewhat like the waves that spread over a surface of water after a stone has been dropped in, but with the difference that the lines reach fixed stationary positions

as soon as the current stops increasing,* whereas the waves keep on moving. When the current decreases in value, the process takes place in a reversed direction: the lines contract inward, and those nearest the path of the current collapse completely.

Many of the most important cases in which electromotive forces are produced by changing magnetic fields are those in which the magnetic fields are due to electric currents, and the changes in the fields are effected by changes in the strength of the currents, without motion of any material bodies. This is the case, for example, in transformers.

18. Self-induction. If the current in any circuit changes, the corresponding change in the magnetic field that links with the current must cause an electromotive force to act in the circuit during the time that the current is changing. This electromotive force is called the *electromotive force of self-induction*; its amount is $-N \cdot \frac{d\Phi}{dt}$, where Φ stands for the total magnetic flux that links with the circuit, and N for the number of turns (or number of times the circuit links with the flux); and its direction is such as to *oppose* the change of current.

That is, if the current is increasing, the electromotive force of self-induction acts in the opposite direction to the current, and opposes its growth. It cannot *stop* the growth of the current, for if the current should stop changing, Φ would stop changing too, and so the electromotive force of self-induction would also stop; but it slows down the growth of the current, and requires that some time be taken for the current to increase.

If the current is decreasing, the electromotive force of self-induction acts in the *same* direction as the current, and thus tends to maintain it. It cannot permanently maintain the current at any fixed value, because in this case the current would stop changing, and then the electromotive force of self-induction would cease; but it slows down the decrease of the current, and requires that some time be taken for the current to decrease.

On account of self-induction a current cannot start or stop instantaneously. However, the time required for a current to start, stop, or change in value may be exceedingly short in many cases.

19. Inductance. The quantity of magnetic flux linking with a circuit that is produced by a current in the circuit depends on the strength of the current, the shape of the circuit, and the

* Except a few lines, negligible in ordinary circuits, which spread outward indefinitely as electromagnetic waves or radiation.

magnetic properties of the materials in the neighborhood. Assuming that the shape of the circuit and the position and properties of the bodies in the neighborhood are unaffected by the strength of the current, Φ will be proportional to the current, and we may write

$$N\Phi = L \cdot I, \tag{22}$$

where L is a factor of proportionality, and N stands for the number of times that the circuit links with the magnetic flux. The product $N\Phi$ is called the number of "flux-linkages," and L is accordingly the number of flux-linkages per ampere. L is called the *self-inductance* (or simply *inductance*) of the circuit.

Since the electromotive force of self-induction equals N times the rate of change of Φ, equation (22) shows that it also equals L times the rate of change of I, or

$$E = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}, \tag{23}$$

where the minus sign before the last term means that the electromotive force is in the direction opposite to that of the rate of change of the current.

In order to make the current in a circuit increase, it is necessary to impart to it (temporarily) an electromotive force sufficient to overcome the electromotive force of self-induction, over and above whatever electromotive force is required to overcome resistance; that is, there must be imparted to the circuit (temporarily) an electromotive force

$$E = L\frac{dI}{dt} \tag{24}$$

in the same direction as the rate of change of the current, over and above whatever electromotive force is required to overcome resistance.

20. Unit of inductance. The C. G. S. electromagnetic unit of inductance is the inductance of a circuit such that an electromotive force of one abvolt is produced in the circuit by a rate of change of current of one abampere per second. This unit is called the *abhenry*.

The practical unit of inductance is called the *henry*, and is equal to the inductance of a circuit such that an electromotive force of one volt is produced in the circuit by a rate of change of current of one ampere per second.

On account of the importance of the quantity self-inductance,

it is desirable to form some idea of its amount for various types of circuits. In general it is small for circuits of small dimensions and few turns, and not linked with iron, and is large for circuits having large dimensions or many turns, or linked with iron. Thus the self-inductance of a single ordinary incandescent lamp is of the order of 10^{-7} henry; the inductance of a mile of transmission line consisting of two conductors eighteen inches apart is about 0.003 henry; the inductance of an ordinary (4-ohm) Morse sounder is about 0.04 henry; and the inductance of the shunt field winding of the 2-kilowatt generator referred to in Problem 4, Chapter II, is about 10 henries.

21. Analogy between self-induction in electricity and inertia in mechanics. In many ways the property of self-induction in electricity is similar to inertia in mechanics, and the quantity inductance is similar to the quantity mass. This may be brought out by the following parallel columns:

Every body remains at rest, or in motion with constant velocity, unless acted upon by some impressed force. (Frictional resistance must be absent, or else counted as an opposing force.)

The motion of a body cannot start, stop, or change instantaneously: some time is always required for a change of velocity to take place.

When a force is impressed on a body, the velocity of the body undergoes a change, such that the rate of change of velocity times the mass of the body equals the force, or, in symbols,

$$F = m \cdot \frac{dv}{dt}.$$

(If frictional resistance is present, it should be counted as an opposing force, to be subtracted from F.)

Every circuit remains without current, or with current of constant intensity, unless acted upon by some impressed electromotive force. (Electrical resistance must be absent, or else its effect RI counted as an opposing electromotive force.)

The current in a circuit cannot start, stop, or change instantaneously: some time is always required for a change of current to take place.

When an electromotive force is impressed on a circuit, the current in the circuit undergoes a change, such that the rate of change of current times the inductance of the circuit equals the electromotive force, or, in symbols,

$$E = L \cdot \frac{dI}{dt}.$$

(If electrical resistance is present, its effect RI should be counted as an opposing electromotive force, to be subtracted from E.)

But the analogy does not hold for one important property of mass: the mass of a body is regarded as a permanent property of the body, characteristic of the quantity of matter in it; whereas the inductance of a circuit varies with the shape of the circuit and the properties of the region surrounding it, and has nothing to do with the quantity of electricity in it.

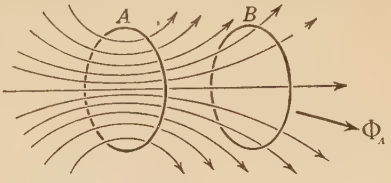


FIG. 10. Magnetic flux for mutual induction

22. Mutual induction. If the magnetic flux due to the current in a circuit A passes in part through a distinct circuit B, as shown by Fig. 10, then any change in the current in A will cause an electromotive force to appear in B, on account of the change in flux through B. Such an electromotive force is called an electromotive force of *mutual induction*, and is evidently proportional to the rate of change of current in A; so that we can write

$$E_B = -M \cdot \frac{dI_A}{dt}, \tag{25}$$

where E_B is the electromotive force in the circuit B due to the change of current in circuit A at the rate of $\frac{dI_A}{dt}$, and M is a factor of proportionality and is called the *mutual inductance*. Evidently mutual inductance is a quantity of the same nature as self-inductance; therefore it is expressed in terms of the same unit, the henry.

The value of the mutual inductance between two circuits is determined by their shapes, dimensions, and relative position, and by the magnetic permeability of the materials in their neighborhood. It is found that the mutual inductance M has the same value when the electromotive force is induced in circuit A by a changing current in circuit B, as when the electromotive force is induced in circuit B by a changing current in circuit A.

23. Electromagnets. Attention was called in section 8 to the fact that a piece of previously unmagnetized iron acts as a magnet when it is placed in a magnetic field such as may be due to a permanent magnet or to electric currents. When the field is due to an electric current a piece of iron magnetized by the field is called an electromagnet. When the iron is removed from the field, it usually loses its magnetism more or less completely; while it is in the field, its magnetism varies with the strength of the field.

Since the strength of the field depends on the strength of the current producing it, it is possible to control the strength of an electromagnet by varying the strength of current, a matter that can easily be accomplished. For this reason among others, electromagnets offer great advantages over permanent magnets for most purposes, and are of fundamental importance in the applications of electricity.

Evidently the most effective way to dispose an electric circuit in order to magnetize a bar of iron would be to place it so that a given current would produce the strongest possible field at the position to be occupied by the iron. This would be accomplished by winding the circuit many times around the bar of iron; that is, the circuit should have the form of a solenoid.

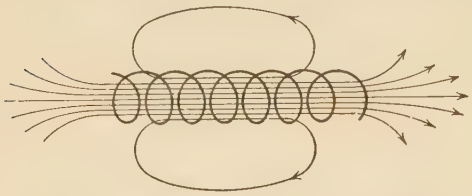


FIG. 11. Magnetic field due to current in a solenoid

From the statements in section 16 it can be seen that a long solenoid produces in air a magnetic field of about the same shape as that due to a bar magnet. When the solenoid is free from iron, the field intensity due to a current of I abamperes in it is $4\pi nI$ gausses at all points within the solenoid, except near the ends, where some of the lines of force stray outward, as shown in Fig. 11. If the area of cross section of the solenoid is A, the total magnetic flux through the solenoid is $4\pi nI \cdot A$ maxwells. Since 4π maxwells correspond to one unit pole, the solenoid is equivalent to a bar magnet having a pole of nIA units at each end.

If, now, a bar of previously unmagnetized iron is inserted into the solenoid, it becomes magnetized, with poles at each end; but the strength of these poles is found to be very much greater than nIA units. This amounts to the same thing as saying that the magnetic flux is very greatly increased by putting iron into a portion of the field previously occupied by air.

According to section 16 the solenoid produces the same field intensity H within it, whatever material may be there. But the flux density B $(= \mu H)$ will be more or less according to the permeability of the material within the solenoid. The great

increase in magnetic flux due to putting iron in place of air is a consequence of the large permeability of iron.

24. Magnetic properties of iron. It is evidently a matter of importance to know how much magnetic flux may be expected to pass through a given piece of iron when placed in a field of known intensity H. It might be thought that a knowledge of a single quantity, the permeability of the iron, would be sufficient for this purpose; but unfortunately the permeability is not constant, but varies with the degree of magnetization of the iron. Accordingly the magnetic properties of a sample of iron cannot be stated in terms of a single number, but require for their statement a set of data. Such data should show the flux density B through the sample for different values of the field intensity H; the data may be presented very conveniently by a curve showing B as a function of H, as is done in Fig. 12. Such a curve is called the magnetization curve for the sample of iron in question.

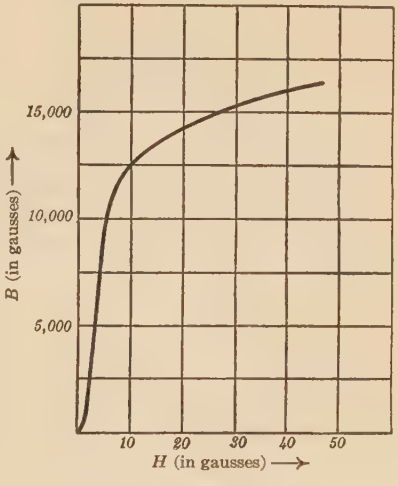


FIG. 12. Magnetization curve for a certain sample of iron

To obtain the data for such a curve, the iron is placed inside of a solenoid, in which the value of H can be computed from the current in the solenoid and the number of turns per unit length. In one method the sample of iron has the form of a closed ring, with the cross section of the iron rather small in comparison with the length of the ring. A winding of many turns of wire is placed about the iron, as shown in Fig. 13, thus forming a solenoid with a closed curve for its axis. The field intensity H within the solenoid can be computed from the strength of current I in the winding, and the number of turns per unit length n, as $4\pi nI$.

To obtain the flux density B through the iron, a test coil C is wound over the ring, as shown in Fig. 13, and is connected to a galvanometer G. The galvanometer is of the type called a ballistic galvanometer, having a rather slow period of swing and not much damping. When the current is started in the solenoid winding, the

change of flux produced in the iron ring causes a momentary electromotive force in the coil C. But the flux Φ is very quickly built up to its final value and then ceases to cause electromotive force any longer; in fact, the flux reaches its final value before the galvanometer has had time to deflect appreciably. But during the short time that the flux is growing, the electromotive force due to it sends a momentary current through the galvanometer and gives the moving element of the galvanometer an impulse, or "kick." The moving element, or needle, is accordingly set in motion, and swings on by means of its inertia after the momentary current has stopped. By reasoning that will not be given here it can be shown that the amplitude of the swing, or "throw," of the galvanometer is determined by the total change in the flux through the coil C. Accordingly the flux can be found by observing the throw of the galvanometer; and the flux density B is equal to the total flux divided by the cross section of the iron.

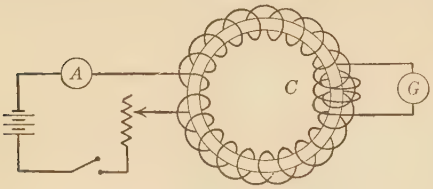


FIG. 13. Connections for determination of magnetization curve by the ring method

The magnetization curves of different samples of iron or steel vary widely in form, according to their constituents and the treatment they have received. It is general, however, for the curve to become less steep with increasing magnetization, and eventually to approximate a straight line of such slope that any increase in H corresponds to an equal increase in B. In other words, for this part of the curve the iron does not act any differently from air as regards increase in its magnetization. When this condition is reached or approximated, the iron is said to be *saturated*. In soft iron, saturation is usually reached rather abruptly, corresponding to a sharp bend in the magnetization curve.

25. Magnetic hysteresis. If a sample of iron, originally unmagnetized, is magnetized to a certain degree, and then the magnetizing field H is decreased back to zero, it is found that the flux density B does not return to zero, but the iron remains somewhat magnetized. The magnetization so retained is called *residual magnetism*. If the magnetization curve is taken for decreasing values of H, as well as for increasing values, a result like that in Fig. 14 is obtained, where the ordinate OP represents the residual flux density.

In order to bring the flux density back to zero, a field of suitable intensity H has to be applied in the opposite direction; this field corresponds to the abscissa OQ in Fig. 15. If H is further increased in the reversed direction, a magnetization

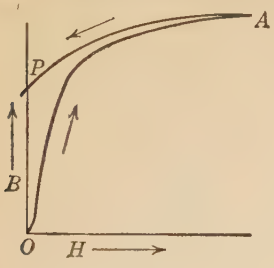


FIG. 14. Curves showing magnetic hysteresis

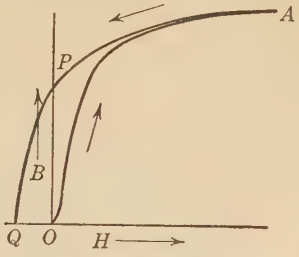


FIG. 15. Curves showing magnetic hysteresis

curve like QC in Fig. 16 is obtained, values of H and B in the reversed direction being counted negative. If H is then brought back to zero, and then increased in its original direction, a magnetization curve like CRA in Fig. 16 will be obtained. If thereafter H is caused to alternate forward and backward between the values corresponding to A and C in Fig. 16, the curves AQC and CRA will be traced over and over. (If H should alternate between different limits, the curves would evidently be different.)

The behavior just described, according to which the magnetization curve for decreasing H differs from that for increasing H, is called *magnetic hysteresis*. The complete curve of Fig. 16 is called a *hysteresis loop*.

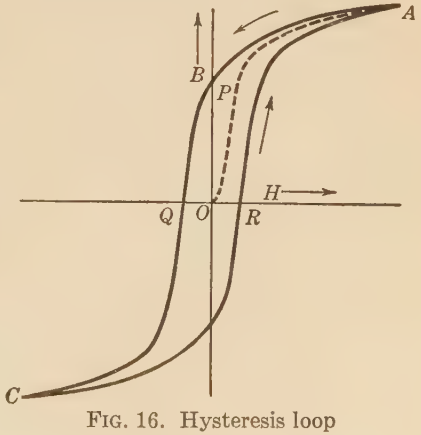


FIG. 16. Hysteresis loop

Hysteresis varies greatly with different kinds of iron or steel and with the limits between which H alternates. In general, soft irons, with moderate limits for the range of H, do not show much hysteresis: the magnetization curve for decreasing H nearly coincides with that for increasing H, and the hysteresis

loop is narrow. Hard irons, on the contrary, show a great deal of hysteresis: the magnetization curve for decreasing H differs widely from that for increasing H, and the hysteresis loop is wide.

Hysteresis occurs in the iron or steel cores of transformers and dynamo armatures, in which the magnetism keeps alternating in direction. It is very important in these and other practical applications, for the reason that it represents a waste of energy.

This may be seen from the following considerations: When a piece of iron is magnetized by passing a current through a coil of wire wound round the iron, the growth of the magnetic flux in the iron induces in the coil a temporary electromotive force in a direction opposing the flow of current. Accordingly the current has to be maintained against an opposing electromotive force during the growth of the magnetic flux in the iron; and this means that energy has to be supplied to maintain the current while the magnetic flux is being built up (over and above any energy required for maintaining the current against electrical resistance). This energy is stored in the magnetic field, mostly within the iron. Now if the current is allowed to decrease, the magnetic flux through the iron also decreases, and the decreasing of the flux induces in the coil a temporary electromotive force in the *same* direction as the current. This means that energy is being given back to the current from the magnetic field (mostly within the iron). But if the flux in the iron fails to decrease to the value from which it started, the energy returned to the current will be less than that which had been supplied by it. Some then remains in the iron. To demagnetize the iron in this case requires current in reversed direction, and this demands more energy *from* the current. Accordingly, every time the iron is carried through a cycle of magnetization, more energy is put into the iron than is returned by it. This energy takes the form of heat in the iron, and is completely wasted so far as any useful effect is concerned.

26. The magnetic circuit. In every magnetic field the lines of force (and the lines of flux density likewise) form closed curves. All the lines that pass through the boundary of a surface transverse to the field form a tube which closes upon itself, and the flux through any cross section of such a tube is the same as that through any other cross section of the tube. Such a tube may be looked upon as a *magnetic circuit*. The tube may be made wide enough to include all the lines of the magnetic field under consideration, and the complete field may then be looked upon as a magnetic circuit.

In studying electric currents it is sometimes useful to fix attention on some limited part of the path of the current. But a steady current always flows in a closed path or circuit, and has the same value throughout the circuit; and it is often useful to consider the current from the point of view of the whole circuit. Thus the current in the circuit can be found by Ohm's law from a knowledge of the total electromotive force and the total resistance of the circuit.

In the discussion of magnetism thus far, attention has been fixed upon conditions at particular parts of the magnetic field. But the field as a whole forms a magnetic circuit; and it is often useful to consider the field from the point of view of the whole circuit. Thus the flux in the circuit can be found from a law whose form is similar to Ohm's law, as will be shown in the next paragraphs.

27. Law of the magnetic circuit. Consider first the case in which the magnetic field is uniform over some cross section of the magnetic circuit. Then the total flux $\Phi = A \cdot B = A \cdot \mu H$, where A stands for the area of the cross section. Suppose, now, that the quantities A, μ, and H remain the same for all other parts of the magnetic circuit, as would be the case with the ring described in section 24. Then, if l is the length of the circuit, we may write for the flux

$$\Phi = A\mu H = Hl \cdot \frac{\mu A}{l} = \frac{Hl}{l/\mu A}. \tag{26}$$

This expression has the same form as Ohm's law — the current, the electromotive force, and the resistance in Ohm's law being replaced by the quantities Φ, Hl, and $\frac{l}{\mu A}$ respectively. The quantity Hl is called the magnetomotive force, and the quantity $\frac{l}{\mu A}$ the reluctance of the circuit, so that we may write equation (26) in the form

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}, \tag{27}$$

where Φ stands for the magnetic flux, $\mathcal{F}$ for the magnetomotive force, and $\mathcal{R}$ for the reluctance.

The expression (27) has been derived for the special case in which H, μ, and A were constant throughout the circuit; but

it still holds true, even when none of these quantities is constant, if proper values are assigned to the magnetomotive force and to the reluctance.

If the magnetic circuit consists of several sections, such that H, μ, and A are constant within any one section but differ from section to section, then we should have

$$\Phi_1 = \frac{H_1 l_1}{l_1/\mu_1 A_1}, \quad \Phi_2 = \frac{H_2 l_2}{l_2/\mu_2 A_2}, \text{ etc.} \tag{28}$$

But Φ is the same for every cross section of the circuit; hence it follows from (28) that

$$\Phi = \frac{H_1 l_1 + H_2 l_2 + \text{etc.}}{l_1/\mu_1 A_1 + l_2/\mu_2 A_2 + \text{etc.}}. \tag{29}$$

In this case, then, the magnetomotive force has the value of the numerator of the fraction in (29), and the reluctance has the value of the denominator.

28. Magnetomotive force. By extending the reasoning of the last paragraph to the case in which H is not constant for any part of the circuit, we could consider the circuit divided into an indefinitely large number of sections, each of length dl, and the magnetomotive force would then equal the sum of such terms as $H \cdot dl$; or, if we use the notation of calculus,

$$\mathcal{F} = \int H \cdot dl, \tag{30}$$

the integral being taken round the complete circuit. This amounts to the same thing as saying that $\mathcal{F}$ is equal to the length of the circuit multiplied by the average value of H round the circuit.

Since H equals the force per unit pole, $H \cdot dl$ must equal the work done in moving a unit pole over the distance dl in the direction of the field; and $\int H \cdot dl$ taken round the magnetic circuit (which equals $\mathcal{F}$) must equal the work done in moving a unit pole completely round the magnetic circuit. Accordingly we have, as a general statement, *the magnetomotive force around any magnetic circuit is equal to the work done in moving a unit pole completely round the magnetic circuit.*

If, in determining the magnetomotive force by the foregoing

relation, H and l are expressed in C. G. S. units (that is, in gausses and centimeters respectively), then $\mathfrak{F}$ will be in C. G. S. units or in ergs per unit pole. The C. G. S. unit for magnetomotive force has received the name of the *gilbert*.

If the field is due to a current of I abamperes flowing in a coil of N turns, it can be shown that

$$\mathfrak{F} = 4\pi NI, \tag{31}$$

where F stands for the magnetomotive force, in gilberts, round any path that links with all the turns of the coil.

Proof. Imagine that a unit magnetic pole starts from any point in the field and is moved along a line of force until it makes the complete circuit of the line, coming back to its starting point. Since every line of force in the field is linked with the coil, the path followed by the unit pole passes through the coil, as shown in Fig. 17. Consider now the magnetic flux emanating from the unit pole. It emanates radially from the pole and is cut by each turn of the electric circuit while the pole is being moved round the path that links with the electric circuit. An electromotive force is accordingly induced in the electric circuit, equal at each instance to the rate of the cutting of the flux from the unit pole, or $N \cdot \frac{d\Phi}{dt}$. Assuming that the direction of motion of the pole is such that this electromotive force has the same direction as the current I, power would be imparted to the electric circuit equal to $N \cdot \frac{d\Phi}{dt} \cdot I$. The work done by this power in any time-interval dt equals the power multiplied by the time, or $N \cdot \frac{d\Phi}{dt} \cdot I \cdot dt$, or $NI \cdot d\Phi$. That is, the work done during any time-interval equals NI times the flux cut by the circuit, regardless of the time required for cutting the flux. But when the unit pole is carried completely round a path linking with the electric circuit, all the flux emanating from the pole will be cut by each turn of the electric circuit. This amounts to 4π maxwells, and the work is accordingly $NI \cdot 4\pi$ ergs. This is the work imparted to the electric circuit by reason of the motion of the unit pole, and accordingly must equal the work done upon the unit pole. But the work done upon the unit pole equals the magnetomotive force; so $\mathfrak{F} = 4\pi NI$, which was to be proved.

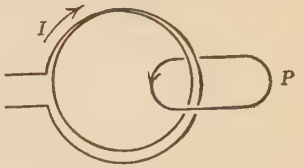


FIG. 17. P is the path of the unit pole

If I is expressed in amperes, the product NI is called the number of *ampere-turns*, and is often used as a unit of magnetomotive force. Then

$$\mathcal{F} \text{ (in gilberts)} = \frac{4\pi}{10} \text{ times the ampere-turns}$$
$$= \text{(approximately) } 1.25 \text{ times the ampere-turns.} \quad (32)$$

The relation (31) or (32) holds for any path that links with all the turns of the electric circuit. Accordingly the magnetomotive force due to a current-carrying coil, round a path that links with the coil, is independent of the position of the coil with respect to the path and of the manner in which the different turns of the coil are placed (whether a long solenoid or a short one, a set of separate coils in series, and so on), provided only that all the turns link with the path in question.

In practical applications the use of current-carrying coils, or "field windings," offers by far the most important source of magnetomotive force. The only other readily available method for obtaining a magnetomotive force is the comparatively unimportant one of using a permanent magnet, which acts as a source of magnetomotive force.

29. Reluctance. The reluctance of a magnetic circuit equals the ratio of the magnetomotive force acting round the circuit to the magnetic flux in the circuit. The C. G. S. unit for reluctance is accordingly the reluctance of a circuit such that one gilbert of magnetomotive force will maintain one maxwell of flux in the circuit. This unit has been named the *oersted*.

According to equations (26)–(29) the reluctance of a magnetic circuit equals the sum of the reluctances of the various parts of the circuit that are in series with one another; and the reluctance of any part is proportional to its length l and inversely proportional to the area A of its cross section, and is given by

$$\mathcal{R} = \frac{1}{\mu}\frac{l}{A}, \quad (33)$$

where $\mathcal{R}$ is the reluctance in oersteds, μ the permeability of the material, l the length in centimeters, and A the area of cross section in square centimeters.

Since μ for air is unity, the reluctance of any portion of a magnetic circuit that passes through air is $\frac{l}{A}$.

30. Units based on the inch. Since the inch is the unit of length most commonly used in machine shops in this country, American electrical manufacturers often use units based on the inch, rather than the international or C.G.S. units. Thus the flux density B is often expressed in maxwells (or lines) per square inch, and the field intensity H in a magnetic circuit is often expressed in ampere-turns per inch.

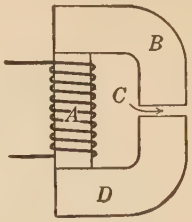


FIG. 18. Example of a magnetic circuit

31. Example of a magnetic-circuit computation. To illustrate the use of the law of the magnetic circuit, let it be required to compute the magnetomotive force necessary to produce 500,000 maxwells of flux in the circuit shown in Fig. 18, the following data being given:

PORTION OF CIRCUIT	MATERIAL	LENGTH IN CENTIMETERS	CROSS SECTION IN SQUARE CENTIMETERS
A	iron	20	40
B or D	iron	25	80
C	air	1	80

The iron is of the same kind as that for which the magnetization curve of Fig. 12 is drawn, from which μ is found to be 1136 for portion A, where the flux density is 500,000/40, or 12,500, gausses; and μ is 1642 for portion B, where the flux density is 500,000/80, or 6250, gausses. The reluctance of each portion of the circuit is then as follows:

$$A \qquad \frac{20}{40 \times 1136} = 0.00044 \text{ oersted},$$

$$B \text{ or } D \qquad \frac{25}{80 \times 1642} = 0.00019 \text{ oersted},$$

$$C \qquad \frac{1}{80} = 0.01250 \text{ oersted},$$

and the reluctance of the whole circuit is $0.00044 + 2 \times 0.00019 + 0.01250$, or 0.01332, oersted. The magnetomotive force to be found equals $\mathcal{R}\Phi$, or $0.01332 \times 500{,}000$, or 6660, gilberts, and this equals 1.25 times the number of ampere-turns. Then the number of ampere-turns required will be 5340; so that if the

coil about A has 1000 turns, a current of 5.34 amperes through it will be required to produce the 500,000 maxwells of flux.

It is noteworthy that the reluctance of the air gap is nearly twenty times that of all the rest of the magnetic circuit.

32. Currents in open circuits; condensers. When electric currents have been spoken of in the preceding discussions, attention has nearly always been fixed upon unvarying or steady currents.

A steady current must necessarily flow in a closed circuit and have the same value at all parts of the circuit. For if the steady current entering a portion P of the circuit from one side should exceed the steady current leaving the portion P on the other side, there would occur an accumulation of (positive) electricity in the portion P, increasing without limit as long as the steady currents continued. On the other hand, if the steady current leaving P should exceed that entering P, there would be a deficiency of (positive) electricity (which is the same as an accumulation of negative electricity) within P, increasing without limit as long as the steady currents continued.

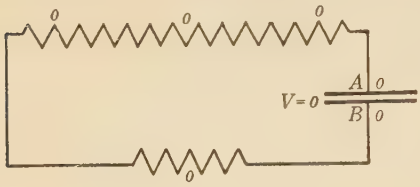


FIG. 19

But although a steady current must have the same value at all parts of its circuit, there is nothing to prevent a temporary current, or a varying current at some instant, from differing in value at different parts of a circuit or from flowing along a path that is not closed. In such cases accumulation of positive electricity in some regions and of negative electricity in other regions always takes place; but the accumulations do not increase without limit, inasmuch as the current does not continue to flow without change.

Consider a conductor not forming a closed circuit, but ending in a large flat plate at each end, as shown in Fig. 19. The plates offer a much larger surface for storing an accumulation of electricity than would be available if the conductor were merely cut off with "dead ends"; and by placing the two plates near to each other the storage of electricity is further facilitated by the electrostatic attraction between any positive electricity that may accumulate on one plate and any negative electricity that

may accumulate on the other. The two plates represent a simple form of the well-known device the condenser.

If the conductor in Fig. 19 does not contain any electromotive force, the potential should be the same at all points of the conductor, and this potential may be taken as zero. Suppose, now, that an electromotive force E is introduced into the conductor as shown in Fig. 20; this will alter the distribution of potential, and a current will be set up, as shown in Fig. 20 for the condition immediately following the introduction of the electromotive force. This current conveys positive electricity to the plate A and takes positive electricity from the plate B (which is the same as conveying negative electricity *to* the plate B) at the rate of one coulomb per second for each ampere flowing.

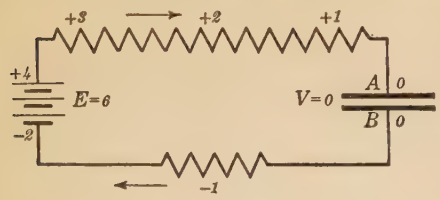


FIG. 20

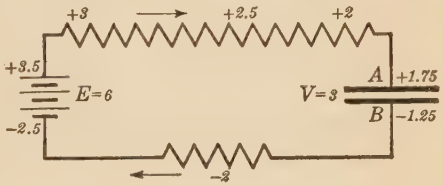


FIG. 21

But the presence of positive electricity on plate A and negative on plate B causes a difference of potential V, because it would now require some work to transfer any (positive) electricity from B to A in opposition to the electrostatic attraction from B and repulsion from A. Evidently this difference of potential is such as to oppose the flow of the current. So the condition represented in Fig. 20 would hold only instantaneously, and would immediately change: the current would decrease, and V would increase until it equaled the electromotive force E, after which the current would cease entirely. Fig. 21 represents the condition at some instant before the final condition is reached, but later than for Fig. 20; and Fig. 22 represents the final condition. With actual

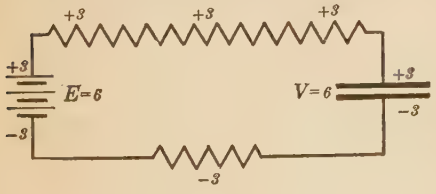


FIG. 22

condensers the time required to attain the final condition may be exceedingly short (perhaps a millionth of a second).

If, after the condition shown in Fig. 22 is attained, the electromotive force is removed from the circuit (for instance, by short-circuiting the battery), then evidently a temporary current will flow from A to B until A and B are brought to the same potential.

33. Capacitance: the farad. It is found that the difference of potential V between the plates of a condenser is proportional to the quantity Q of electricity stored on one* of them, so that we may write

$$Q = CV, \tag{34}$$

where C is a constant of proportionality, and is called the *capacitance* of the condenser. The capacitance is accordingly equal to the quantity of electricity on one plate of the condenser per unit of difference of potential between the plates.†

The practical unit of capacitance is the *farad*, which is defined as the capacitance of a condenser such that one coulomb of electricity (on each plate) causes a difference of potential of one volt between the plates. The largest condensers in actual use have capacitances very much less than one farad; for this reason capacitance is nearly always expressed in millionths of a farad, or *microfarads* (abbreviated *mf.*).

The notion of capacitance has a definite meaning for other conductors, as well as for those that form the plates of condensers; but in such cases the conductor in question should be thought of as representing one plate of a condenser, the other plate being represented by some other conductor in the neighborhood. Thus we may have the capacitance of one main of a two-wire transmission line, the other main representing the opposite plate of a condenser. In all such cases the capacitance will evidently differ, according to the particular conductor chosen to represent the opposite plate of the condenser. Thus we may have the capacitance of one transmission-line main with respect to the ground; and this capacitance will differ from that of the one main with respect to the other.

* By reasoning that will not be given here it can be shown that the quantity of negative electricity on one plate must equal the quantity of positive electricity on the other.

† Formerly the term *capacity* was used for the quantity just defined as capacitance. The older term is still sometimes used, but this usage is objectionable, because the word *capacity* also has other meanings, technical as well as nontechnical.

PROBLEMS

Any problem preceded by an asterisk may be omitted without loss of continuity.

1. A bar magnet 20 cm. long is placed with its center at the origin of coördinates, its north pole at the point (10, 0), and its south pole at the point (− 10, 0). Each pole has a strength of 200 units. Find the field intensity and direction at the points (+ 20, 0), (− 20, 0), and (0, + 10). (The distances are expressed in centimeters.)

2. Two bar magnets are placed as shown in the accompanying figure. Bar *A* has poles of strength 250 units, and bar *B* has poles

of strength 90 units. Find the magnitude and direction of the total force acting on bar *B* from bar *A*. The poles are at the ends of the bars.

3. A steel needle 8 cm. long is suspended by a fiber passing through an eye near the center of the needle. When unmagnetized the needle assumes a horizontal position. The needle is now magnetized, so that a pole of 20 units appears at each of its ends. The needle now dips, but it is provided with a 50-milligram weight which can be slid along the needle and adjusted so as to counteract the tendency to dip. How far from the center must this weight be placed to make the needle stand horizontally? (Given: vertical component of the earth's field, 0.45 gauss; value of g, 980 dynes per gram.)

4. A certain room is 39 ft. (or 12 m.) long, 33 ft. (or 10 m.) wide, and 13 ft. (or 4 m.) high. The length of the room is in the (magnetic) north-and-south direction. Find the total magnetic flux from the earth's field through the floor, ceiling, and each of the walls of the room, the horizontal and vertical components of the earth's field intensity being 0.25 and 0.45 gauss respectively.

5. A certain overhead cable is supported on poles 125 ft. apart and carries 200 amperes. Find the magnitude and direction of the horizontal force from the earth's field upon one span of this cable, in which the current is flowing westward. The vertical component of the earth's field is 0.45 gauss. Express the force in pounds.

6. In a certain electric generator one of the armature conductors is 1 m. long, and carries 100 amperes while in a field whose strength is 11,000 gausses at right angles to the conductor. Find the force, in pounds, acting upon the conductor.

*7. By using equation (8) derive a formula for the field intensity due to a current I in an infinitely long straight conductor, at a point P distant D from the conductor.

SUGGESTION. Count the distance l along the conductor from the foot of the perpendicular from P upon the conductor; then

$$dH = \frac{Idl}{D^2 + l^2} \cdot \frac{D}{\sqrt{D^2 + l^2}},$$

which can easily be integrated by expressing l and dl in terms of D and the angle θ between l and r.

*8. Using the result of Problem 7, derive a formula for the force, per unit of length, acting between two long straight parallel conductors at distance D from each other and carrying currents of I_1 and I_2 amperes respectively. What will be the direction of this force? How great will it be between two of the armature conductors of Problem 6 if D is 2 cm.?

9. The temperature of an electric flatiron rises from 20° to 110° C. during the first four minutes after turning on its current of 5 amperes. Make an approximate estimate of its resistance by assuming that half the heat generated in the first four minutes is stored in the iron itself, whose mass is 4 lb. (or 1800 g.). (Take the specific heat of iron as 0.11.)

10. A magnetic needle is suspended at the center of a circular coil of wire, the plane of the coil being vertical and in the magnetic meridian. If the horizontal component of the earth's magnetic field is 0.20 gauss, and the needle is deflected 45° by a current in the coil, how strong is the field due to the current? How strong is the current if the coil has 10 turns and the average radius of a turn is 20 cm.?

11. A tube 20 cm. long and 3 cm. in diameter is wound uniformly with 1000 turns of wire. Compute the field intensity in the tube when the winding is carrying 0.5 ampere.

12. A piece of wire, of resistance 0.1 ohm, is wrapped loosely 100 times round the middle of the bar magnet of Problem 1, and its ends are connected together. The magnet is then suddenly jerked out of the coil. Find the average electromotive force and current in the coil during the motion of the magnet, which occupied $\frac{1}{30}$ sec.

13. Compute the amount and direction of the electromotive force induced in the trolley pole of an electric car by moving eastwardly through the earth's magnetic field at the rate of 20 mi. per hour (or 900 cm. per second). (Take the length of the pole as 9 ft., its angle with the horizontal as 45°, and the horizontal intensity of the earth's field as 0.20 gauss.)

14. To investigate the possibility of using the earth's magnetic field as the magnetic field for an electric generator, suppose that a coil of 100 turns of wire is rotated at the rate of 1800 revolutions per minute about a horizontal east-and-west axis. Suppose that the coil is square and measures 50 cm. on each edge. Compute the average of the electromotive force induced in it in a quarter of a revolution (beginning with no flux through the coil), taking the total intensity of the earth's field as 0.5 gauss.

15. A current of 3 amperes, flowing in a field winding whose inductance is 10 henries, is interrupted by opening a switch. A spark takes place between the jaws of the switch. The spark looks to be about 1 cm. long before it ceases on account of the increasing length of the air gap between the switch jaws. The voltage required for a disruptive discharge through 1 cm. of air is about 30,000 volts. Compute how quickly the current would have had to be reduced to zero, in order that the electromotive force of self-induction should be 30,000 volts, and compare the result with the time you would estimate to be occupied in moving the switch 1 cm. in the course of a brisk opening of the switch.

16. 100 volts is applied for 0.02 sec. to a coil of which the inductance is 0.1 henry and the resistance is negligible. What current flows at the end of 0.02 sec.? The electromotive force is then suddenly reversed without interrupting the current. Find the current at 0.01, 0.02, and 0.03 sec. after the reversal of the electromotive force.

17. Compute the self-inductance of the solenoid of Problem 11, there being no iron near it. Assume that all the flux links with all the turns.

*18. A switch is closed, connecting a constant electromotive force E to a circuit whose resistance is R and whose inductance is L. Prove that $i = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$, where i is the current flowing at the time t after closing the switch. (Note that the total electromotive force acting in the circuit at any instant is the difference between E and the electromotive force of self-induction $L \cdot \frac{di}{dt}$.)

*19. Compute the values of i for $t = 0.1$ sec., 1 sec., and ∞, with $E = 100$ volts, for (a) the field winding of Problem 15, whose resistance is 50 ohms, and (b) an incandescent lamp whose resistance is 150 ohms and whose inductance is 10^{-7} henry.

20. A round bar of iron 1 in. in diameter and 39 in. (or 1 m.) long is bent into a closed ring and wound helically with 2000 turns of wire. When 0.5 ampere flows in the winding, it is found that the flux through the iron is 100,000 maxwells. Compute H, B, and μ.

21. Compute the magnetomotive force and the reluctance for the ring in Problem 20.

22. If the ends of the bar in Problems 20 and 21 were not brought into contact, but were left separated by an air gap 0.5 cm. long, what current would then be required in the winding to produce 100,000 maxwells of flux through the iron?

23. With a bar of the same shape and dimensions as that of Problem 22, but made of the iron whose properties are given in Fig. 12, find the flux when 2 amperes flow in the winding. (To do this it will be necessary first to compute an approximate value of Φ by using an assumed value for μ; then the value of μ corresponding to the approximate value of Φ can be obtained and be used for computing a more exact value for Φ.)

24. Find the charge on one plate of a 2-microfarad condenser when the difference of potential between the plates is 100 volts. How long could this charge supply a current of 1 milliampere, assuming the discharge to take place at a uniform rate?

25. Suppose the plates of a 2-microfarad condenser are at a potential difference of 110 volts at one instant, and at 110 volts in reversed direction $\frac{1}{120}$ of a second later, as would be the case if the condenser were connected across 60-cycle alternating-current mains. What would be the average value of the current flowing from the condenser during the $\frac{1}{120}$ sec.?

26. A storage battery giving about 6 volts and capable of storing 40 ampere-hours costs about $20. A 2-microfarad condenser costs about 80¢. Find how many 2-microfarad condensers would be required to store 40 ampere-hours having a potential difference of 6 volts between plates, and compare the cost of the condensers with that of the storage battery.

*27. A switch is closed, connecting a constant electromotive force E to a circuit the resistance of which is R and which contains a condenser of capacitance C, initially without charge. Prove that the instantaneous value q of the charge is $q = CE\left(1 - \epsilon^{-\frac{t}{RC}}\right)$ and that the instantaneous value i of the current is $i = \frac{E}{R} \cdot \epsilon^{-\frac{t}{RC}}$, where t is the time after closing the switch. (Note that $i = \frac{dq}{dt}$ and also $i = \frac{E - \frac{q}{C}}{R}$; i can be eliminated, and the resulting equation integrated for q.)

*28. Compute the current when 100 volts is connected to a circuit containing a 2-microfarad condenser and having a resistance of 5 ohms, when $t = 0$, 0.00001 sec., 0.0001 sec., and ∞.

CHAPTER II

THE DYNAMO: GENERAL PRINCIPLES

34. Sources of electrical energy. The utility of electricity lies in the fact that it furnishes very convenient means for the transmission and transformation of energy.

Electrical energy does not occur as such in nature, in forms suitable for practical applications (in the present state of our knowledge); but it can be obtained from other forms of energy. Of the multitudinous forms of energy found in nature, only two are found practically useful at present for the large-scale generation of electricity — the potential energy of elevated masses of water, and the chemical energy of fuels. Neither gives electrical energy directly.* The former gives mechanical power through the agency of hydraulic machinery; the latter produces heat by combustion of the fuel, a part of which in turn gives mechanical power through the agency of steam engines (or other heat engines). In both cases, then, the flow of energy drawn from the stores of nature assumes the form of mechanical power. Electrical power (except in some small-scale applications) is always obtained by conversion from mechanical power, whatever the original source of energy; and this is always effected by some form of dynamo.

35. Dynamo; generator; motor. The term *dynamo* refers to a machine which may be driven by mechanical power and yield electrical power, or may be driven by electrical power and yield mechanical power. In the former case it is said to be operated as a *generator*; in the latter as a *motor*. Any dynamo is capable of being operated either as a generator or as a motor, although most practical dynamos are intended for operation in only one of these ways.

* When electricity is obtained from a battery, chemical energy is converted directly to electrical, but this is on a small scale. The amount of electrical energy obtained from batteries is insignificant in comparison with that obtained from dynamos. As yet no practical method has been found for converting the chemical energy of fuels directly into electrical energy.

36. Principle of action of the dynamo. Essentially, a dynamo consists of a conductor or set of conductors, capable of motion relative to a magnetic field and capable of connection to an outside circuit. In actual dynamos a set of conductors is used rather than a single conductor; but the principles of action can be most conveniently discussed by fixing attention on a single one of the conductors, arranged for connection with an external circuit as shown in Fig. 23. The conductor, *ab*, is fastened to an axis *cd*, about which rotation can take place; it is straight and parallel to the axis; the end *b* is connected with the metallic slip-ring *f*, which is insulated from the axis; the end *a* is connected through the axis with the slip-ring *e*. *N*, *S* represent the poles of a magnet, between which a field exists.

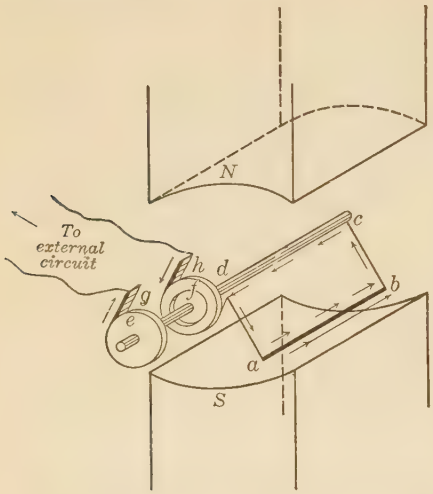


FIG. 23. Model to illustrate the action of a dynamo

The long arrow denotes electromotive force; the short arrows denote current (if any) in generator action

37. Action as a generator. If, by applying outside mechanical force, the conductor *ab* is moved across the magnetic field, then, by the fundamental physical principle discovered by Faraday and discussed in section 17, an electromotive force is induced in it equal at any instant to lBv abvolts, where l is the length of the conductor in cm., v its velocity in cm./sec., and B is the magnetic flux density in gausses, at right angles both to the conductor and to the direction of its motion (or the component of flux density mutually perpendicular to l and v, when the field does not already have this direction).*

* In case the field is not uniform, B should be taken to stand for the average value, along the conductor, of the component of flux density mutually perpendicular to l and v. If the conductor should not be parallel to the axis, the formula above would not apply without modification; but this does not need to be considered here, since in all modern dynamos the conductors are parallel to the axis, or very nearly so.

This electromotive force may or may not cause a current to flow, according as the conductor *ab* is or is not part of a closed circuit. The circuit may be closed by connecting a conducting path between slip-rings *e* and *f* by means of the sliding contacts *g* and *h*.

If no current flows, the electromotive force merely maintains a difference of potential between *g* and *h*; no electrical power (= current × electromotive force) is delivered, and the dynamo is said to be unloaded.

If a conducting path (or "load") is connected between *g* and *h*, electrical power is then delivered, because the electromotive force can then drive a current through the completed circuit, which includes, of course, the wire *ab*. But when a current-carrying conductor lies in a magnetic field, it receives from the field a mechanical force. In this case the force must evidently be in such direction as to oppose the motion — otherwise a "perpetual motion machine" would result. Accordingly, when current is delivered by the dynamo, mechanical force (in excess of that required to overcome friction) has to be applied in order to maintain the motion of the machine, and the work done by this force supplies the energy appearing in electrical form in the circuit. That is, the dynamo is capable of transforming mechanical energy (in the work done in turning its axle) into electrical energy (in the current flowing under the electromotive force generated).* This is the operation of the dynamo as a generator.

38. Action as a motor. Suppose, now, that some outside electromotive force (such as that from a battery) is applied to the terminals *g* and *h* of the dynamo, and causes a current to flow in the conductor *ab*. Then we have a current-carrying conductor lying in a magnetic field; accordingly, by the fundamental principle of physics discussed in section 11, a mechanical force will act upon it, in a direction perpendicular both to it and to the magnetic field, and of strength (in the direction in which the conductor can move) of lBI dynes, where I is the current in abamperes, and l and B have the same meaning as before.

* Electromotive force times current gives electrical *power*, or rate of delivery of electrical energy; this power must be multiplied by the time to obtain the electrical *energy*.

If this mechanical force sets the wire *ab* in motion, *ab* will then cut across the magnetic field and will therefore have an electromotive force induced in it. Evidently the direction of this electromotive force must be such as to oppose the current — otherwise a "perpetual motion machine" would result. To maintain the current against this counter-electromotive force, the electromotive force applied from outside must exceed the value required merely to maintain the current through the resistance of *ab*. The (electrical) work done in driving the current against this counter-electromotive force supplies the energy which is transmitted in mechanical form to the shaft of the dynamo. That is, the dynamo is capable of transforming electrical energy (in current flowing against its electromotive force)* into mechanical energy (in work done in turning its shaft). This is the operation of the dynamo as a motor.

39. Generality of the principles of action of the dynamo. If, in Fig. 23, the conductor *ab* were held stationary, while the poles *N* and *S* were free to move (of course carrying the magnetic field with them), the dynamo would work just the same, both as generator and as motor.† All phenomena would be essentially the same as just discussed. In dynamos the relative motion between armature conductor and field is the essential thing; it is immaterial which of them is stationary and which movable.

The principles discussed in sections 37, 38, and the present section are quite general, and apply to all practical dynamos as well as to the simplified model shown in Fig. 23. On account of their generality, it seems worth while to restate and develop them a little at this point, before discussing the construction of practical dynamos.

40. Production of electromotive force in a dynamo. The field being excited,‡ if the moving part (which may be an armature carrying conductors, or may be the field) of a dynamo of any kind whatever is set in rotation, an electromotive force is generated in its armature winding. If the speed is increased,

* See footnote at the end of section 37.

† If a current is flowing in *ab*, the mechanical force upon it due to the magnetic field acts between the current-carrying conductor and the poles from which the field proceeds, the force upon conductor and that upon poles standing in the relation of action and reaction.

‡ In most actual dynamos the field is produced by electromagnets, which may be excited to give a field of greater or less strength.

the electromotive force (depending on lBv) is increased in the same proportion, provided the strength of field (and other conditions) is not changed. If the strength of the magnetic field is changed, the electromotive force is changed in the same proportion, provided the speed (and other conditions) remains the same.

These statements are true, whether the dynamo is operated as a generator or as a motor. In the former case the electromotive force is in the direction of whatever current the machine delivers; in the latter it opposes the current through the armature.

41. Electrical output and mechanical input in a generator. If, when operating as a generator, the dynamo is driven on open circuit, so as not to deliver any current, then the force (or torque) required to drive it is merely that to overcome friction (and other similar forces that dissipate energy within the machine). But if the generator delivers current (or is "loaded"), a greater force (or torque) is required to drive it.

In actual generators, running at full load, a comparatively small part of the driving torque (perhaps 5 per cent, being more or less according to the size and type of machine) is consumed in overcoming friction (and similar forces); the greater part is used in driving the generator against the opposing torque due to current in its armature. The work done against this latter torque represents mechanical energy converted into electrical.

42. Production of torque in a dynamo. The field being excited, if a current flows in the armature winding of a dynamo of any kind whatever, then mechanical forces are produced which result in a torque acting on the moving part of the dynamo (which may be an armature carrying conductors, or may be the field). If the current increases, the torque (depending on lBI) is increased in the same proportion, provided the strength of magnetic field (and other conditions) is unchanged. If the strength of magnetic field is increased, the torque is increased in the same proportion, provided the current (and other conditions) is unchanged.

These statements hold true, whether the dynamo is operated as a motor or as a generator. In the former case the torque is in the direction of the motion (maintains the motion); in the latter it opposes the motion.

43. Mechanical output and electrical input in a motor. If, when connected to operate as a motor, the dynamo is clamped motionless, so as not to deliver any mechanical power (although exerting a torque), then the electromotive force at its terminals is all used in driving current through its internal resistance.* But if the motor turns, thus delivering mechanical power, a greater electromotive force is required at its terminals than merely that to drive current through its internal resistance.†

In actual motors under normal running conditions, a comparatively small part of the terminal electromotive force (perhaps 5 per cent, being more or less according to size, type, and loading) is required for overcoming resistance (or impedance); the greater part is used in driving the current against the opposing electromotive force generated in the motor. The electrical work done against this counter-electromotive force represents energy converted from electrical to mechanical form.

44. Reversibility of dynamos. The same dynamo will operate either as a generator or as a motor. This is a perfectly general rule, although some types of dynamos require special conditions to be met in order to act reversibly.‡

PROBLEMS

1. The flux density in the magnetic field of ordinary dynamos is something like 10,000 gausses. Find the electromotive force generated in a conductor 30 cm. long when it is moved across such a field by the motion of an armature 30 cm. in diameter making 1500 revolutions per minute. How many such conductors would need to be connected in series to obtain 110 volts?

2. If the conductor of Problem 1 carries a current of 10 amperes, find the force in pounds with which it would be acted on by the field.

3. A certain direct-current generator is rated to give 80 amperes at 125 volts. Find its rating in kilowatts. If its efficiency is 85 per cent, find the horse power of the engine required to drive it.

* In alternating-current machines, against its *impedance*.

† In practice the applied electromotive force is usually approximately constant. If the motor is clamped, the constant electromotive force drives a very heavy current through it; if in motion, this same electromotive force drives a much smaller current through the motor, most of the applied constant electromotive force being then balanced against the counter-electromotive force of the motor.

‡ The usual types of alternating-current generators will run as motors only when brought to a definite speed, in synchronism with the alternating voltage of the supply. Induction motors will operate as generators only when connected to a line kept alive from some other source.

4. A certain dynamo in the laboratory of the University of Florida runs at 1400 revolutions per minute, being driven by belt on a pulley of 4 in. diameter. When generating 110 volts but delivering no current, the tension in the tight side of the belt is about 2 lb. greater than in the slack side. Find how much the tension in the tight side exceeds that in the slack side when the machine delivers 20 amperes at 110 volts.

SUGGESTION. The kilowatts of electrical power delivered must come from additional mechanical power put into the machine. In each revolution the foot-pounds of mechanical work equal the circumference of the pulley, times the difference in tension between the tight and slack sides of the belt.

5. A certain direct-current motor is exerting a torque of 1000 pound-inches while taking a current of 200 amperes. What torque will it exert if it takes 300 amperes at the same time that its field strength is increased 50 per cent?

6. A certain alternating-current generator gives 2300 volts when running at 360 revolutions per minute. Suppose its speed drops to 350 revolutions per minute, by what percentage will the magnetic field strength have to be increased so that the voltage generated may remain 2300?

CHAPTER III

CONSTRUCTION OF DYNAMOS

For discussing the fundamental principles of the action of the dynamo it has been convenient to make use of a simplified model (Fig. 23). We shall now consider the construction of actual dynamos, giving attention in this chapter to features which are common to most of the different types. In later chapters various different types will be more fully discussed.

45. Production of the magnetic field. The magnetic field is always obtained by the use of electromagnets rather than by permanent magnets, excepting sometimes in very small dynamos. (A small dynamo depending on permanent magnets for its field is known as a *magneto*.)

Electromagnets offer two advantages over permanent magnets: a stronger field can be obtained than from permanent magnets of the same size, and the strength of the field is controllable by varying the strength of the exciting current.

46. The magnetic circuit. Fig. 24 shows a form of electromagnet which is sometimes used (in small machines). The dotted lines show the path of the magnetic flux, or the magnetic circuit. To obtain as strong a field as possible the reluctance of this circuit should obviously be made as small as possible. For this purpose the space between the poles should be filled as nearly as possible with soft iron, which is accomplished by making the core of the armature of soft iron and by making

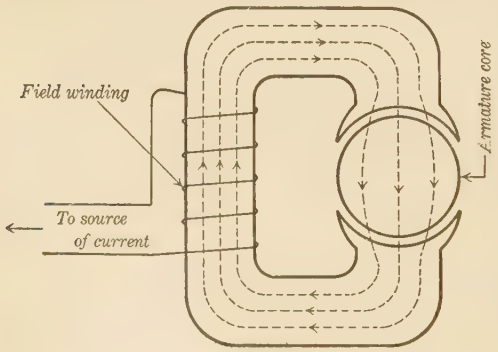


FIG. 24. Magnetic circuit of a dynamo

the air gaps between the pole faces and the armature core narrow. By placing the armature conductors in slots between teeth, as shown in Fig. 25, the air gaps can be made narrower than would otherwise be possible. This arrangement also has the advantage of permitting very firm attachment of armature conductors to the core — an important matter, since the force (lBI) from the magnetic field upon the conductors tends to slide them tangentially along the armature. In actual dynamos the air gap is sometimes as short as 1/32 inch in small machines, and does not often exceed 0.4 inch in large ones. Even with the shortest air gaps used, the greater part (perhaps 80 per cent) of the reluctance of the magnetic circuit is located in the air gaps rather than in the iron parts. The flux density in the air gap ranges from about 5000 to 14,000 gausses in actual machines.

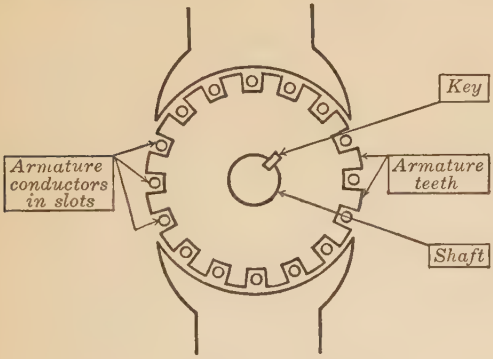


FIG. 25. Armature teeth and slots

47. Construction of armature cores. In small machines the armature core is usually keyed to the shaft, as shown in Fig. 25; in larger machines the core is made hollow inside, as shown in Fig. 26, thus saving material, lessening the weight of the machine, and facilitating the escape of the heat which always appears in an armature when it is in operation. The arrangement which supports the core and attaches it to the shaft is called a *spider*.

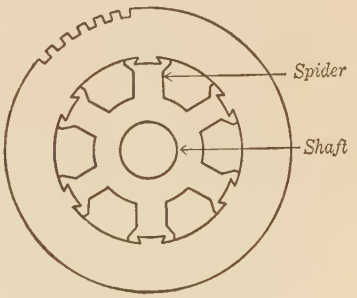


FIG. 26. Armature core

For simplicity of drawing, the armature teeth are not shown all around the periphery, but only in the upper left-hand part

48. Lamination of armature cores. When the iron core of an armature rotates in the magnetic field, electromotive forces are

induced in the core as well as in the armature conductors attached to it, and tend to make currents flow in the inside of the core, as shown in Fig. 27. Such currents are known as *eddy currents*. They are not only useless but actually detrimental since they waste energy and produce heat in the armature core. Their magnitude may be very greatly decreased by laminating the core; that is, building it up out of a large number of thin sheets of iron insulated from one another, as shown in Fig. 28. The advantage of this arrangement may be understood by considering Fig. 29, which is drawn like Fig. 27, but with the armature core constructed in four sections; in any one section the electromotive force is evidently only one fourth of that in the whole core, whereas the resistance of one section is greater than that of the whole core. The current is accordingly much less than it would be in an undivided solid core of the same material and thus less energy is wasted.

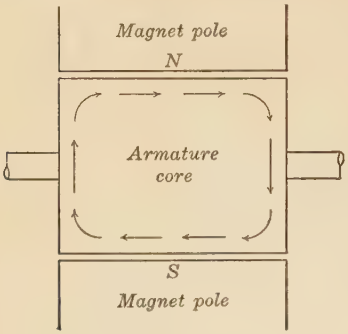


FIG. 27. Eddy currents in a solid armature core

Section through axis

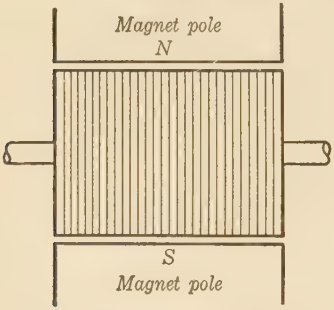


FIG. 28. Laminated armature core

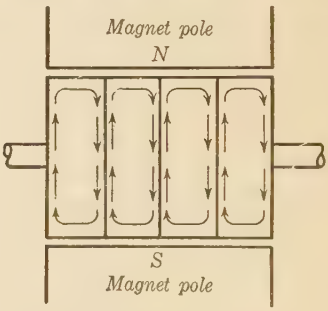


FIG. 29. Effect of lamination on eddy currents

When the laminations are thin, the electromotive force tending to make current pass from any lamination to the next is very small. Accordingly very weak insulation between laminations is sufficient, such as can be obtained by a thin coating of shellac.

Sometimes nothing more is used than the slight film of oxide which naturally forms on the surface of iron exposed to the air.

Evidently the lamination of the core offers no obstruction to the magnetic flux, since the planes of the laminations are parallel to the directions of the lines of magnetic induction.

FIG. 30. Armature disk

Courtesy of the Reliance Electric and Engineering Company

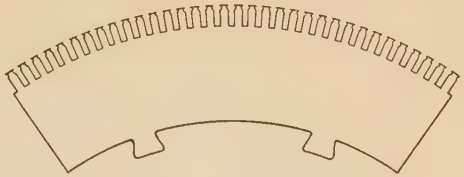


FIG. 31. Segmental punching

Courtesy of the Westinghouse Electric and Manufacturing Company

The thickness of laminations used in actual dynamos in this country is generally 0.014 inch.

In constructing an armature core the laminations are punched out of sheet iron (by use of a special die, giving the shape of lamination desired) and are then assembled by hand on their shaft or spider, pressed tightly together, and clamped or bolted in place (see Figs. 30 and 31).

49. Bipolar and multipolar dynamos. A dynamo whose field magnet has two poles only (as in Fig. 24) is called *bipolar*; one whose field magnets have four poles (as in Fig. 32) or more than four poles is called *multipolar*. The multipolar construction has the advantage that less iron is required for the field structure than in the bipolar type, thus making the dynamo cheaper, lighter, and more compact.

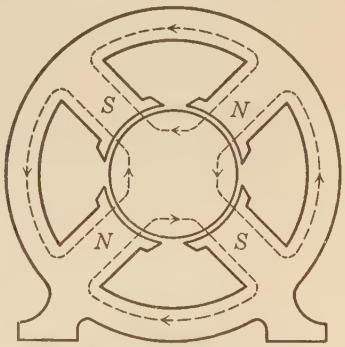


FIG. 32. Magnetic circuits in four-pole dynamo

To see how the economy of iron is effected, compare the bipolar dynamo of Fig. 24 with the multipolar machine of Fig. 32. In Fig. 24 a long field core is required, of sufficient cross section for the total magnetic flux. In Fig. 32 half the total flux leaves each of the north poles and enters each south pole, but only

one fourth of the total flux passes through the yoke connecting one pole with the next. Accordingly this yoke need have only one fourth of the cross section of the field core of the bipolar machine if the total flux is to be the same in both machines. On the other hand, there are four of these yokes (or yoke sections) in Fig. 32 compared with only one field core in Fig. 24. But the length of one of the yokes of Fig. 32 is less than half that

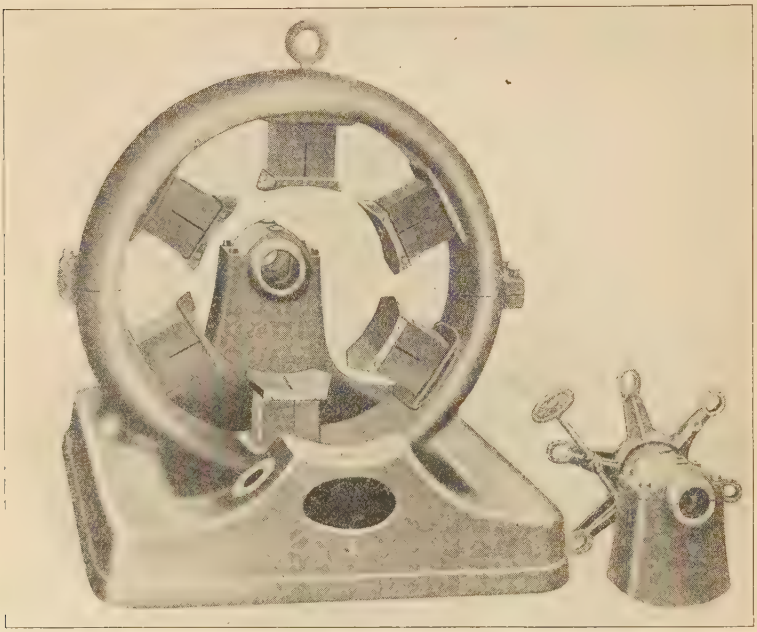


FIG. 33. Frame of a six-pole dynamo

of the field core of Fig. 24. So the total iron required for the yokes of Fig. 32 will be less than half that required for the field core of Fig. 24.

For the same total magnetic flux still further economy in field iron can be obtained by the use of six poles, or eight, or still more. On the other hand, magnetic leakage becomes greater as the number of field poles is increased; and the machine becomes more complex, with consequent increase in cost of manufacture.

For small machines, where the total quantity of material concerned is insignificant, two poles are usually used; for somewhat

larger machines (with a capacity, say, of from 2 to 100 kilowatts) four poles are now usual; and for still larger ones a larger number of poles are provided, as in the examples in Figs. 33 and 34. Large alternating-current generators are sometimes constructed with as many as 100 poles.*

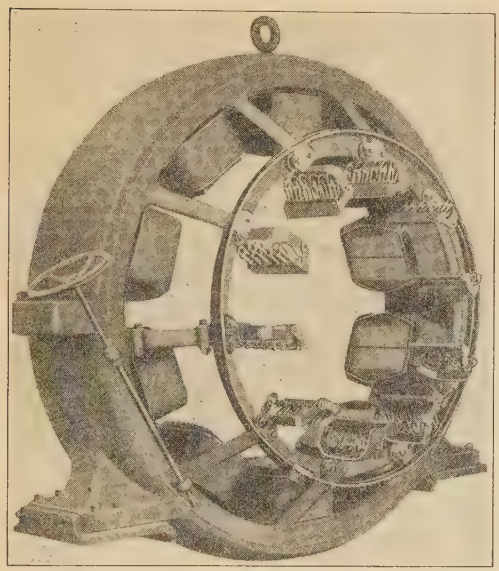


FIG. 34. Field structure (with brushes and brush supports) of a ten-pole generator

50. Construction of field magnets. Fig. 35 shows the usual construction of the field magnets. The yoke, of cast iron, serves not only as part of the magnetic circuit but also as the frame of the machine. In larger machines it is usually made in two pieces as shown, so that the upper piece can be raised if it becomes necessary to get access to the armature. The cores are usually of cast steel, bolted to the yoke; and the pole shoes are of soft iron, usually laminated, bolted or screwed to the cores. But sometimes the pole shoe and the core are made in a single piece.

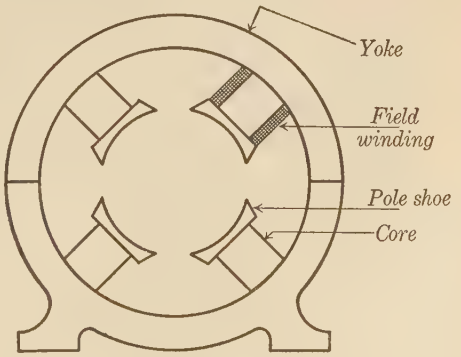


FIG. 35. Construction of field magnets

51. Revolving-field dynamos. As has already been pointed out, the action of the dynamo depends on the *relative*

* In such machines the number of poles is determined by the desired frequency at some assigned speed, rather than by considerations of economy.

motion between armature and field. In the last few paragraphs the field has been assumed stationary and the armature movable;

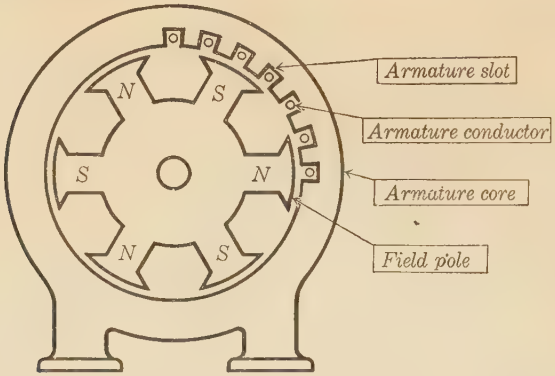


FIG. 36. Arrangement of parts in a revolving-field dynamo

For simplicity of drawing, the armature slots and conductors are shown for only a part of the circumference of the armature

but a dynamo would work just as well, in principle, if the armature were stationary and the field movable. In actual dynamos

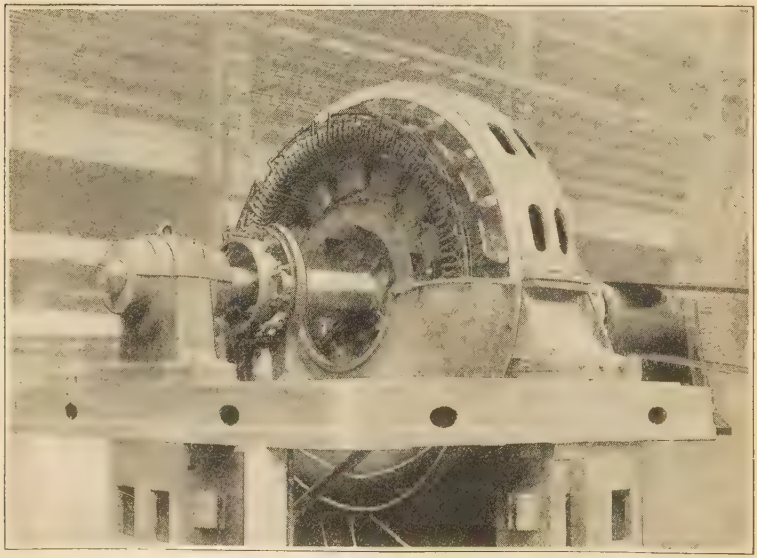


FIG. 37. Revolving-field generator

Courtesy of the Westinghouse Electric and Manufacturing Company

both the revolving-armature type and the revolving-field type are in use in present-day practice.

In revolving-field dynamos the field structure is placed inside, upon the shaft, and the stationary armature is placed outside. The arrangement is illustrated in Fig. 36. Except for this inversion in position between armature and field, the construction of the revolving-field type is much like that of the revolving-armature type, since the same principles govern both. In both types the armature core is laminated, the conductors are placed in slots, the air gap is made narrow, etc.

Alternating-current generators are more frequently of the revolving-field type than of the revolving-armature type. Fig. 37 shows a revolving-field alternating-current generator. Direct-current dynamos of the revolving-field type are theoretically possible, but are not used in present practice.

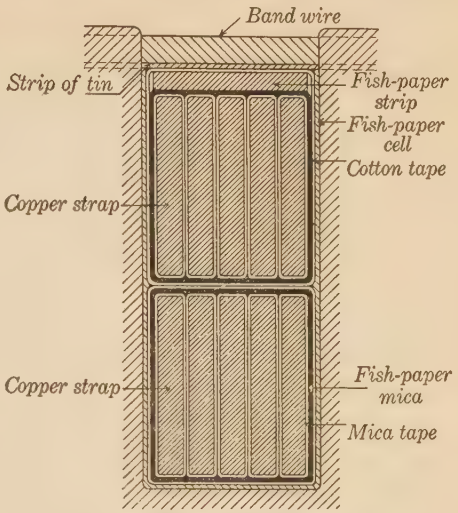


FIG. 38. Cross section showing armature conductors in slot

Courtesy of the Westinghouse Electric and Manufacturing Company

52. Armature conductors. The armature conductors consist of copper wire in small machines and of copper bars in large ones, the cross section being determined by the current that the conductor is expected to carry. The bars are often of rectangular cross section (for economy of space), as shown in Fig. 38. Usually several conductors, insulated from one another, occupy the same slot.

The electromotive force induced in a single conductor is small. To obtain sufficient voltage for practical purposes a number of conductors are connected together in series.

53. Drum armatures and ring armatures. One armature conductor can be connected with another by a connection across

the end of the armature, as in Fig. 39, or by a connection carried through a hollow inside the armature, as in Fig. 40. An armature in which the connections between conductors are made across the ends is called a *drum armature*; one in which they are made through a hollow interior is called a *ring armature*.*

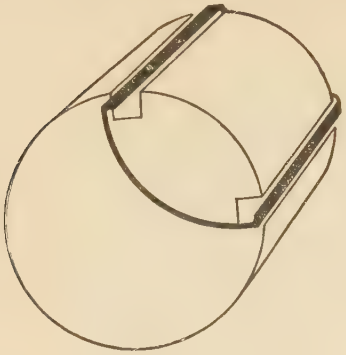


FIG. 39. Drum-armature method of connecting one armature conductor to another

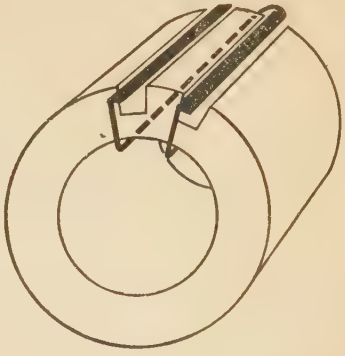


FIG. 40. Ring-armature method of connecting one armature conductor to another

Ring armatures are seldom used in modern dynamos, the principal reason being that drum armatures are easier to manufacture and to repair.

In both types the connections from one armature conductor to another contribute nothing to the electromotive force in generator action, or to the torque in motor action, for the reason that they are outside the magnetic field. (The hollow interior of the core of a ring armature is not a magnetic field, since the magnetic lines follow the iron in preference to air, as shown in Fig. 41.) Even if the magnetic field extended beyond the ends of the armature core, the position of the end connections is such that when the armature revolves they would

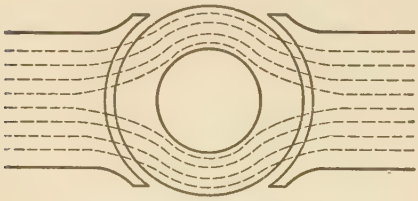


FIG. 41. Arrangement of magnetic flux in a hollow armature core

* Still another type of armature, no longer used, is the *disk armature*, in which the conductors were arranged radially, like the spokes of a wheel. The magnetic field through which they cut was parallel to the axis.

slide between the magnetic lines rather than cut across them. Since these connections add to the resistance of the armature without contributing to the electromotive force (or to the torque in motor action), it is desirable to make them short. In a machine having few poles, large diameter, and short axial length, they will be shorter for a ring armature than for a drum armature; but the reverse will be the case in a machine having many poles, small diameter, and great axial length.

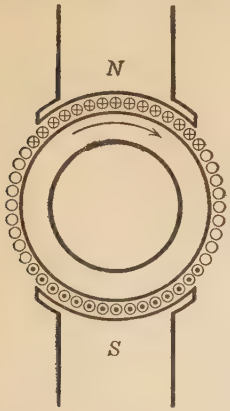


FIG. 42. Direction of electromotive force in different armature conductors

Drum armatures are often made with hollow cores similar to those of ring armatures, but they must not on that account be mistaken for ring armatures. The distinction is not according to whether the core is solid or hollow, but according to whether the connections between the armature conductors are carried across the end of the armature or are threaded through a hollow interior.

54. Armature windings. The term *armature winding* is used to denote the aggregate of all the armature conductors, including their connections with one another.

When a number of conductors are connected in series with one another, they form a path through the armature. The order in which they are connected is usually such that all those in any one path contribute electromotive force in the same direction relative to that path.

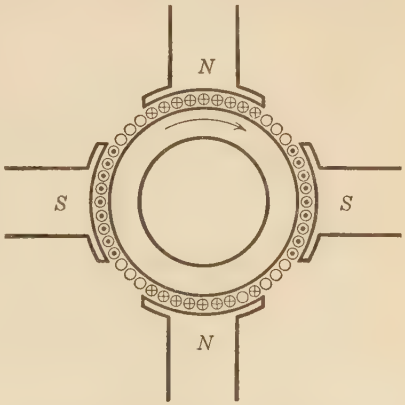


FIG. 43. Direction of electromotive force in different armature conductors

Figs. 42 and 43 show the direction of the electromotive force at a given instant in the different conductors of an armature. The symbols ⊙ and ⊕ indicate conductors whose electromotive

force is toward the reader and away from him respectively; the dots being intended to suggest the points of arrows, and the plus signs, the butts. In the conductors lying (at the instant) between the poles little or no electromotive force is generated, in those next to the north poles the electromotive force is toward the reader, and in those next to the south poles it is away from him.

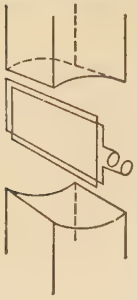


FIG. 44. A possible simple type of drum-armature winding

In a drum armature, then, each of the end connections should lead between a conductor next to a north pole and a conductor next to a south pole. This is illustrated for a bipolar machine in Fig. 44, which represents a possible simple type of drum-armature winding. It will be understood, of course, that an armature need not necessarily have its conductors distributed uniformly about its periphery, as in Fig. 42, but may have them confined to bands.

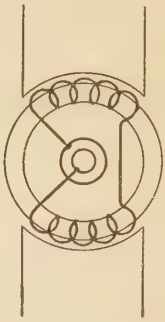


FIG. 45. A possible simple type of ring-armature winding

In a ring armature each conductor should be connected with another one next to the *same* kind of pole. This is illustrated,

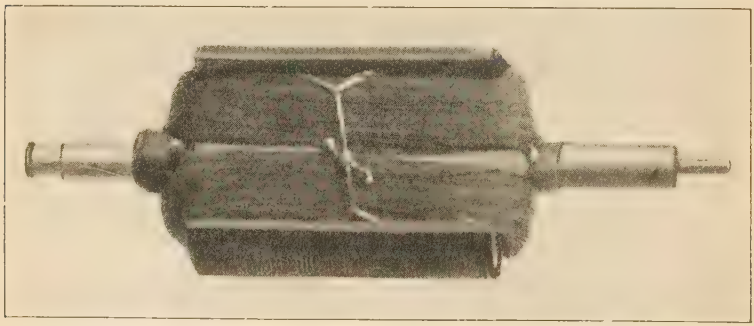


FIG. 46. Armature of telephone magneto

for a bipolar machine, in Fig. 45, which represents a possible simple type of ring-armature winding.

The winding shown in Fig. 44 is applied practically (but with a larger number of conductors) in the telephone magneto,

used for "ringing up" on rural lines and in those towns which have not yet installed central-battery telephone exchanges. The armature of a telephone magneto is shown in Fig. 46.

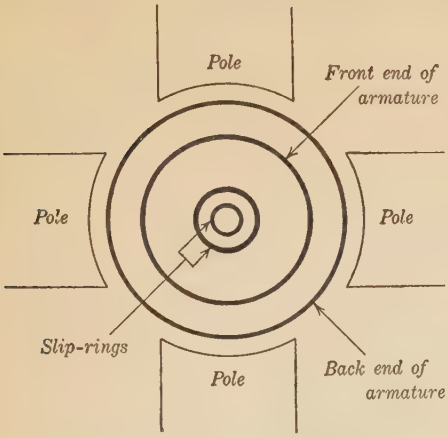


FIG. 47. To explain diagrammatic representation of drum armature

55. The diagrammatic representation of drum-armature windings. Connections between conductors in multipolar drum armatures are shown most conveniently by using a diagrammatic scheme, illustrated in Figs. 47 and 48. In this scheme the armature is supposed to be viewed from the slip-ring (or commutator) end; but the back end is drawn larger than the front end, and connections across the back are drawn *outside* the circle representing the back end, instead of inside it. On such diagrams the armature conductors appear as radial lines.

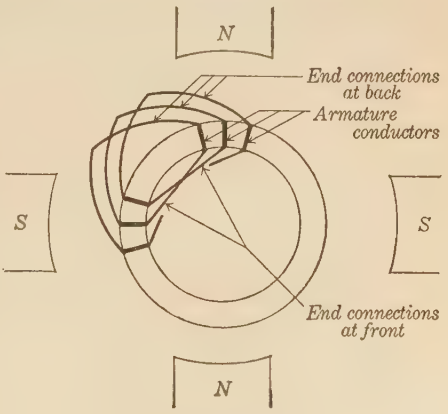


FIG. 48. To explain diagrammatic representation of drum armature

56. Practical construction of armature windings. In earlier times the winding was actually wound on the armature out of a single long piece of wire; at present the "winding" of a drum armature is accomplished by placing coils like that shown in Fig. 49 upon it, as shown in Fig. 50. After the coils are in place, their ends are soldered together in proper order. This process is easier and more economical than that of actually winding wire on the armature, and it has the further advantage

of facilitating repair work. In case of accident the coils affected can be replaced by new ones, without the need of tearing out the whole winding.

The coils themselves are made by winding wire (or bending bars) about frames, so shaped as to give the coil the proper form to fit on to the armature for which it is intended. In large machines a coil sometimes consists of only one turn (two conductors), but in smaller machines each coil contains a larger number of turns. Insulated wire is then used; and after the coil is wound, insulating tape or fabric is wrapped over it, binding the different conductors together, as shown in Fig. 49. It is common to place inside each armature slot an insulating casing, within which the coil is placed. The insulation between the different conductors of the same coil does not need to be as heavy as that between the coil and the armature core, because the difference of potential between conductors within a coil is comparatively small, being due to the electromotive force induced in only a few conductors. The arrangement of conductors and insulation within a slot is illustrated in Fig. 38.

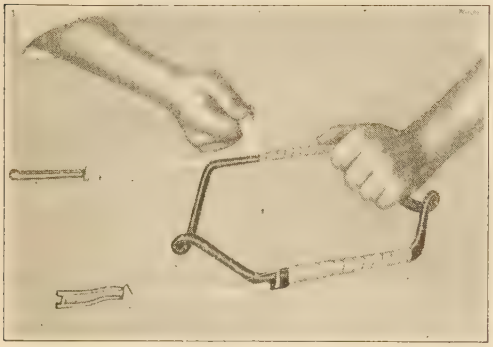


FIG. 49. Armature coil

Courtesy of the Reliance Electric and Engineering Company

A good many different materials are used as insulating materials for armature conductors and coils. Among these may be mentioned cotton fabric treated with linseed oil, shellac, bakelite,* micarta,* and flexible mica. There is no such limitation of choice as there is in the material for the conductors themselves, for which copper has no rival; or for the material for magnetic parts, which must always be some sort of iron. It is not improbable that additional insulating materials will be developed.

* Bakelite is a synthetic organic substance produced from phenol and formaldehyde. Micarta consists of heavy duck fabric bonded with bakelite.

57. Alternations of electromotive force in armature conductors. It is evident that the electromotive force in an armature conductor is generated first in one direction and then in the opposite direction, as the conductor moves first past a north pole and then past a south pole. The same thing is true for the total electromotive force generated in a group of conductors in series, since (whatever the distribution of the conductors within the group) the group moves alternately past poles of opposite polarity. Accordingly, when the ends of an armature winding

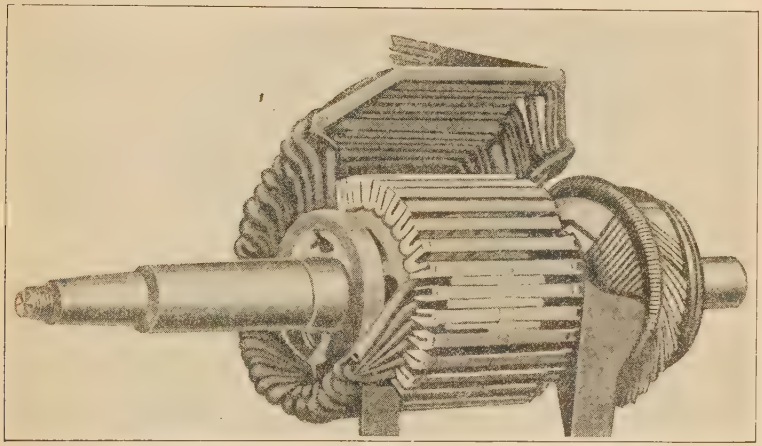


FIG. 50. An armature partly wound

Courtesy of the Westinghouse Electric and Manufacturing Company

are connected to slip-rings (as has been assumed in all the discussion thus far), the terminal electromotive force appearing between the slip-rings is an alternating electromotive force giving alternating current when applied to a circuit.

For many applications alternating current is entirely satisfactory, but for others direct current is necessary.* To obtain direct current from a dynamo some device must be provided by which the electromotive force between terminals is always

* For heating, lighting, and power, alternating currents and direct currents can be used about equally well (ignoring advantages in favor of the one or the other in special cases). For charging storage batteries and most other electrochemical applications, and for exciting the fields of dynamos, direct current must be used. For transmission of energy, except over very short distances, alternating currents possess great advantages over direct currents.

in the same direction, notwithstanding that the electromotive force in each armature conductor alternates. The device by which this is accomplished is the *commutator*, which will be discussed in the following chapter.

Broadly speaking, a direct-current generator is essentially the same as an alternating-current generator provided with a commutator, notwithstanding radical differences in many details. The only difference in principle is the addition of the commutator in the case of the direct-current machine; within the direct-current machine the electromotive force and current alternate just as in an alternating-current machine. That is, the electromotive force contributed by any direct-current armature conductor is generated first in one direction along the conductor and then in the opposite direction; and the current in any conductor flows first in one direction and then in the opposite direction. It is only after passing the commutator that the electromotive force and current are direct.

58. Instantaneous values of electromotive force. If the instantaneous values of the electromotive force generated in an armature conductor are plotted, the result will be a curve somewhat like that of Fig. 51. The exact form of the curve is determined by the manner in which the magnetic field varies along the path swept over by the conductor in its motion (since $e = lBv$, and l and v are constant). Since just as much magnetic flux enters the south poles as leaves the north poles, the area between the negative half-waves and the horizontal axis must equal that between the positive half-waves and the horizontal axis; but the shape of the negative half-waves need not necessarily be the same as that of the positive ones.

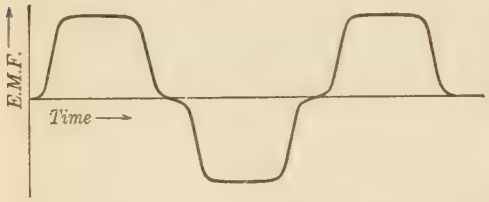


FIG. 51. Approximate manner of variation of electromotive force in an armature conductor

59. Open-coil and closed-coil armature windings. Every armature winding hitherto considered (Figs. 23, 44, 45) starts from one terminal (or slip-ring) and ends at another. A winding of this kind is called an *open-coil winding* in contrast to a *closed-coil winding*, which is closed upon itself and has no free

ends. Open-coil windings are used a great deal in alternating-current machinery, and they have been used in some types of direct-current generators which are now obsolete.* Closed-coil windings will be discussed in the next chapter.

PROBLEMS

1. A certain direct-current motor delivers 20 horse power when it is connected with 220-volt mains. If its efficiency is 85 per cent, find how much current it takes.

2. Four amperes of the current received by the motor of Problem 1 pass through its shunt field, and the rest through its armature, whose resistance is 0.2 ohm. Find the counter-electromotive force generated in its armature.

SUGGESTION. Armature current equals 220 volts minus the counter-electromotive force, divided by the resistance.

3. The armature winding of the motor of Problem 1 consists of two paths in parallel, so that each armature conductor carries half the total armature current. Find the force (in pounds) with which each armature conductor presses against the side of the armature slot in which it lies, the length of the armature being 30 cm. and the field intensity 11,000 gausses.

4. The magnetic flux through the armature of a small 2-pole 1500-r.p.m. 1-kw. dynamo is 800,000 maxwells. The armature is 10 cm. long and 9 cm. in diameter. If the armature core were not laminated, find the electromotive force that would drive an eddy current in it, and estimate the amount of the current and the power required to maintain it. (A rough estimate of the resistance of the path of the eddy current may be made by assuming that the length of the path is half the distance round the outside of the armature ($\frac{1}{2}$ of $(10 + 9 + 10 + 9)$), and the cross section half the axial section of the armature ($\frac{1}{2}$ of 10×9). The resistivity of iron is 0.000011 ohm for a centimeter cube.)

5. Find the difference of potential between a point on an armature winding and an adjacent point on the next turn of the same armature coil. The coil belongs to a drum armature whose length is 40 cm., diameter 30 cm., and speed 1200 revolutions per minute. The field intensity is 10,000 gausses.

6. How many times per second does the electromotive force reverse in an armature conductor of a 4-pole 1400-r.p.m. direct-current generator?

* Brush arc dynamo and Thomson-Houston arc dynamo.

CHAPTER IV

DIRECT-CURRENT GENERATORS

60. Direct-current armature windings. In all ordinary direct-current dynamos of present practice the armature windings are of the closed-coil type.

The arrangement of a closed-coil armature winding can be understood best by considering it first in connection with a ring armature. Let a continuous winding be placed on such an armature and closed on itself, as shown in Fig. 52. If this armature is rotated in a bipolar magnetic field, electromotive force will be generated as shown by the arrows. Starting from a point A (fixed in space, not moving with the armature), electromotive force will be generated toward the point B, both along the path ANB and the path ASB. The electromotive force along ANB must be exactly equal to that along ASB, since all the lines of flux entering the armature from the north pole must again leave it to reach the south pole. Accordingly there is no tendency for current to flow *round* the winding; although closed on itself, it is not short-circuited on itself.

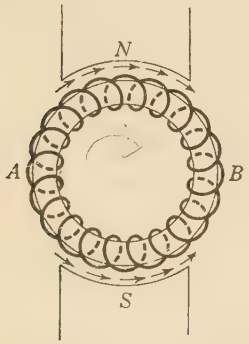


FIG. 52. Closed-coil winding on ring armature

If brushes should be made to touch the winding at the points A and B, so as to be always in contact with the parts of the winding passing A and B at the moment, then an electromotive force of constant direction would be maintained between the brushes, equal to that generated in the paths ANB and ASB (which are in parallel with each other with respect to an external circuit connected with the brushes). This might be arranged by placing the brushes so that they would touch the outside of the armature between the poles, the insulation being

scraped off from the winding at the points which would pass A and B in the course of rotation. In practice this is not done; but the same result is attained by connecting wires (called *armature leads*) to the winding at frequent intervals, each lead ending in a commutator bar, as shown in Fig. 53. The brushes then rest against the commutator; and when a brush touches a commutator bar, its connection with the armature winding (through commutator bar and lead) is practically the same as if it actually touched the winding itself.

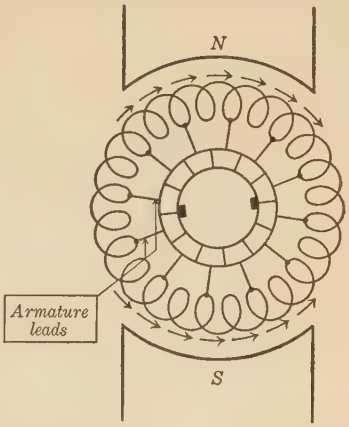


FIG. 53. Method of connecting armature winding to commutator

The brushes should be located so as to touch those bars of the commutator which are connected with portions of the winding midway between the poles. In actual machines it is not always easy to locate these points accurately by inspection, since the leads are frequently curved (see Fig. 54), in which case the proper positions (called *neutral points*) may lie more nearly opposite the poles than between them. On a generator in operation (but

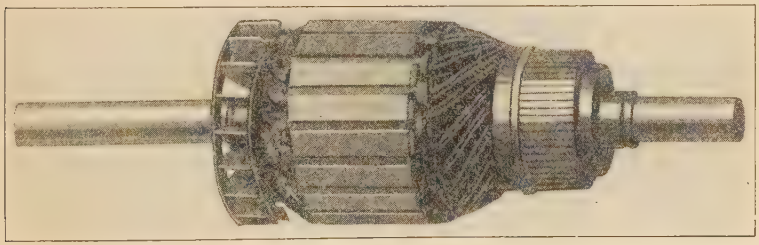


FIG. 54. Armature with curved leads

Courtesy of the Westinghouse Electric and Manufacturing Company

unloaded) it is easy to locate the neutral points by connecting a voltmeter between brushes and shifting the brushes till the position for maximum voltage is found. It is easy to see that if the brushes are placed off from the neutral points, as shown in Fig. 55, the voltage would be less than that along the path

ANB (or *ASB*), being equal to volts *ANC* minus volts *AD* (or volts *DSB* minus volts *BC*).

61. Closed-coil windings on drum armatures. Suppose that in Fig. 52 or 53 the end of a given armature conductor is connected with the beginning of the next, not by threading the wire through the hole of the ring but by carrying it across the back of the armature, then along the face of the armature next to the opposite pole, and then across the front of the armature, as shown in Fig. 56. Then plainly the winding will act the same as before, except that additional electromotive force will be generated, in proper direction, in the portion of the winding transferred from inside the ring to outside. If connections like that of Fig. 56 are substituted throughout for the connections through the hole of the ring, then we have the armature transformed to the drum type, with closed-coil winding; and it will evidently act, in all important particulars, just like the closed-coil winding on the ring armature.

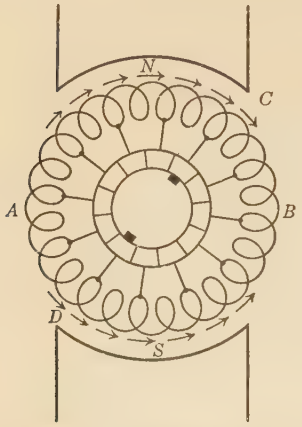


FIG. 55. To illustrate how the voltage between brushes is decreased when they are displaced from the neutral points

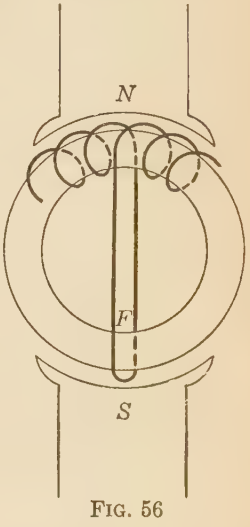


FIG. 56

For the majority of purposes, both in theory and in operation, it is unnecessary to consider whether an armature is of the ring type or of the drum type: in their broader aspects the behavior of the two types is identical.

62. Armature windings for multipolar fields. If the attempt were made to use the drum armature shown partly developed in Fig. 56 with a four-pole field, it would fail to act properly, since the return connection *F*, which ought to come next to a south pole, would in fact be next to a north pole and accordingly would generate electromotive force in the wrong direction. To

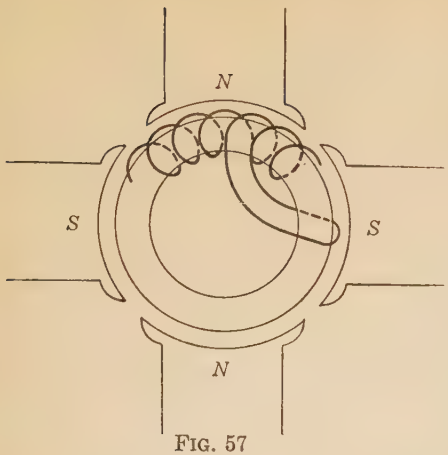


FIG. 57

make a drum winding which will work with a field of four poles, it would be necessary to place the return connection next to a south pole, as shown in Fig. 57. The arrangement of the complete winding for four poles is shown in Fig. 58, which uses the diagrammatic scheme explained in Figs. 47 and 48.

Fig. 58 may be considered typical of multipolar drum windings in general. Drum windings are also used which differ in various respects from that of Fig. 58; some of these are illustrated and briefly explained in Figs. 59–61.

One of the few particulars in which ring windings act quite differently from drum windings is that the same ring armature can be used with a field of any number of poles, whereas a drum armature (as brought out above) will not work with a field having a different number of poles from that for which the armature is wound. Let the ring armature of Fig. 53 be used, as shown in Fig. 62, with a field of four poles instead of two. The only difference in its behavior is that electromotive force is now induced in four sections of the winding, next to each of the four poles, instead of in two only. There are accordingly four neutral points instead of two, and four brushes are required; two of the brushes, *A* and *C*, are at the same potential,

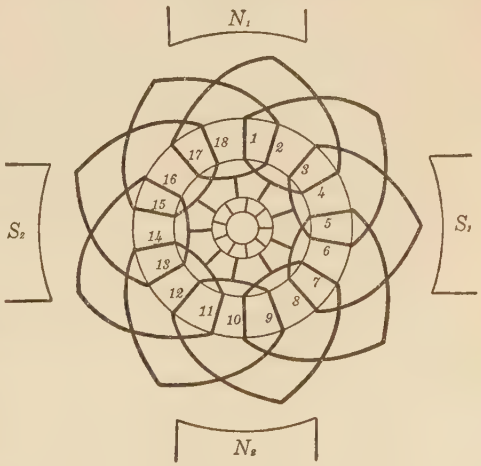


FIG. 58. Four-pole drum-armature winding

Lap winding

and the other two, B and D, are at a potential higher by the electromotive forces generated in the paths AN_1B, AS_2D, CS_1B, and CN_2D, all of which should be equal.* Accordingly brushes B and D should be connected in parallel with the positive side of the external circuit, and brushes A and C with the negative side, as shown; and the armature will then work satisfactorily with a four-pole field. By providing a proper number of brushes it can be made to work equally well with a field of any other number of poles.

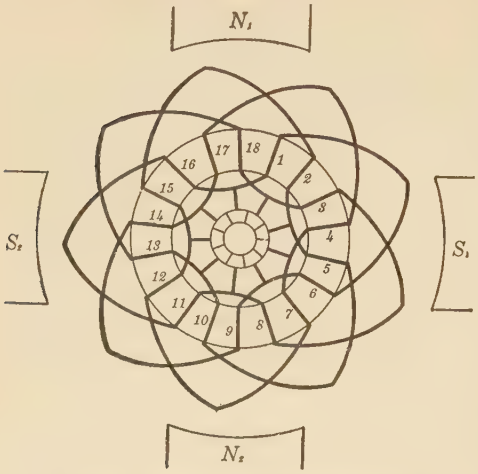


FIG. 59. Four-pole drum-armature wave winding

In this winding the back of conductor No. 1 is connected to the back of conductor No. 6 just as in Fig. 58, but the front of No. 6 is not returned to No. 3 (next to pole N_1) as in Fig. 58, but instead is carried to No. 9 (next to pole N_2)

63. Construction of commutators. The construction of a commutator calls for a number of conducting bars insulated from one another and from the shaft and mounted on the shaft.

The bars are wedge-shaped pieces of hardened copper, usually having about the form shown in Fig. 63. They are held in place by means of rings at the ends, which engage with the grooves of the bars, as shown in Fig. 64, a layer of insulation being placed between the rings and the bars.

Since sparking sometimes occurs at the commutator, it is essential that the insulating material between the bars should be fireproof. Since some wear occurs where the commutator slides past the brushes, it is desirable that the insulation should not differ greatly in hardness from the copper bars. The insulating material which best meets these conditions is mica, and it is the one generally used between commutator bars.

* They will be exactly equal if the magnetic flux at each pole exactly equals that at the other poles, a condition which is approximated but not perfectly realized in practice. While fundamental principles require that as much flux should leave the closed winding as enters it, it does not follow that the flux entering *each* south pole must equal that leaving *each* north pole, but merely that the sum of the fluxes entering all south poles must equal the sum of those leaving all north poles.

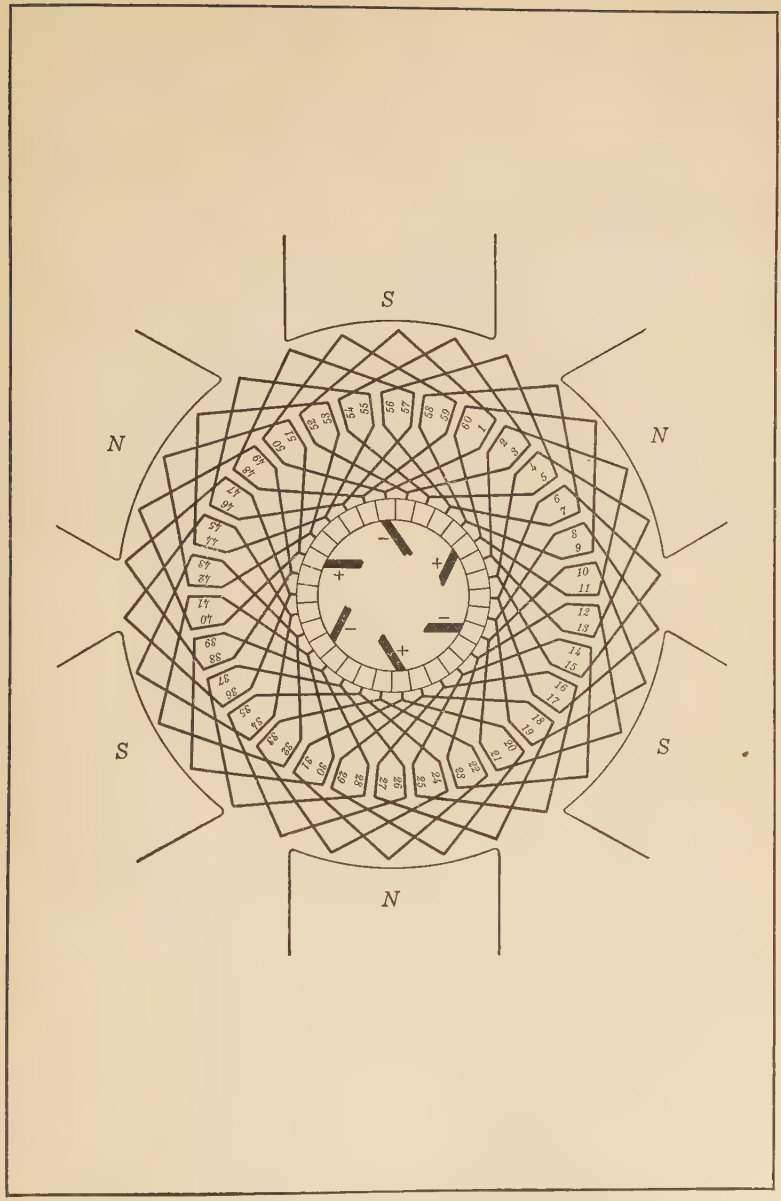


FIG. 60. Six-pole lap winding

Just like Fig. 58, except for the greater number of poles, conductors, commutator bars, and brushes. There are six paths from the positive to the negative terminal

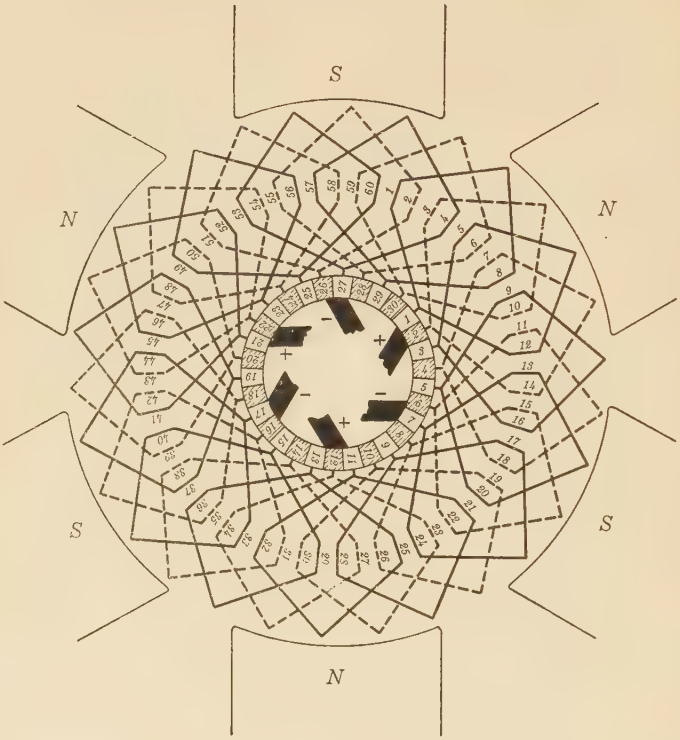


FIG. 61. Doubly reëntrant winding

The winding of this figure is equivalent to two separate windings superposed, so that each conductor of one lies between two conductors of the other, and each commutator bar of one between two of the other. Each brush is wide enough to reach from one commutator bar to the second one from it. The number of paths connecting a positive terminal with a negative is evidently twice that of one of the superposed windings by itself

After the commutator has been assembled and mounted on the shaft of its dynamo, its outer surface is made true by turning it down with a turning-tool, applied to it while the machine is rotated.

64. The construction and mounting of brushes. In present practice the brushes used with commutators are always made of carbon. It is possible to obtain carbon of varying degrees of hardness, which permits choice of different qualities of carbon for different conditions.

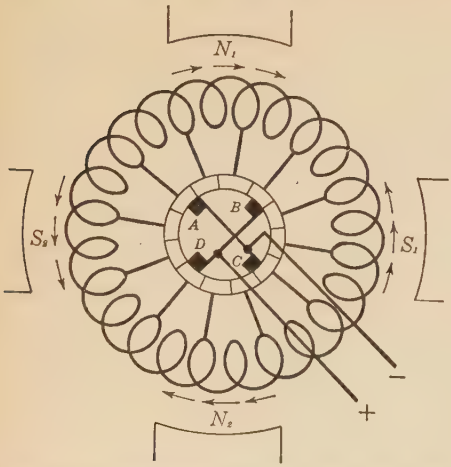


FIG. 62. Ring armature with four-pole field

The cross section of a carbon brush is so designed that when the machine is fully loaded the current density in the brush shall be from 30 to 85 amperes per square inch, according to the quality of carbon used. The thickness of a brush is made not less than that of one commutator bar; often it is made equal to that of one commutator bar, but sometimes it is considerably greater, so that the brush touches three or four bars at the same time.

The force with which the brush presses against the commutator should be great in order to make good contact, but should be small in order to avoid mechanical friction. A compromise has to be effected between these contradictory requirements; it is found empirically that the best results are attained when the pressure of brush against commutator is between 1.5 and 5 pounds per square inch, according to the kind of carbon in the brush. To maintain the proper pressure, the brush is not mounted rigidly, but is placed in a brush-holder (see Fig. 65) in which its fits loosely, while a spring *s* presses against the top of the brush. The pressure from the spring can be adjusted by the handle *h* as the brush wears down.

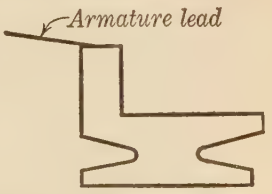


FIG. 63. Commutator bar

There is some variety in the details of brush-holders in use. Fig. 66 shows a type in which the spring is so designed as to give nearly the same pressure when the brush is worn down as before. Figs. 67 and 68 show other types. For machines intended to turn always in the same direction the brushes are often held slanting; for those intended to turn in either direction the brushes are held radially, as in Figs. 66 and 68.

Carbon is a fairly good conductor of electricity, but its hardness and brittleness introduce difficulties in making contact

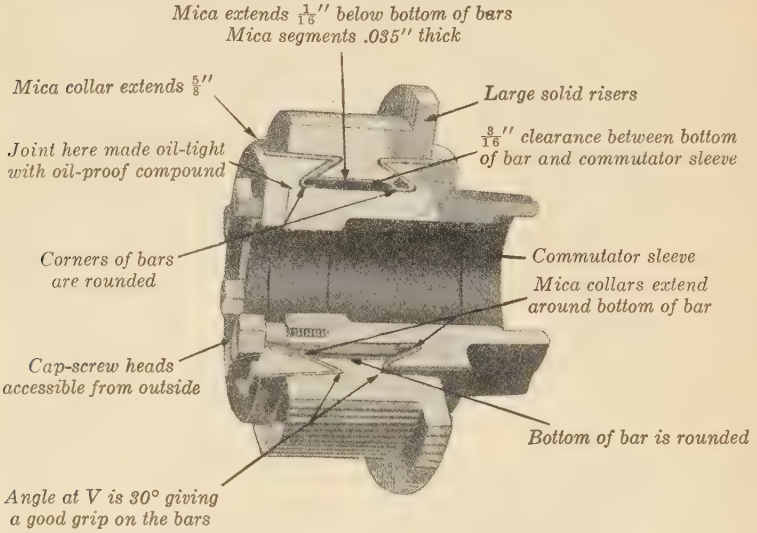

Mica extends $\frac{1}{16}$" below bottom of bars
Mica segments .035" thick
Mica collar extends $\frac{5}{8}$"
Large solid risers
Joint here made oil-tight with oil-proof compound
$\frac{8}{16}$" clearance between bottom of bar and commutator sleeve
Corners of bars are rounded
Commutator sleeve
Mica collars extend around bottom of bar
Cap-screw heads accessible from outside
Bottom of bar is rounded
Angle at V is 30° giving a good grip on the bars

FIG. 64. Sectional view of commutator

Courtesy of the Reliance Electric and Engineering Company

with it. If pressed on a piece of copper, it tends to make contact at only a few points, rather than over an extended surface. To lessen this difficulty where brushes touch commutators, it is customary (except in small dynamos) to use brush sets consisting of several separate small brushes, as shown in Fig. 69, instead of single large brushes.

The connection of the brush with the external circuit is effected by means of flexible stranded wire, as shown in Figs. 65–69. To obtain a good contact between the wire and the brush, the outer end of the brush is copperplated; the wire is then clamped to the copperplated part of the brush or sometimes soldered to

it. The spring might be used to carry current from the brush to the external circuit if it were not for the heating effect of the current, which is likely to draw the temper of the spring.

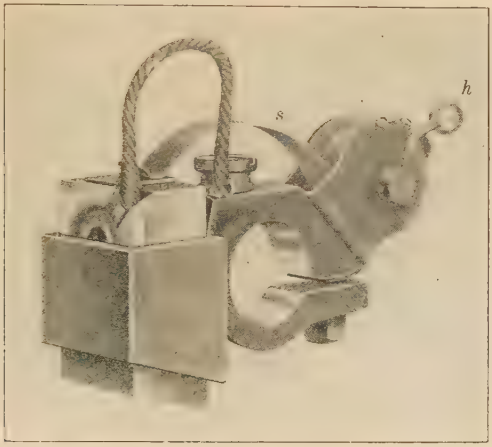


FIG. 65. Brush-holder

Courtesy of the General Electric Company

In earlier times, before the advantages of carbon as a brush material were established, copper brushes were universally

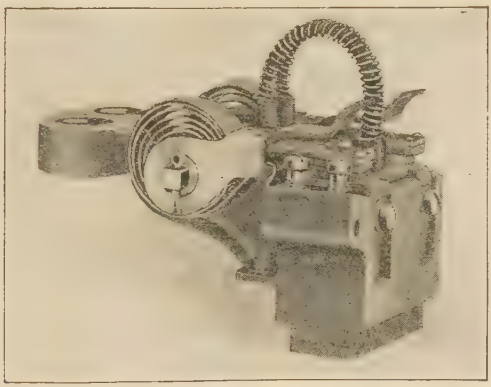


FIG. 66. Brush-holder

Courtesy of the Westinghouse Electric and Manufacturing Company

used; and such brushes are still used to some extent with the slip-rings of alternating-current machinery, although not with

commutators. The copper brush is built up of a number of thin springy strips of copper, the elasticity of the brush itself furnishing the means of holding it against the slip-ring with proper

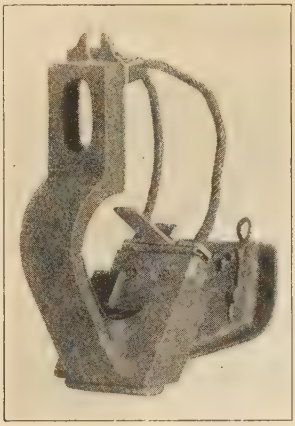


FIG. 67. Another type of brush-holder

Courtesy of the Westinghouse Electric and Manufacturing Company

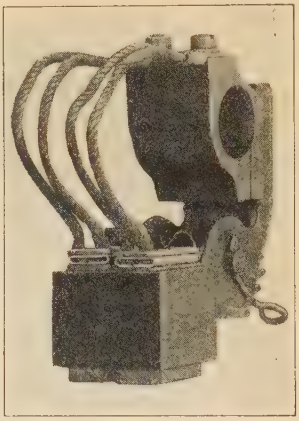


FIG. 68. Still another type of brush-holder

Courtesy of the Westinghouse Electric and Manufacturing Company

pressure. It has been found that the surface under a copper brush is much more likely to roughen as it wears down than the same surface under a carbon brush.

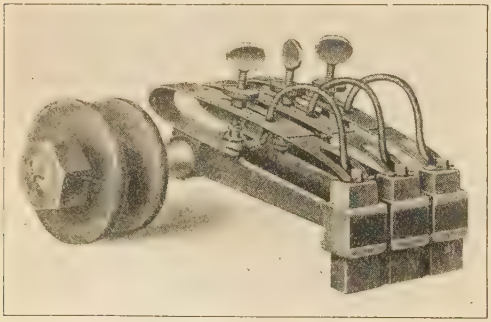


FIG. 69. Brush set, consisting of three separate brushes in one holder

Courtesy of the General Electric Company

65. The mounting of brush-holders. In most direct-current dynamos all the brush-holders are attached to an arrangement known as a *rocker arm* or *rocker ring* (Figs. 70 and 71), which can be rotated about the shaft to any desired position and clamped there. This has the advantage of making it possible to shift all the brushes round the commutator simultaneously.

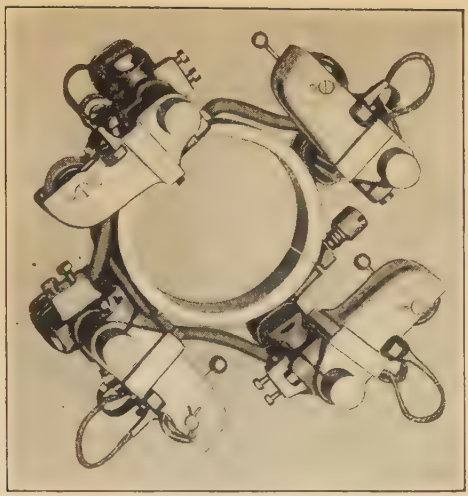


FIG. 70. Rocker ring with brush-holders
Courtesy of the Westinghouse Electric and Manufacturing Company

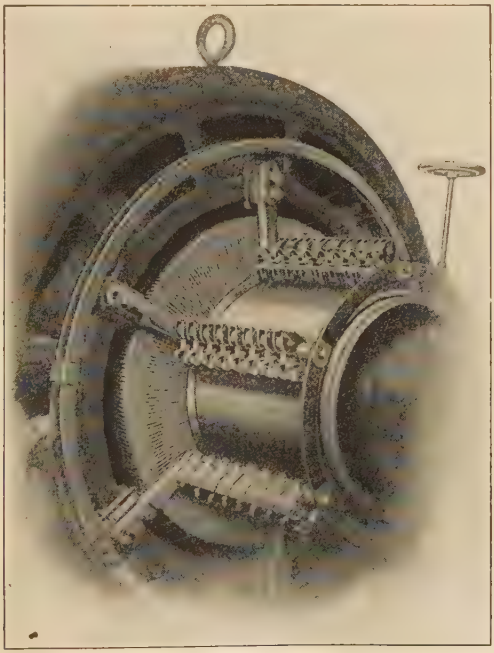


FIG. 71. Rocker ring (not that of Fig. 70) in place, with brush-holders supporting brushes in contact with commutator
Courtesy of the Westinghouse Electric and Manufacturing Company

In some cases, however, the brush-holders are fastened rigidly to the frame of the machine, without provision for shifting the brushes. This is the usual practice in small motors.

66. Sparking at commutators; interpoles; armature reaction. There is a tendency for sparking to take place at the contacts

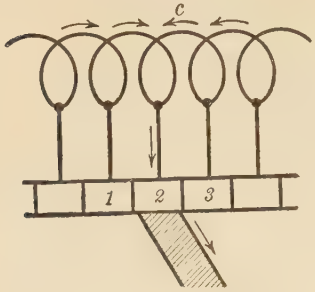


FIG. 72

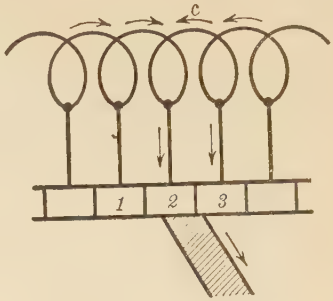


FIG. 73

between brushes and commutator, which is objectionable because it causes excessive wear and roughening of commutator and brushes, introduces fire risks, wastes energy, and heats the commutator (sometimes sufficiently to melt the solder with which the armature leads are attached to it). In early

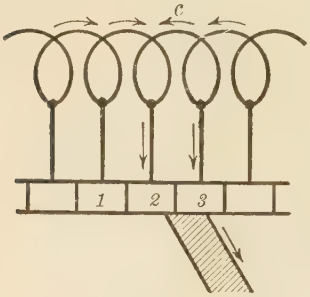


FIG. 74

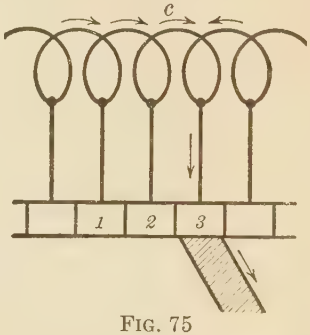


FIG. 75

dynamos much trouble was experienced from this cause, which used to offer one of the most serious difficulties in the way of practical use of direct-current dynamos; but in modern machines it has become possible by skillful design to eliminate this trouble very completely (provided, of course, that the machine is operated properly and is not out of order).

Sparking at the commutator may be due to a number of causes, both in the design and in the operation of the dynamo. It would be beyond the scope of this book to go into the theory of sparkless commutation at all fully, but two of the features of this subject will be briefly discussed here.

1. *Commutation when the brushes are at the neutral points.* Let Fig. 72 represent a portion of an armature winding and commutator with brush. The arrows show the path of the current, which, in every closed-coil winding, enters the commutator lead from two directions. Figs. 73–75 show the same winding at later instants, as the commutator has moved forward under the brush. In Fig. 75 the direction of the current in coil *c* must be the reverse of what it was in Fig. 72; that is, the current in this portion of the armature winding must be reversed during the time the brush passes from one commutator bar to the next — a few thousandths of a second. But on account of the inductance of the coil *c*, the current in it tends to continue in the same direction. This does no harm in Fig. 73; but in Fig. 74, where the brush is just leaving commutator bar *2*, the continuance of current in the same direction in coil *c* causes a spark to pass between bar *2* and the brush. The spark can be prevented if an electromotive force can be generated in coil *c* sufficient to reverse the current in it during the time that the brush passes from bar *2* to bar *3*. This is accomplished in many machines by the use of *interpoles* (also called *commutating poles*). These are small poles placed between the main poles of the dynamo, as shown in Figs. 76 and 77, and so arranged as to apply a magnetic field to those armature coils which are undergoing commutation. The direction and strength of this field are such as to induce in the coils undergoing commutation an electromotive force sufficient to reverse the current in a coil during the time that the brush passes from one commutator bar to the next.

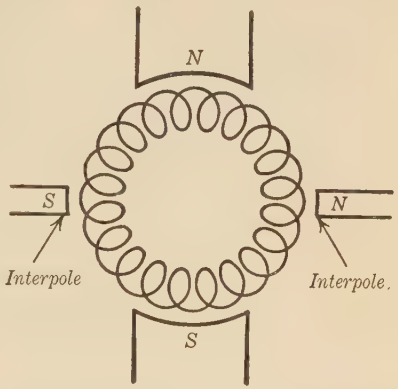


FIG. 76. Diagram to show location of interpoles midway between main poles

2. *Sparking when the brushes are not at the neutral points.* Let Fig. 78 represent a portion of the commutator and armature winding, which is not at a neutral point. Electromotive force is being generated in this portion of the winding. Accordingly, if commutator bars 2 and 3, which are connected to the ends of coil *c*, are short-circuited, a current will be started in the local circuit 2–3–*c*–2. Such short-circuiting takes place every time that in the revolution of the commutator a brush bridges across the insulation between bars 2 and 3. As the resistance of the local circuit 2–3–*c*–2 is very low, the short-circuit current may be large even when the electromotive force induced in *c* is small. As the commutator moves on and bar 2 breaks connection with the brush, this current is broken, resulting in a spark. This effect is, of course, independent of that discussed in the last paragraph, which takes place whether or not the brush is placed at a neutral point.

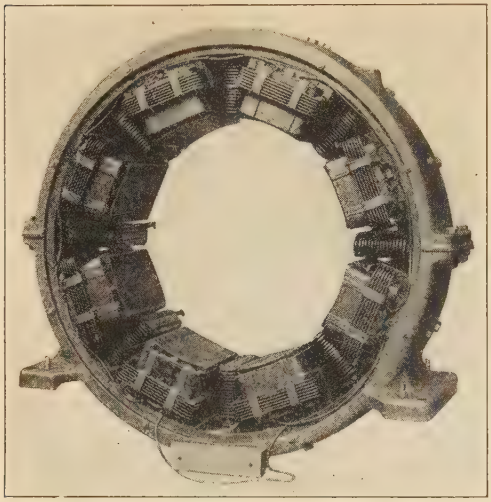


FIG. 77. Frame of eight-pole generator, showing interpoles between main poles

Courtesy of the Westinghouse Electric and Manufacturing Company

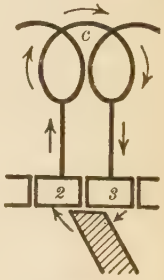


FIG. 78

It might be thought that if the brushes are once set at the neutral points, no trouble could be encountered from this cause; but unfortunately the position of the neutral points tends to shift with the strength of current in the armature, so that they may occupy a different position at full load from that at no load. This is caused by the effect called *armature reaction*, due to the fact that the current in the armature produces a magnetic field (whose strength varies with that

of the armature current) and superposes this field upon that maintained by the field poles, as illustrated by Figs. 79–81.

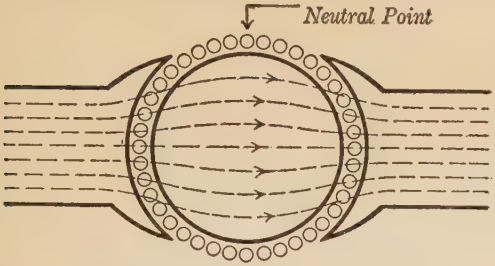


FIG. 79. Magnetic field due to field magnets alone

The neutral points are determined by the positions where the armature conductors are not cutting across any magnetic lines; these positions may be shifted when an additional field is superposed upon the main field.

In early direct-current generators the shifting of the neutral points due to armature reaction was sufficient to necessitate shifting of the brushes whenever any considerable change in the armature current took place. In modern machines the effects of armature reaction have been brought so completely under control by skillful design that it is not usually necessary to shift the brushes when a change of load occurs.

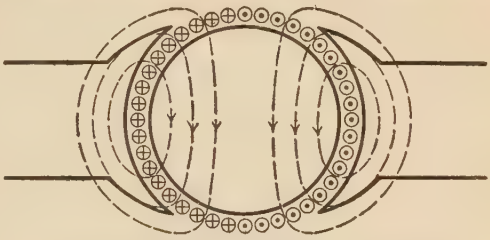


FIG. 80. Magnetic field due to armature currents alone

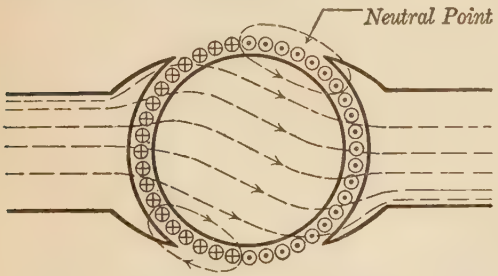


FIG. 81. Resultant magnetic field, due to field magnets and armature currents together

67. Care of commutators. On account of the injury that may be done by commutator sparking, it is necessary in the practical operation of dynamos to watch the behavior of the commutator with somewhat the same vigilance as the lubrication of the bearings. Whenever sparking is observed, steps should be taken to stop it. A few of the most common causes of sparking, with their remedies, are listed below.

1. *Sparking due to faulty operation.* *a.* Brushes not in neutral axis. Shift the rocker arm to the position in which the sparking is a minimum (none at all, if faulty brush position was the only cause of the sparking).

b. Overload. When a machine has been successfully designed for sparkless commutation under its normal running conditions, of course it may not operate sparklessly under the abnormal condition of excessive armature current. The obvious cure is to remove the overload; if this cannot be done at once, partial relief may be obtained by shifting the brushes.

c. Insufficient pressure of brushes. Adjust the springs which press the brushes against the commutator.

2. *Sparking due to faulty condition of the machine.* *a.* Rough or irregular commutator. Through accident, abuse, accumulation of dirt, or excessive wear, a commutator sometimes becomes so rough or irregular that the brushes cannot rest against it with steady pressure during its motion, but shake up and down, with resultant sparking. If the roughness is slight, it may be corrected by sandpapering the commutator (emery paper must not be used, since particles of emery are likely to embed themselves permanently in the copper and cause excessive wear). If the roughness is bad, the only cure is to turn down the surface of the commutator with a turning-tool.

b. Moisture or dirt in the commutator insulation. This may result in more or less completely short-circuiting two or more adjacent bars, or even in short-circuiting the whole path from a positive to a negative brush, thus causing a "flash-over." The commutator should be thoroughly wiped over with a clean dry cloth.

c. Brushes not fitted to the commutator. This is a common condition with new brushes, and tends to correct itself with time. Slight sparking from this cause may be safely ignored. Bad sparking may be corrected or diminished by laying sandpaper between the commutator and the brush (with the sand side next to the brush) and grinding down the face of the brush.

d. Short-circuit, ground, or open-circuit in the armature winding. Repair the winding.

68. Excitation of the magnetic field. The electromagnets that supply the field of a dynamo are excited by passing current through windings surrounding their cores. In a direct-current generator the exciting current may be obtained from the generator itself, in which case the machine is said to be "self-excited"; or it may be obtained from some separate source, such as a battery or an independent generator, in which case the machine is said to be "separately excited." Self-excitation is nearly

always used in actual direct-current generators, and is accomplished by one of three schemes of connections: by shunt winding, by series winding, or by compound winding.

In the shunt-wound dynamo the field winding is connected in shunt across the terminals of the armature, as shown

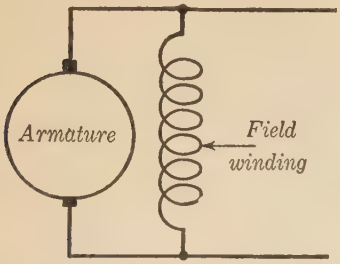


FIG. 82. Armature and field connections for shunt-wound generator

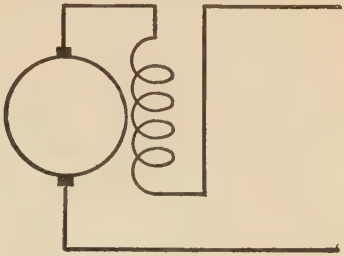


FIG. 83. Armature and field connections for series-wound generator

diagrammatically in Fig. 82; in the series-wound dynamo the field winding is connected in series with the armature, as shown in Fig. 83. The compound-wound dynamo is merely one provided with two field windings: one a shunt winding, and the

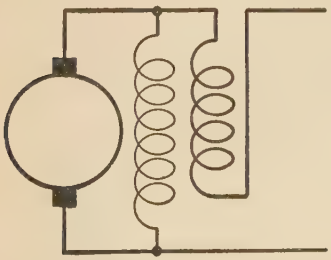


FIG. 84. Armature and field connections for compound-wound generator

Short shunt

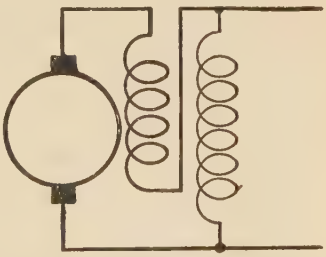


FIG. 85. Armature and field connections for compound-wound generator

Long shunt

other a series winding. The shunt winding may be shunted inside the series winding (Fig. 84), giving a *short-shunt* machine; or outside of it (Fig. 85), giving a *long-shunt* machine.

69. Construction of field windings. In a shunt field winding, in order to save loss of energy, the current permitted to flow should be made as small as other conditions allow; the required

magnetomotive force (which is proportional to the ampere-turns) can still be obtained by using a sufficiently large number of turns. To keep the field current small the field winding must have high total resistance, but compactness requires low *specific resistance* of the material of the winding. These conditions are met in actual shunt field windings by making them (as is always done) of many turns of copper wire, sufficiently small to give the winding the necessary total resistance.

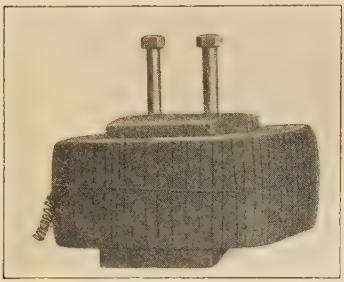


FIG. 86. Field coil on pole core

Courtesy of the Westinghouse Electric and Manufacturing Company

In series windings the current is determined by that flowing in the armature. With full-load current comparatively few turns in the series winding are sufficient to supply all the magnetomotive force desired (up to or beyond that which causes saturation of the magnetic circuit). To save loss of energy the resistance of the series field winding should be made as small as other conditions permit; accordingly series field windings are made of heavy copper wire or bar, and the number of turns is comparatively small.

FIG. 87. Field coil wound with conductor of rectangular cross section

Courtesy of the Westinghouse Electric and Manufacturing Company

In actual dynamos the field winding is distributed symmetrically over all the poles, each pole having a field coil about its core, as can be seen in Fig. 34 or Fig. 77. The various coils of a field winding are connected together in series. A field coil is manufactured by winding insulated wire or bar upon a form of

the same shape as the pole core. When sufficient turns have been wound, the coil is slipped off the form and wrapped over with insulating tape or fabric. It is then slipped bodily on to its pole core, as shown in Fig. 86. After the field is put together, the core is held in place by the pole shoe at one end and the yoke at the other end.

To economize space in field windings, conductors of rectangular cross section are often used, as shown in Fig. 87; and a thin coating of enamel is often used for the insulation about the conductor material. Since the potential difference between adjacent turns is comparatively small, a comparatively weak insulation between turns is sufficient, provided adjacent *layers* are more strongly insulated from one another, and the winding as a whole is well insulated from the cores.

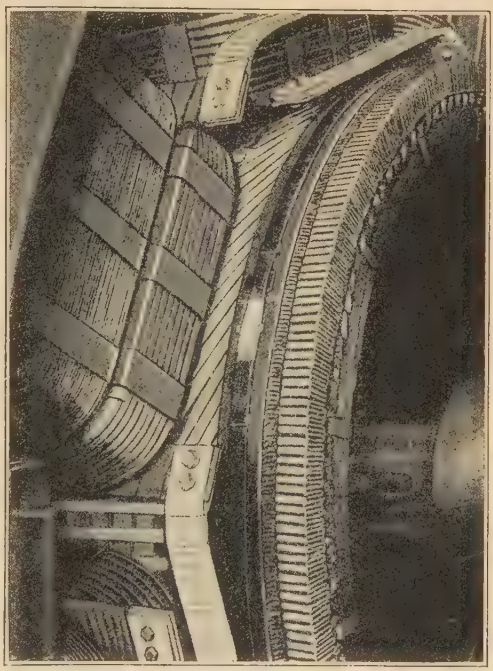


FIG. 88. Series field coil and shunt field coil on the same pole core

Courtesy of the Westinghouse Electric and Manufacturing Company

In compound-wound machines the series field coils are made entirely distinct from the shunt field coils; each magnet core holds one coil of each winding, as shown in Fig. 88.

70. The "building up" of generators. When a self-excited generator is stopped, there is no longer any current to excite its field, which therefore becomes demagnetized. If the demagnetization is complete, evidently no electromotive force can be generated when the armature is again set in motion. But if some little residual magnetic flux exists in the field, — even a minute

amount, — then some little electromotive force will be generated when the armature rotates; this small electromotive force will send a small current through the field winding; this small current should cause an increase in the field flux; this in turn will cause more electromotive force to be generated, thus more current will flow, further increase of flux will take place, and so on until an equilibrium is reached.* This process is called *building up*. It requires an appreciable interval of time, of the order of half a minute.

In practice there is nearly always sufficient residual magnetism to enable a generator to build up, except occasionally in newly built machines or in machines which have met with rough treatment (as in transportation). In such cases some initial magnetism has to be imparted to the field by separate excitation, to start the process of building up.

In starting a generator it is possible that the field current due to residual magnetism may flow in the wrong direction and thus diminish the magnetic flux instead of increasing it, making the machine "build down" instead of build up. A machine which builds up when the armature rotates in one direction will always "build down" when the armature rotates in the opposite direction. This is because reversing the motion reverses the direction of the electromotive force and therefore the direction of the field current also. The remedy is to disconnect the field and reverse its connections to the armature.

Reversing the direction of the residual magnetism (as sometimes happens from the accidental occurrence of some powerful outside magnetic field) does not prevent a generator from building up, but causes it to build up with reversed polarity; that

* The point of equilibrium, for a shunt machine, can best be found graphically. In Fig. 89 the ordinates of curve A represent the terminal electromotive force observed on open circuit when the field is separately excited (the armature rotating at its proper speed), and the abscissas represent the field current. In curve B the abscissas also represent the field current, and the ordinates represent the values $R_f I_f$ (field resistance × field current), or the electromotive force required to drive the field current through the field resistance. Where these two curves intersect is evidently the point of equilibrium. The machine builds up until its terminal electromotive force equals that for the point of intersection of curves A and B, and no further.

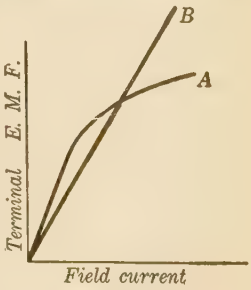


FIG. 89

is, the terminal which was the positive one before the reversal becomes the negative one after the reversal.

The failure of a generator to build up is a common trouble, and may be due to a good many causes, of which the commonest are mentioned here, approximately in the order of the frequency of their occurrence. (It is assumed that the machine is not out of order with broken field windings or similar defects, but that the trouble comes from faulty operation.)

1. Reversed field connections, as just discussed.

2. Brushes displaced far from the neutral axis, thus receiving only a small fraction of the volts generated.

3. In shunt machines too high field resistance (due often to dirty contacts with the field circuit or at the commutator surface).

4. An open circuit (or very high-resistance circuit) across the terminals of a series-wound generator, or a short circuit (or very low-resistance circuit) across the terminals of a shunt-wound generator. The series machine will not build up unless an appreciable current can flow in the external circuit, and the shunt machine will not build up unless an appreciable difference of potential (which a short circuit prevents) can exist at its terminals and drive current through the field winding.

5. Too weak residual magnetism, as discussed previously.

71. Fundamental equations of direct-current dynamos. It was pointed out in Chapter II, sections 37 and 38, that the fundamental principles of action of the dynamo (whether operated as generator or as motor) are contained in the two general physical laws

$$E = lBv \qquad (19)\ (35)$$

and

$$F = lBI, \qquad (9)\ (36)$$

where the symbols have the same meanings as in sections 37 and 38.

In studying direct-current dynamos it is convenient to use some equations derivable from these principles and true for all direct-current dynamos * but not applicable to alternating-current machinery.

1. *Equation for electromotive force.* First an equation for the electromotive force of the direct-current dynamo will be derived.

Let Z be the total number of conductors on the outside of the armature, and let p' be the number of paths in parallel in the armature winding between the positive terminal of the

* Except homopolar dynamos, which are very rarely met with.

dynamo and the negative terminal. Then $\frac{Z}{p'}$ is the number of conductors in series in each path; and the total voltage E, generated in the armature, is $\frac{Z}{p'} \cdot e_{av}$, where e_{av} means the average of the electromotive forces in the various single conductors in the path. According to (35), e_{av} is equal to $lB_{av}v$, where B_{av} is the average of the flux density across which a conductor cuts* in the course of a revolution; l and v are constant. Let n be the number of revolutions of the armature per second, and r its radius; then $v = 2\pi rn$, and

$$e_{av} = lB_{av} \cdot 2\pi rn. \tag{37}$$

Let Φ be the total magnetic flux from one pole, and let p be the number of poles; then $\frac{2\pi r}{p}$ is the length of armature periphery belonging to each pole, and B_{av} equals Φ divided by the area $l \cdot \frac{2\pi r}{p}$. Substituting this value of B_{av} in (37), we have

$$e_{av} = p\Phi n \tag{38}$$

and

$$E = \frac{Z}{p'} \cdot p\Phi n. \tag{39}$$

This equation holds true, whether the dynamo is operated as a generator or as a motor. When operated as a generator, E drives the current (if any); when operated as a motor, E opposes the current, and is called the *counter-electromotive force.* In both cases p, p', and Z are fixed by the construction of the machine, whereas Φ and n may vary during its operation.

If Φ is expressed in maxwells (or lines) as usual, E comes out in abvolts. Reducing to volts, we have

$$E \text{ (volts)} = \frac{Zp}{p' \times 10^8} \Phi n. \tag{40}$$

This, or its equivalent, equation (39), is the fundamental equation for the electromotive force of the direct-current dynamo.

It should be clearly understood that E means the electromotive force *induced in the armature,* and not the electromotive force appearing at the terminals of the machine. The latter will differ from E whenever current is flowing in the armature,

* That is, the average of the component of flux density perpendicular both to the conductor and to the direction of its motion.

since some electromotive force is required to drive the armature current through the resistance of the armature. Let I_a be the armature current* in amperes, and R_a the resistance of the armature between terminals† in ohms. Then, by Ohm's law, the "armature drop," or voltage required for maintaining the armature current through armature resistance, is $R_a I_a$; and the volts E_t appearing across the terminals must equal the induced electromotive force diminished (or increased) by this armature drop, or

$$E_t = E \text{ (volts)} \mp R_a I_a$$
$$= \frac{Zp}{p' \times 10^8} \cdot \Phi n \mp R_a I_a. \tag{41}$$

Evidently the minus sign is to be used for a generator, since in this case E must supply the armature drop as well as the terminal voltage; and the plus sign is to be used for a motor, since in this case E_t, maintained from some outside source, must supply the armature drop as well as overcome E.

R_a is fixed in the construction of the machine (except as its value varies slightly with the temperature), whereas I_a is variable during its operation.

2. *Equation for torque.* An equation will now be derived for the torque exerted about the armature shaft when current flows in the armature.

The contribution made by one armature conductor to the total torque is, by the definition of torque, equal to Fr, or, by equation (37), $lBI \cdot r$, where r is the radius of the armature. Since there are Z conductors in all, and since I, the current in one conductor, equals $\frac{I_a}{p'}$, we have, for the total torque T,

$$T = Z \cdot lB_{av} \frac{I_a}{p'} \cdot r, \tag{42}$$

* That is, the total current entering or leaving the armature, not the current flowing in one armature conductor. Since there are p' paths between brushes, the current in one conductor equals $\frac{I_a}{p'}$.

† If R stands for the resistance of the armature winding, beginning at one point and passing through the whole winding back to the same point, then the resistance of any one of the paths through the armature is $\frac{R}{p'}$; and since all these paths are in parallel, the resistance R_a of the whole armature between terminals is $\frac{R}{(p')^2}$.

where B_{av} is the average value of B for the different conductors. Substituting for B_{av}, as before, its equal $\frac{\Phi p}{2\pi rl}$, and rearranging the terms, we have

$$T = \frac{1}{2\pi} \cdot \frac{Z}{p'} \cdot p\Phi I_a. \tag{43}$$

This equation holds true, whether the dynamo is operated as a generator or as a motor. When operated as a generator, T resists the motion and must be overcome by an outside source of mechanical power; when operated as a motor, T maintains the motion (if any). In both cases Z, p', and p are fixed by the construction of the machine, whereas Φ and I_a may vary during its operation.

T will come out in dyne-centimeters if Φ is expressed in maxwells (or lines) and I_a in abamperes. It is common in this country to express torque in terms of pound-inches. Since 1 pound-inch $= 1.13 \times 10^6$ dyne-centimeters, and 1 ampere $= 0.1$ abampere, we have

$$T \text{ (pound-inches)} = \frac{\frac{1}{2\pi} \cdot \frac{Z}{p'} p\Phi \frac{I_a(\text{amperes})}{10}}{1.13 \times 10^6}$$

$$= 1.41 \cdot \frac{Zp}{p' \times 10^8} \cdot \Phi I_a. \tag{44}$$

This, or its equivalent, equation (43), is the fundamental equation for the torque in the direct-current dynamo.

It should be clearly understood that T means the torque *developed in the armature*, and not the torque appearing at the pulley of the machine. The latter will differ from T by an amount T_f necessary for overcoming friction (and other resisting forces). Denoting the torque appearing at the pulley (or other mechanical transmission device) of the dynamo by T_t, we have

$$T_t \text{ (pound-inches)} = T \pm T_f$$

$$= 1.41 \cdot \frac{Zp}{p' \times 10^8} \cdot \Phi I_a \pm T_f. \tag{45}$$

Evidently the plus sign should be used for generators, and the minus sign for motors.

Equations (40) and (44) are of such frequent use that the student is advised to memorize them, as well as to study their derivation.

It is noteworthy that the expression $\frac{Zp}{p' \times 10^8}$ (which contains all the terms determined by the construction of the machine as distinguished from the manner of operating it) occurs both in equations (40) and (44). Using the abbreviation Z' to stand for this expression, equations (40) and (44) can be simplified to

$$E \text{ (volts)} = Z' \cdot \Phi n, \qquad (40a)$$

and

$$T \text{ (pound-inches)} = 1.41\ Z' \cdot \Phi I_a, \qquad (44a)$$

where Z' is a constant belonging to the particular machine under consideration.

PROBLEMS

1. A certain small direct-current generator has 31 commutator bars and runs at 1400 revolutions per minute. How much time is available for reversing the current in an armature coil during commutation? What is the average rate of change of current in an armature coil during commutation at full load, the full-load armature current being 18 amperes, and the armature having two paths between brushes?

2. What should be the resistance of the shunt field of a 600-volt 150-kilowatt generator if the shunt field excitation is to consume 3 per cent of the output of the generator?

3. Find the terminal voltage of the generator of Problem 1 at full load, the number of armature conductors being 248, the number of poles 4, the magnetic flux per pole 1,200,000 maxwells, and the armature resistance 0.5 ohm.

4. Compute the torque required to drive this machine at full load, there being required a torque of 12 pound-inches for overcoming friction.

5. Suppose the terminals of this generator are accidentally short-circuited while the machine is running. Assuming that the speed is unchanged but that the flux per pole drops to 700,000 maxwells under these conditions, find the armature current and the torque required to keep the generator in motion.

6. In designing a certain 25-kilowatt 220-volt generator the shunt field winding is to be permitted to take 2 per cent of the output of the generator. Find the current in the shunt field winding and its resistance. To produce the required magnetizing effect, the winding must supply a magnetomotive force of 5000 ampere-turns. Find the number of turns and the length and size of (copper) wire required, the average length of a turn being 18 in. (see table on copper wire, p. 337).

CHAPTER V

TRANSMISSION AND DISTRIBUTION OF ELECTRICAL ENERGY IN DIRECT-CURRENT SYSTEMS

72. Advantage of high voltage in electrical transmission. In Fig. 90 let G represent a generator, and let L represent a (concentrated) load, such as a motor or a set of lamps, which is to receive power at some distance from the generator. G and L must be made parts of the same circuit, which is accomplished

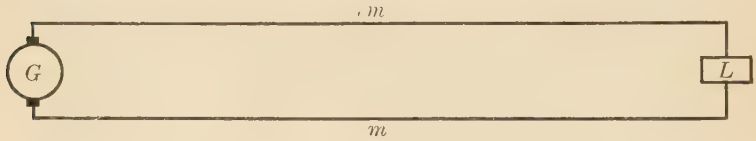


FIG. 90. Simple transmission line

by means of the mains m, m (one main, however, may be replaced by the ground). Let V_G be the voltage at the terminals of the generator, and let I be the current in amperes; then power is delivered to the line at the rate of $V_G I$ watts. V_L, the voltage at the load, will be less than V_G by the amount RI, where R is the resistance of both mains. Accordingly the power delivered to L is $V_L I = (V_G - RI) \cdot I = V_G I - RI^2$; or the power delivered by the line is less than that put into it, by the amount RI^2, which is the line loss.

It is evidently important to make the line loss small. It is always possible, theoretically, to make it small by making R small; but this might require the use of such large and heavy conductors as to be prohibitively expensive in practice, especially in long lines.

The line loss can also be made small if the current can be made small. In many cases it is just as satisfactory to have the power $V_L I$ delivered by a small current at high voltage as by a large current at low voltage. The power, or rate of delivery of electrical energy, is the same either way, but in the former

case the line loss is much less for the same line resistance. Or, if we permit the same line loss, higher-resistance mains (that is, smaller ones) can be used, with consequent economy in cost of construction.

Unfortunately it is not always satisfactory or feasible to receive electrical energy at a voltage as high as would be desirable from the point of view of economical transmission. For household appliances safety requires that the voltage be kept low; incandescent lamps cannot easily be constructed for high voltages; it is difficult to construct generators and motors for more than a few thousand volts.

In alternating-current systems the difficulty due to this situation is overcome by using high voltage for transmission and transforming to lower voltage at the point of delivery of the energy, but in direct-current systems no sufficiently convenient devices for transforming the voltage are available. Accordingly direct-current systems are pretty much limited to situations in which the distance of transmission is small, not more than a few miles; for long-distance transmission alternating currents are almost always used.*

To illustrate the difficulty of transmitting electrical energy at low voltage, consider what would happen if transmission should be attempted at 110 volts (generator voltage). Suppose the generator supplied the comparatively small power of 110 kilowatts; the current would then have to be 1000 amperes. Suppose copper mains were used with a cross section of 1,000,000 circular mils; the resistance of both mains per mile would then be approximately 0.11 ohm. Accordingly the voltage drop in a single mile would be 110 volts, and the whole output of the generator would be consumed in line loss in one mile, and no power would be available for useful purposes one mile from the generator. The mains assumed would cost, in 1927, about 40 cents per foot, or, for a mile of both mains, about $4000. To transmit the 110 kilowatts one mile with a line loss of 10 per cent (generator voltage being 110), the mains would have to have ten times the cross section assumed above, and cost about $40,000.

73. Systems of distribution. When electrical energy is to be distributed to a number of separate appliances rather than transmitted to a single load as in Fig. 90, there are two methods

* But there are a few long-distance direct-current systems in use in Europe, with voltages above 50,000, transmitting over distances greater than 100 miles. The system used is a constant-current system, developed by Thury.

in use in direct-current systems: the constant-voltage system and the constant-current system. Of these the constant-voltage system is much more widely used. It is the system used in house-wiring and for all ordinary types of motors, including railway motors.

74. The constant-voltage system. In the constant-voltage system (also called the parallel system) the lamps, motors, and

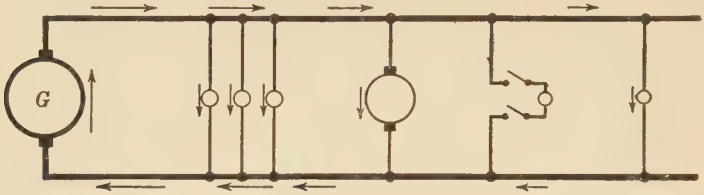


FIG. 91. Constant-voltage system of distribution

other appliances which receive energy are all connected in parallel across mains between which the voltage is maintained as nearly constant as practical operation permits. Fig. 91 shows such a system in its simplest form, the arrows showing

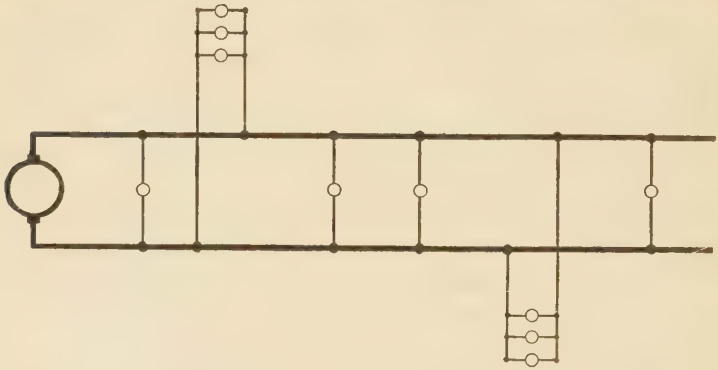


FIG. 92. Constant-voltage system of distribution, with branches

the paths followed by the current. To disconnect any appliance a switch is opened, thus stopping the flow of current through it; in practice double-pole switches are nearly always used, so as to disconnect from both mains at the same time. Branch lines of mains are led off as shown in Fig. 92.

Variations in the rate of output of electrical energy (EI) are accomplished by variation of the current delivered by the

generator, its electromotive force remaining the same; when unloaded, the generator delivers no current, but maintains the voltage across mains.

Fig. 93 shows the arrangement when more than one generator supplies energy to the system from the same central station. In this case the terminals of all the generators are connected

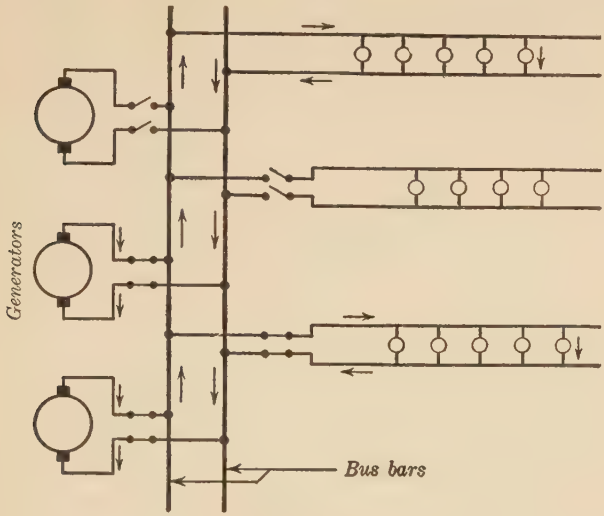


FIG. 93. Constant-voltage system of distribution, with more than one generator

to "bus bars," which are heavy copper bars placed behind the station switchboard, and the various lines of mains are led out from the bus bars.*

In electric-railway circuits the trolley wire (or third rail) forms one main, and the rails (and ground) form the other.

75. Standard voltages. In this country the following standard voltages are in common use for the distribution of direct-current electrical energy by the constant-voltage system: for small isolated systems (domestic lighting plants, car-lighting plants, etc.), 32 volts between mains; for incandescent lighting and household appliances, 115 volts; for motors, 115, 230, and, less often, 550 volts; for street-car circuits, 550 volts. For

* In most actual station switchboards there are more than two bus bars, Fig. 93 being a simplification of actual arrangements.

longer lines of electric railways the voltage in direct-current systems is usually 1200, 2400, or 3000.

It is not practically possible to maintain the voltage absolutely constant at all times and throughout all parts of a distribution system. The service standards which have been set up by public-utility commissions usually specify the maximum permissible variation above and below the nominal value of the voltage. This is usually from 3 to 5 per cent on lighting circuits during lighting hours, and 10 per cent for all other conditions.

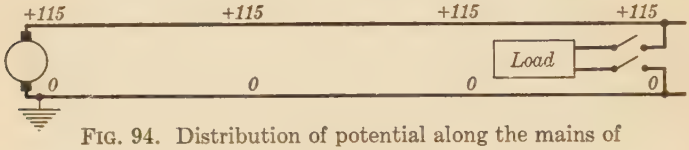


FIG. 94. Distribution of potential along the mains of an unloaded line

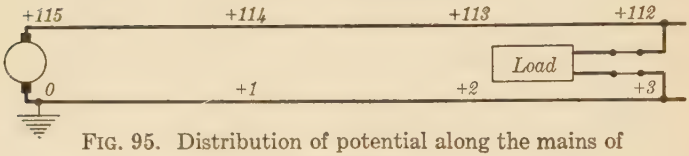


FIG. 95. Distribution of potential along the mains of a loaded line

The variation is caused partly by fluctuation in the voltage maintained at the station bus bars, but much more by voltage drop in the line.

On some systems the nominal voltage differs a little from the standards mentioned above, which are those used by manufacturers of electrical equipment. For example, on lighting circuits, the nominal voltage is sometimes 110 volts or 117 volts, or some other figure not widely different from 115.

76. Voltage drop in distribution circuits. As long as there is no load across a pair of live mains, the potential of each main is the same throughout its length, as in Fig. 94; but as soon as the line is loaded, a difference of potential must exist between any two points of the same main, equal by Ohm's law to RI, where R is the resistance between the two points, and I is the current. The station voltage being unchanged, this leads to a distribution of potential along the line like that shown in Fig. 95.

The voltage across mains accordingly decreases with increasing distance from the station.

For a given current the voltage drop can be kept as small as we please by making the resistance of the line sufficiently small; that is, by using sufficiently heavy mains. But when the line is long, the cost of such mains becomes prohibitive, and a different system of transmission (probably an alternating-current system) has to be used. The constant-voltage direct-current system is suitable only when the distances of transmission are small.

77. The Edison three-wire system. The constant-voltage system of distribution permits the use of several mains, each maintained at a different (approximately) constant potential, instead of only two. A common arrangement for general distribution is the Edison three-wire system, shown in Fig. 96. The middle, or "neutral," wire is usually grounded, thus making its potential 0 and that of the two outer wires +115 and −115 respectively.* When the load is equally divided between the two sides, — a condition at least approximated in actual operation, — no current flows in the middle main,† and the current in each of the outer mains is only half as great as if the whole load were connected in parallel across two mains at 115 volts. Accordingly smaller mains can be used than in the two-wire system.‡

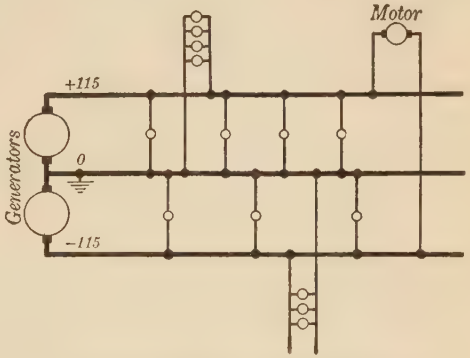


FIG. 96. Edison three-wire system of distribution

* Of course slight deviations from all these values occur by reason of voltage drop when the line is loaded.

† Except locally, to pass from a load on one side to the next load on the other side.

‡ To transmit the same power with the same voltage drop, three-wire mains need to have only one fourth the cross section of two-wire mains. The current is half as great; the voltage between outside mains is twice as great, thus permitting twice as great voltage drop, of which one half would appear on each side; accordingly, for the same voltage drop on one side, the resistance of the mains may be four times as great, or their cross section one fourth as great.

78. Distribution networks. For general distribution in cities the arrangement is somewhat different from that shown in Fig. 92, although on the same principle. A pair of mains (or set of three mains, when the Edison system is used) is run along each street, each main being connected at each corner with the corresponding main of the intersecting street, so that a complete network is formed. At certain points (called centers of distribution) a pair (or set) of heavy conductors called "feeders" is connected with the mains and brings current to them from the central station. The arrangement is illustrated in Fig. 97, where, for simplicity of drawing, a pair (or set) of mains is represented by a single full line, and a pair (or set) of feeders is represented by a single dotted line.

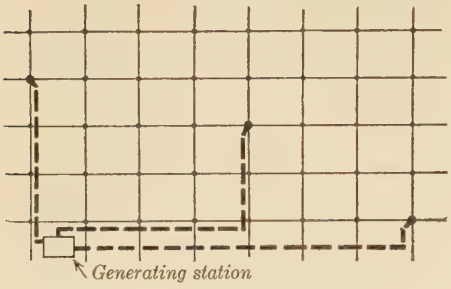


FIG. 97. Distribution network

79. The constant-current system. In the constant-current system of distribution (also called the series system), which has been the standard system for arc-lamp circuits, the various lamps and other appliances are all connected in series in a circuit in which the current is kept as nearly constant as practical operation permits. Fig. 98 shows the arrangement. To cut any lamp out of service, it is short-circuited, thus causing the current to flow round it rather than through it, after which it may be completely disconnected.

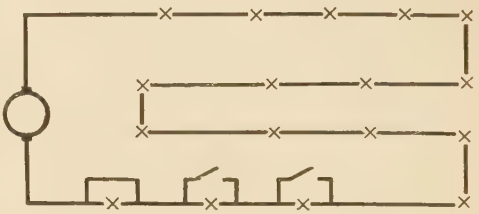


FIG. 98. Constant-current system of distribution

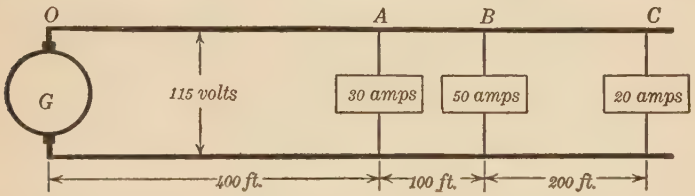
Variations in the rate of output of electrical energy (EI) are accomplished by variation in the electromotive force of the generator, the current delivered remaining the same. When unloaded, the generator gives no terminal voltage and runs on short circuit.

If two or more generators contribute energy to the same circuit, they are placed in series with each other.

The constant-current system permits the economies of high-voltage transmission without requiring a high voltage across the terminals of each appliance placed in the circuit; but since the main is (for the most part) at a high difference of potential from the ground, considerations of safety prevent the use of the system for the delivery of electrical energy within ordinary buildings.

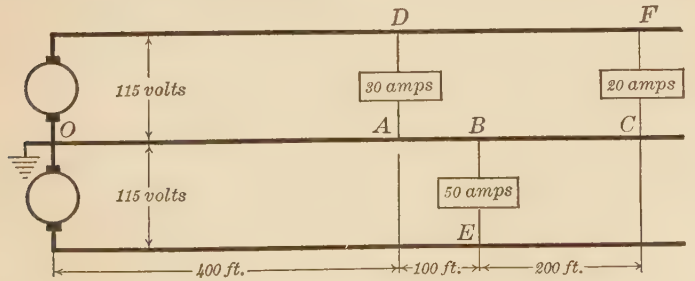
PROBLEMS

1. The figure below represents an arrangement for supplying power to three loads requiring 30, 50, and 20 amperes respectively. Each main is of No. 0 wire, having a resistance of 0.1 ohm per 1000 feet.



Find (*a*) the current in each section *OA*, *AB*, and *BC* of the main; (*b*) the voltage across the mains at *A*, *B*, and *C*; (*c*) the power loss in the mains.

2. The figure below represents the Edison three-wire system for supplying energy to the same loads as in Problem 1. Each main is now of No. 5 wire, having a resistance of 0.3 ohm per 1000 feet.



Find (*a*) the amount and direction of current in each section *OD*, *DF*, *OA*, *AB*, *BC*, and *EO* of the system; (*b*) the potential at the points *A*, *B*, *C*, *D*, *E*, and *F*, the point *O* being grounded; (*c*) the power loss in the mains.

3. With the arrangement used in Problem 1, find the voltage across mains at B: (a) when the load at A is turned off (the loads at B and C remaining connected), (b) when the load at C is turned off (the loads at A and B remaining connected), and (c) when the loads at A and C are turned off (that at B remaining connected).

4. With the arrangement used in Problem 1, it is required to select a size of wire for the mains such that the voltage between mains will lie between 110 and 115 at all points and at all times, no matter what combination of the three loads may be turned on. Find the smallest size of wire which will accomplish this.

SUGGESTION. The lowest voltage will occur at C when all three loads are turned on. The voltage drop from O to C is then 5 volts for the two mains. This drop can be expressed in terms of the currents, the distances, and the resistance r per 1000 feet of the wire. After solving for r, the corresponding size of the wire can be found from the table on page 337.

CHAPTER VI

OPERATING CHARACTERISTICS OF DIRECT-CURRENT GENERATORS

80. Characteristic curves. For studying the suitability of different types of generators for different services it is convenient to make use of curves in which the abscissas represent the current output of the generator, and the ordinates its terminal voltage. Such curves are called *characteristic curves.**

According to equation (41) the value of the terminal voltage varies not only with the current output I, but also with the speed of rotation n and with the magnetic flux per pole Φ. In actual generators, however, n is held as nearly constant as possible by the governor of the prime mover that drives the generator.† Throughout this chapter the speeds of generators are always assumed constant. Φ is then a function of I; so that under these conditions a definite value of the terminal voltage is determined by each value of I.

81. The separately excited generator. Fig. 99 shows the characteristic curve for a separately excited generator. OP is the terminal voltage on open circuit; with increasing current the voltage falls off a little, rated full-load current being reached at F; with further increase of current output the voltage falls off rather more rapidly. It would finally reach zero if the terminals were short-circuited, under which condition the current OM would be the maximum possible for the machine

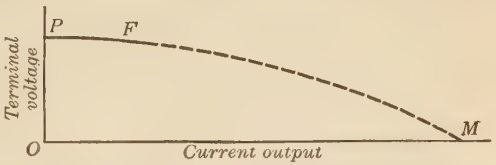


FIG. 99. External characteristic curve of a separately excited generator

* More strictly, *external characteristic curves*, the term *characteristic curve* being in use also for certain other curves descriptive of the operation of generators.

† Exceptions in which n is not constant occur in those generators which are driven from the axles of railroad cars, for car lighting; or by variable-speed engines, as on automobiles.

in question; but the dotted portion of the curve could not be realized in practice without injury to the machine from excessive current.

The falling off of the terminal voltage with increasing current output is caused (1) by the increase in armature drop ($R_a I_a$ of equation (41)) and (2) by armature reaction, the magnetic flux due to armature current being superposed on the (in this case constant) flux contributed by the poles, and the total resultant flux (Φ of equation (41)) being less as the current is greater. The second effect is negligible in practice at light loads, so that the curve starts as a straight line; but the effect of armature reaction becomes very appreciable at heavy loads.

Since the voltage changes very little with change of load, this type of generator might be used for supplying energy to a constant-voltage distribution system; but it would be inconvenient, since it requires a separate source of supply for exciting its field.

82. The shunt generator. The characteristic curve for a shunt-wound generator is shown in Fig. 100. In this case the terminal voltage falls off more rapidly with increasing current than in the case of the separately excited generator. This is because, in addition to the causes mentioned above as effective in the separately excited generator, the flux due to the field poles decreases — the value of this flux being determined by the field current, which is equal to the (varying) terminal volts divided by the field resistance.

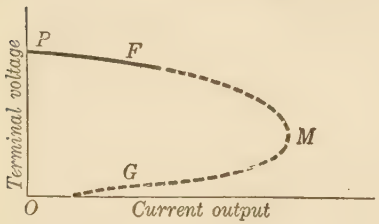


FIG. 100. External characteristic curve of a shunt-wound generator

In Fig. 100 OP represents the voltage on open circuit, and F is the point at which rated full load is reached. If the current is increased beyond that of full load, the curve droops more and more steeply and would become perpendicular at M, at which point the current would be the maximum possible from the machine in question. But the dotted portion of the curve represents conditions that would not be met in ordinary operation, the part FG corresponding to currents beyond the safe load of the machine. The curve continues beyond M, turning

backward with both current and voltage decreasing, and reaching the horizontal axis near O.

The explanation of the return branch of the curve can best be understood by thinking of the resistance of the external circuit as the independent variable. At the point P, corresponding to open circuit, this resistance R is infinite. By giving it some large value less than infinity, some current I flows. The armature drop $R_a I_a$ is then increased, with consequent decrease in terminal volts E_t and therefore in field current I_f and in Φ, resulting in less generated electromotive force. The external current I always equals $\frac{E_t}{R}$. When R is decreased, E_t *always* decreases; but I may either increase or decrease, according as the effect of decreasing the denominator of $\frac{E_t}{R}$ exceeds that of decreasing its numerator, or not. By calculus

$$dI = \frac{R \cdot dE_t - E_t \cdot dR}{R^2},$$

in which dE_t always has the same sign as dR, but dI will be positive or negative according as $R \cdot dE_t$ is greater than or less than $E_t \cdot dR$. From P to M of the curve, $R \cdot dE_t$ is greater than $E_t \cdot dR$; but after reaching M, any further decrease in R causes the current output to decrease rather than increase. That is, after the point M is reached, any further decrease in R causes so great a decrease in E_t that the resulting value of E_t is unable to maintain as large a current as before, even through the decreased value of R.

A small current flows in the armature even when the terminals are short-circuited, in which case there is no voltage between terminals, and therefore nothing to maintain an exciting current through the shunt field winding; but residual magnetism furnishes sufficient flux to generate the electromotive force necessary to maintain a small current through the low resistance of the armature.

The characteristic curve shows that the shunt-wound generator (unless designed with impracticably small armature resistance and armature reaction) would not be entirely suitable for supplying electrical energy to a constant-voltage distribution system. The terminal voltage varies rather too much between the limits of no load and full load. For constant-current distribution it is wholly unsuitable.

83. The series generator. The characteristic curve of a series-wound generator is shown in Fig. 101. When the machine runs on open circuit, there is no current in the field winding, and

therefore no field flux except a little from residual magnetism. This little causes a small electromotive force (OP of the figure) to be generated. As more load is connected, causing more current output, the flux Φ increases, its value being roughly proportional to the current until the iron of the magnetic circuit is saturated. After saturation is reached, Φ increases at a much smaller rate with increasing current. The total induced electromotive force ($Z'\Phi n$), being proportional to Φ, should accordingly be given by a curve like PT in Fig. 101.* But the terminal voltage E_t ($= E - R_a I_a$), on account of armature drop,† does not increase indefinitely with increasing current output, but reaches a maximum and then falls off with further increase in the current. It would finally reach zero, corresponding to short circuit of the generator, when the current would have the greatest value possible for the given machine. This is represented by OM in Fig. 101; but for most machines the dotted portion FM of the curve would correspond to currents in excess of the safe capacity of the machine.

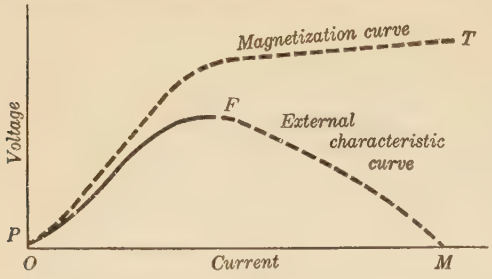


FIG. 101. Magnetization curve and external characteristic curve of a series-wound generator

Evidently the series-wound generator is wholly unsuited to constant-voltage operation.

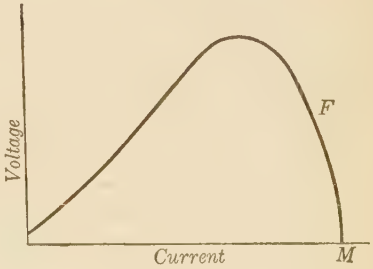


FIG. 102

By intentionally designing a series-wound generator so as to have large armature resistance and armature reaction, it is possible to make the characteristic curve take a form like that

* This curve is called the magnetization curve of the dynamo; its shape is determined by the magnetic characteristics of the magnetic circuit of the dynamo.

† Also on account of armature reaction, which causes Φ to be less than it otherwise would be.

of Fig. 102. If such a machine is operated in the region FM, its current output will be nearly constant as the load in the outside circuit varies; if the resistance of the load is increased, the voltage of the machine increases also and maintains the current approximately constant. Accordingly the series-wound generator is a type that can be adapted for use in the constant-current system of distribution.

84. The compound-wound generator. Fig. 103 shows the characteristic curve of a compound-wound generator. The compound-wound generator may be regarded as a shunt generator which has a few turns of series winding superposed on its shunt winding. As a shunt generator, Φ (and consequently the terminal voltage) would fall off with increasing current, but the series turns contribute to Φ an amount which increases with increasing current. The number of series turns may be made such that, at full load, their contribution to the magnetic flux just compensates for the decrease in Φ that would take place in their absence. In that case the electromotive force generated in the armature at full load will equal that generated at no load. By providing a few more series turns, the armature drop R_aI_a can also be compensated for, making the terminal voltage the same at full load as at no load. For intermediate loads the voltage will be nearly the same, though it is not found possible in actual machines to make it quite the same.* For overloads the flux required to effect compensation would correspond to oversaturation of the magnetic circuit; accordingly the curve droops, and would finally reach the horizontal axis at M, corresponding to the maximum current possible from the given machine. In practice, OM would be far beyond the safe current.

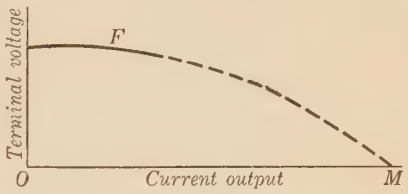


FIG. 103. External characteristic curve of a compound-wound generator (flat-compounded)

* If the magnetization curve of the iron parts were a straight line (that is, if Φ were exactly proportional to the ampere-turns), then it would be possible to obtain perfect compensation all along the curve. The armature drop R_aI_a is exactly proportional to I_a; but the additional flux to compensate for it is only approximately proportional to I_a, since the magnetization curve is only approximately a straight line. Thus the compensation is only approximate, except at one particular load.

Evidently the compound-wound generator is well suited for supplying energy to constant-voltage distribution systems, and it is the type of generator ordinarily used for that purpose.

If the series winding is provided with more than enough turns to compensate for the decrease in voltage that would take place in their absence, the terminal voltage will rise with increasing current, and the characteristic curve will be like PA in Fig. 104. A compound-wound generator having such a characteristic is called *over-compounded*; one in which the series turns are too few for complete compensation, corresponding to the curve PB in Fig. 104, is called *under-compounded*; and one in which the series turns cause the full-load voltage to equal the no-load voltage, corresponding to the curve of Fig. 103, is called *flat-compounded.*

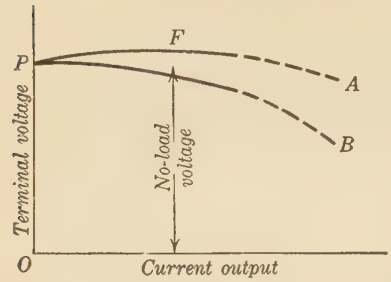


FIG. 104. Characteristic curves for an over-compounded and an under-compounded generator

The over-compounded generator finds practical application in constant-voltage distribution systems. It is often desirable to try to maintain the voltage constant at some distant center of distribution, rather than at the terminals of the generator. To do this, it is necessary to compensate for the line drop in the feeders, whose value $R_L I$ is proportional to the current output of the generator. Evidently this can be accomplished by providing such a degree of over-compounding that the full-load terminal voltage will exceed the no-load terminal voltage by the amount $R_L I$.

PROBLEMS

1. In Problem 1, Chapter V, the 30-ampere load at A is disconnected, and the terminal voltage of the generator, which is over-compounded, then becomes 114. Under these conditions find the voltage across mains at A, B, and C, and compare with the values found in Problem 1, Chapter V.

2. A certain 5-kilowatt generator is flat-compounded for 250 volts. Its shunt field takes a current of 2 amperes. Find the armature

current at no load and at full load. In what ratio is the full-load field flux increased over the no-load field flux, the resistance of armature and series field combined being 0.5 ohm?

3. The small generator of Problems 1 and 3, Chapter IV, is connected for separate excitation. If its flux per pole then amounts to 1,200,000 maxwells at all loads, find how much the full-load terminal voltage differs from the no-load terminal voltage.

4. The same machine is now connected for shunt excitation, and gives the same terminal voltage (139) at no load as before. Find its terminal voltage at full load (armature current 18 amperes), if the flux per pole varies with the terminal voltage according to the following table:

Terminal Voltage	Flux per Pole in Maxwells
139	1,200,000
135	1,181,000
130	1,155,000
125	1,128,000
120	1,100,000
115	1,071,000
110	1,041,000

CHAPTER VII

THE CONTROL OF ELECTRICAL ENERGY IN DIRECT-CURRENT SYSTEMS

The control of electrical energy requires appliances for connecting and disconnecting a circuit from the source of energy, for varying the strength of current in a circuit, for varying the voltage applied to a circuit, and for observing the values of current, voltage, and power.

85. Switches. Of the various possible devices for connecting and disconnecting electrical circuits at will, almost the only ones of practical importance are switches. The well-known knife switch (Fig. 105) is typical, and is the type of switch generally used for all circuits in which the current is more than a few amperes. The knife switch consists essentially of a blade* of drawn copper hinged at one end, the other end fitting tightly between springy clips of copper ("break jaws") when the switch is closed. The hinge and the clips are so designed as to provide large area of contact with the blade, and are attached to copper blocks, with which are connected the wires of the circuit controlled by the switch. In small switches (for not more than 25 amperes) the wires are connected by binding screws; in larger ones the ends of the wires are soldered into lugs (see Fig. 106), the binding screws or nuts taking hold of the lugs and making a more reliable contact than when the wire itself is wrapped under a binding screw. The free end of the blade is furnished with an insulating handle, and the whole

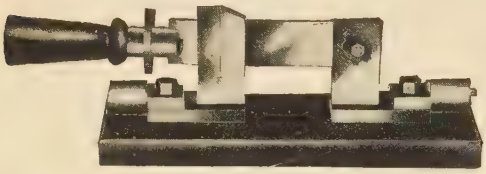


FIG. 105. Knife switch
Courtesy of the Crouse-Hinds Company

* In switches for heavy currents, the blade may be made up of several strips of copper in parallel, for better dissipation of heat.

arrangement is mounted on a base of insulating and fireproof material, such as slate or marble.

Very often it is desirable to connect or disconnect two or more wires simultaneously. For instance, in cutting off current from a motor, while it would be possible to stop the motor by breaking the connection to one terminal only, it is better to break the connection to both terminals and thus avoid having any live wire on the motor when it is not running. To do this, it is convenient to provide the necessary two knife switches with a single handle and mount them on a single base. Such an arrangement is called a *double-pole* switch. Three-pole and four-pole switches are also frequently used.

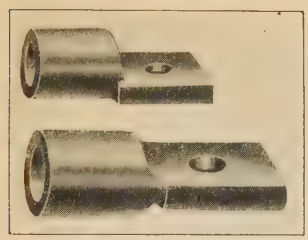


FIG. 106. Lugs

Sometimes it is desirable to use the same switch to accomplish either one of two different connections; this can be done by providing clips at each end of the base, so that when the blade is thrown to one side it connects with one circuit and when thrown to the other side it connects with another. Such an arrangement is called a *double-throw* switch.

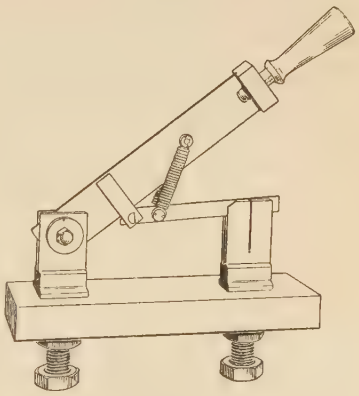


FIG. 107. Quick-break switch

If a switch is opened while carrying a heavy current, a heavy spark occurs between blade and break jaw, which tends to persist as an arc until the blade is opened so wide that its distance from the jaw exceeds the length of arc that can be maintained by the supply voltage. This is likely to melt, burn, and roughen the surfaces of the blade and jaw. To avoid this trouble, switches which must be opened under full current are often provided with quick-break attachments, as shown in Fig. 107. By this arrangement the duration of the arc is so greatly shortened that it does not have time to do appreciable damage.

As far as possible, the opening of switches under heavy current should be avoided. In circuits protected by circuit-breakers (see section 299) disconnection can be accomplished by tripping the circuit-breaker. It is often possible to bring the current to a small value before opening the switch. For example, in a motor the current is many times as great at full load as at no load, so that by removing the load before opening the switch the current can be made small enough to avoid any possible injury to the switch.

The underwriters' rules (see section 282) require that the rating of a knife switch be marked upon it, showing the maximum current and voltage for which it is intended to be used. The current rating is determined by the maximum current which the switch will carry continuously without undue heating of the contacts,* and the voltage rating by the minimum voltage which could under any circumstances maintain an arc between the jaws of the switch. In designing switches the current determines the cross section of the conducting parts, and the voltage determines the length of blade (or distance between jaws) and, in multiple-pole switches, the distance between the different blades.

In installing switches precaution should be taken against the possibility of accidental closing by gravity. Thus a single-throw switch should preferably be mounted in a vertical position, with the hinge at the bottom; then, if the blade falls at all, it falls so as to open the switch.

86. Controllers. Cases frequently occur in which a definite sequence of changes in electrical connections has to be repeated whenever a given appliance is used. An example is the starting of a street car, in which the motors are connected first in series and later in parallel. In all such cases it is much more convenient to effect the whole set of changes by the motion of a single handle, rather than by manipulating a number of switches in succession. A device by which this is accomplished is called a *controller*.

Controllers are most often so constructed that the operating handle turns a spindle or drum carrying a number of copper contact members, each of which, in certain positions of the

* According to the standards of the American Institute of Electrical Engineers their temperature should not rise higher than 70° C., when that of the ambient air is 40° C.

drum, engages with a stationary contact member corresponding to it and fastened to the inside of the controller case. By proper arrangement and connection of the contacts, the controller can be constructed so as to accomplish any desired sequence of

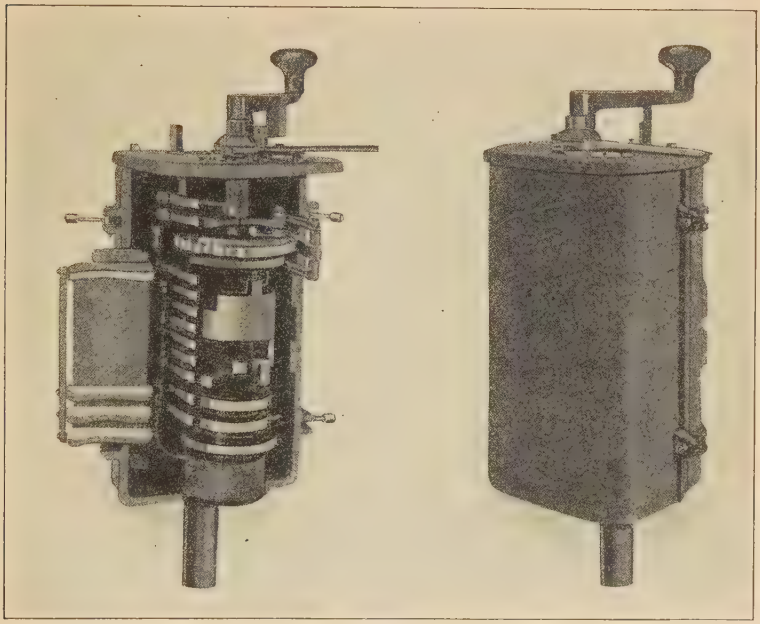


FIG. 108. Controller

Courtesy of the Westinghouse Electric and Manufacturing Company

changes in connections of the circuits connected with it. The general plan of construction can be readily understood from Fig. 108.

87. Control of strength of current; rheostats. The strength of current in any circuit may be modified either by changing the impressed voltage or by changing the resistance* in the circuit. In direct-current constant-voltage systems it is not feasible to change the voltage of the supply for the benefit of some one circuit in the system, hence the only method available is that of varying the resistance. This can easily be accomplished by placing in series with the circuit a rheostat, or adjustable resistance.

* In alternating-current circuits the impedance.

In most rheostats the adjustment is accomplished by turning a conducting arm about an axis at one end, which is connected with one terminal of the rheostat, while the other end makes contact with a succession of "points," each of which is connected with the next through a resisting path, or "resistor." This arrangement is shown in Fig. 109. The points are usually small brass or copper blocks mounted on a plate of some insulating and fireproof material, such as slate, the resistors and all connecting wires being placed behind the plate. The arm ends in springy copper, insuring sufficient pressure against the points for good contact. The parts behind the plate are inclosed in a casing to prevent accidental contact, the casing being so constructed as to permit free ventilation through the interior to aid in the dissipation of heat. In the figure, if the arm is moved to the extreme left, whatever current flows has to pass through all the resistors in series; with the arm at the extreme right, all the resistors are out of the circuit.

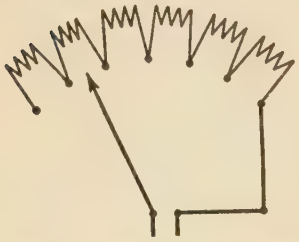


FIG. 109. Diagram to show construction of a rheostat

A rheostat must not only have sufficient resistance to accomplish the purpose for which it is used, but also sufficient current-carrying capacity to carry the current it is intended for without overheating. Other things being equal, the current-carrying capacity will be greater the faster heat can be dissipated from the rheostat; accordingly a rheostat should preferably be placed in a cool, well-ventilated location.

A rheostat, starting cool, can, of course, carry a larger current for a short time than it could carry continuously without overheating. Many rheostats are designed only for intermittent service (for example, starting rheostats for motors), and caution should be used not to keep such a rheostat in use too long at a time, as its normal current will overheat it if continued very long.

While rheostats accomplish admirably their purpose of controlling the strength of currents, they do so at the cost of some waste of power (RI^2), which may be very appreciable in the case of large currents. This somewhat limits their field of

usefulness to the control of comparatively small currents, or to the control of larger currents during small intervals of time.

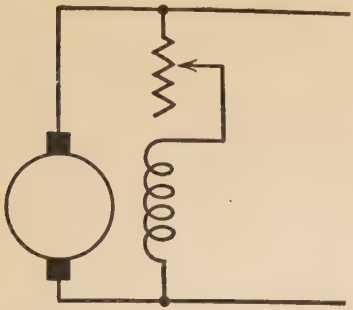


FIG. 110. Connection of rheostat to control the voltage of a shunt generator

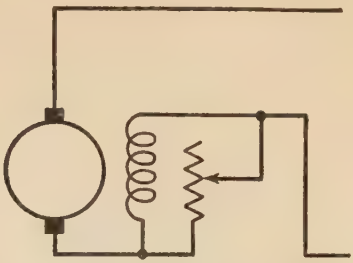


FIG. 111. Connection of rheostat to control the voltage of a series generator

88. Control of supply voltage. In direct-current systems there is no very simple way of varying, for any one circuit, the voltage from mains maintained at constant difference of potential. It

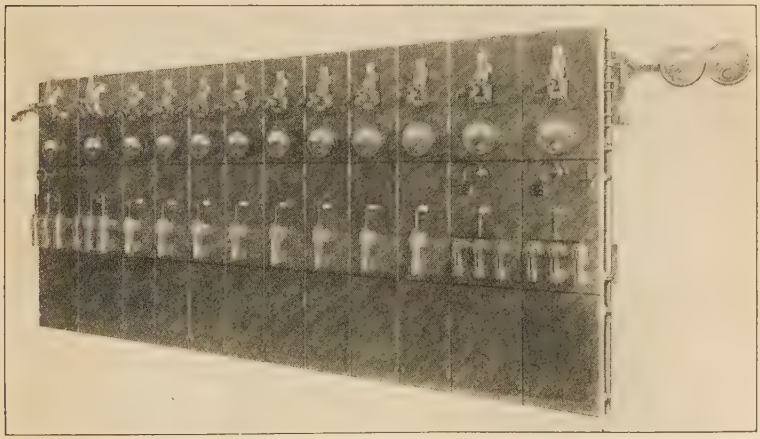


FIG. 112. A direct-current switchboard
Courtesy of the General Electric Company

is possible, of course, to let a current flow through a resistance placed across the mains, and to tap off from this resistance any desired fraction of the line voltage; but this method would in most cases be too wasteful of energy and would in no case

permit obtaining a voltage higher than that of the line. It is possible, also, to connect a motor with the supply mains and let it drive a generator giving any desired voltage, thus obtaining for some special circuit a voltage different from that of the mains: but this method is expensive and cumbersome and involves some loss of energy.

Though it is not easy to vary the voltage for some one circuit in a direct-current system, it is quite easy to vary the voltage of a generator. If this were the generator supplying a direct-current constant-voltage system, of course the voltage in the whole system would be affected simultaneously.

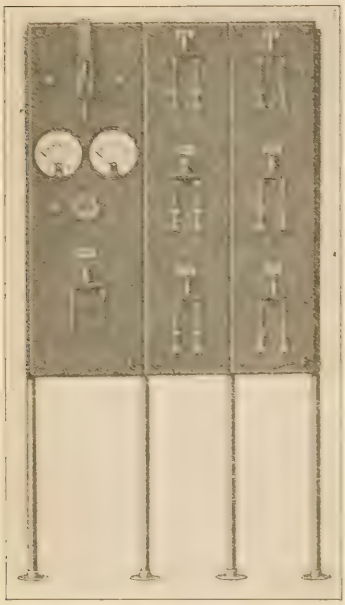


FIG. 113. "Type JD" direct-current switchboard panels

Courtesy of the Westinghouse Electric and Manufacturing Company

The control of the voltage of a generator is accomplished by varying the strength of its field flux. This can easily be done by varying the strength of current in the field winding, which is usually accomplished by means of a rheostat. In the case of a shunt-wound generator the rheostat is connected as shown in Fig. 110; in a series-wound generator, it is connected as in Fig. 111, diverting a part of the current which would otherwise pass through the field. In a compound-wound machine control is obtained principally by a shunt field rheostat, but a rheostat may also be used with the series field winding, as in Fig. 111.*

Field rheostats form part of the regular equipment of direct-current generators.

89. Switchboards. For convenience of operation it is standard practice to mount all the control apparatus, instruments, and

* If either one of the rheostats belonging with a compound-wound generator is changed without changing the other, the degree of compounding of the generator will be changed.

protective devices belonging to any machine or circuit together on a plate of insulating material (usually marble or slate).* Such a plate is called a *switchboard panel.* It need not necessarily be placed very close to the appliance controlled by it. When several machines or circuits are to be controlled by the

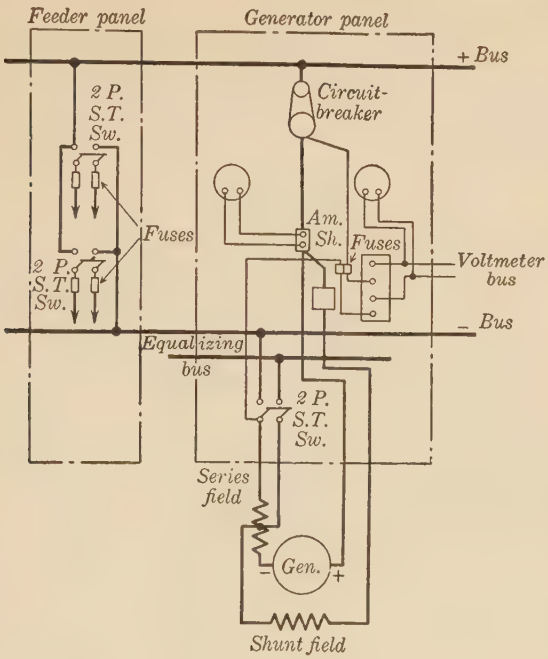


FIG. 114. Diagram of connections for "Type JD" switchboard panels

Courtesy of the Westinghouse Electric and Manufacturing Company

same operator, as in a central station, all the switchboard panels are placed next to one another, and the whole group forms the *switchboard.*

In central-station switchboards there is one panel for each generator and one for each line of feeders. It is customary to place all the generator panels at the left end and all the feeder

* Sometimes, especially when large currents or high voltages are concerned, some of the switchboard appliances are not actually mounted upon the switchboard, although they are controlled from it. For example, a heavy rheostat may be placed upon the floor behind the switchboard rather than mounted upon it, being manipulated in either case by a handle on the front of the switchboard. Large circuit-breakers are usually placed in fireproof cells rather than mounted upon the back of the switchboard.

panels at the right end. The general appearance of a switchboard arranged in this manner is shown by Fig. 112.

While great variety is possible in the choice and arrangement of the switchboard appliances to be used with any given machine or circuit, the construction of switchboards has been so far systematized that standard panels are available for all ordinary purposes. For example, Fig. 113 shows the "Type JD" switchboard made by the Westinghouse Electric and Manufacturing Company and designed for direct-current central stations of moderate size. Fig. 114 shows the diagram of connections.

PROBLEMS

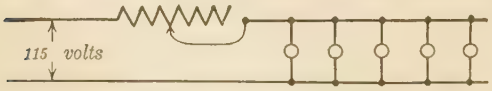
1. A certain rheostat is intended to be connected in series with the shunt field of a dynamo, the terminal voltage of which is 220 volts. The resistance of the shunt field winding is 55 ohms. The rheostat is to be constructed so that the current through the shunt field will depend on the position of the rheostat arm as follows:

Rheostat arm at point 0; current, 2.5 amperes
Rheostat arm at point 1; current, 2.55 amperes
Rheostat arm at point 2; current, 2.60 amperes
Rheostat arm at point 3; current, 2.65 amperes

and so on. Find the resistance that must be provided (*a*) between points 0 and 1, (*b*) between points 1 and 2, and (*c*) between points 2 and 3.

2. A rheostat is to be designed for use with (that is, shunted across the terminals of) the series field of a generator. The series field carries 100 amperes and has a resistance of 0.01 ohm. The rheostat is to divert no current when its handle is at point 0; it is to divert 0.1 of the current when its handle is at point 1, 0.2 when its handle is at point 2, 0.3 when its handle is at point 3, and so on. Find the resistance and current-carrying capacity of the resistor between points 0 and 1, between points 1 and 2, and between points 2 and 3. (You will need to draw a sketch of connections.)

3. To be able to vary the brightness of a set of lamps by controlling the voltage at their terminals, a rheostat is connected as shown in the figure (principle of the theater-dimmer). (*a*) How much resistance is required in the rheostat to secure 110 volts across the lamps if the lamps take 100 amperes? (*b*) With the rheostat unchanged, what will the voltage across the lamps become if half the lamps are turned off?



CHAPTER VIII

DIRECT-CURRENT MEASURING INSTRUMENTS

90. Importance of instruments. In electrical engineering, as in most other fields of endeavor, the development of satisfactory measuring instruments has been of fundamental importance. Without them many of our most successful applications of electricity would be impossible, and most of the rest would be both ineffective and inefficient. Among other uses, they are indispensable for the satisfactory control of electrical energy, to enable the operator to judge accurately the situation with which he has to deal.

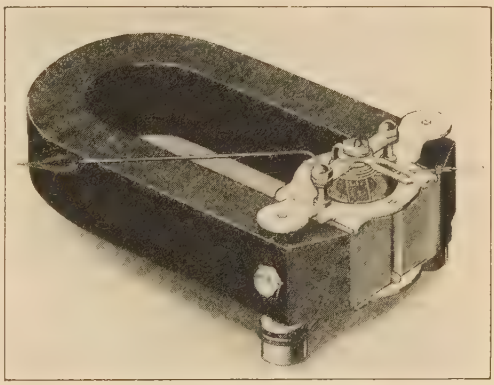


FIG. 115. Movable coil between poles of permanent magnet, in a direct-current ammeter

Courtesy of the Jewell Electrical Instrument Company

91. Direct-current ammeters. With few exceptions direct-current ammeters act on the principle of the D'Arsonval, or suspended-coil, galvanometer. The current (or some definite fraction of it) is made to pass through a coil of wire suspended between the poles of a permanent magnet, as shown in Fig. 115. A torque then acts on the coil, just as if it were the armature of a motor, the torque being greater as the current is greater. The coil is mounted on a spindle, to which spiral springs are attached, as can be seen in Fig. 115. When the coil turns, the elasticity of these springs supplies a torque opposing the displacement of the coil and increasing with the angle of displacement. There is, for any given current, a definite angle of displacement at which the torque from the

magnet is balanced by the torque from the springs; the coil accordingly deflects to this angle when the given current traverses it. A pointer is attached to the spindle and indicates on a scale the value of the current in amperes. The scale is graduated empirically by comparison with a standard instrument. There is usually a stationary core of soft iron inside the coil, which makes the magnetic field stronger and more uniform than would be the case in its absence. The coil is wound on a frame of aluminium; eddy currents are formed in this frame when it moves through the magnetic field, and these damp the vibrations of the moving element which otherwise would take place whenever the deflection changes.

Ammeters are constructed with as little resistance* as possible, to avoid heating and loss of power (RI^2), and so that the insertion of an ammeter in a circuit shall not materially affect the state of affairs which would exist in its absence. The low resistance between terminals of the instrument is attained by connecting a low-resistance shunt in parallel with the moving coil, the resistance of the moving coil itself being comparatively high.

92. Direct-current voltmeters. When a current flows through an ammeter, there must exist across its terminals a definite voltage equal to the current times the resistance of the instrument. Accordingly, for every type of ammeter, it is possible to construct a corresponding type of voltmeter by replacing the ammeter scale with a scale which indicates for each position of the pointer the corresponding voltage across the terminals of the instrument. Such a voltmeter indicates volts only by permitting the volts to send a current through it, the volts at terminals then being equal to this current multiplied by the resistance of the instrument. Evidently the resistance of the voltmeter, in contrast with that of the ammeter, should be as high as possible, to avoid heating and loss of power $\left(\frac{E^2}{R}\right)$, and so that connecting it across a circuit shall not materially affect the state of affairs which would exist in its absence.

Direct-current voltmeters, with few exceptions, act on the D'Arsonval principle, as just described for the ammeter. All essential features are the same as for the ammeter, except that

* For example, the resistance of an ammeter of this type, with a range of 0 to 25 amperes, would be of the order of 0.005 ohm.

the scale is graduated in volts and the voltmeter has high resistance.* Various details, however, differ in the two instruments; for instance, the terminals and current-carrying parts must be made much heavier in an ammeter than in a voltmeter. The high resistance of the voltmeter is usually attained by connecting a stationary high-resistance coil in series with the moving coil.

An ammeter is, of course, intended to be connected *in* a circuit, as shown in Fig. 116, whereas a voltmeter is intended to be connected *from one to the other of the points* between which the voltage is to be measured. Since the two instruments usually are somewhat alike in their external appearance, beginners in electrical work sometimes get them confused and connect the voltmeter where the ammeter should be, or the ammeter where the voltmeter should be. The effect of the former blunder is merely to prevent the passage of the current through the circuit (the voltmeter resistance being high); but the effect of the latter is to make a short circuit (the ammeter resistance being nearly zero) and allow an excessive current to traverse the ammeter, resulting often in the destruction of the instrument before the circuit-breakers or fuses have time to act.

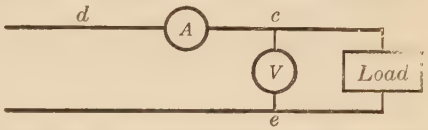


FIG. 116. Connections of voltmeter and ammeter to mains

93. Use of alternating-current instruments with direct currents. The instruments just described will not respond to alternating currents (or voltages), but all alternating-current ammeters and voltmeters (except those acting on the induction principle) respond with substantial correctness to direct currents (or voltages) as well as to alternating currents. There are, however, two objections to using alternating-current instruments for direct-current measurements: first, the deflection is always in the same direction, thus giving no indication of the direction of the current (or voltage); secondly, the scale divisions are very small for currents (or voltages) less than about half the maximum for which the instrument is intended,† thus making

* For example, the resistance of a voltmeter of this type, with a range of 0 to 150 volts, would be of the order of magnitude of 10,000 ohms.

† This is due to the fact that in alternating-current instruments the deflection is proportional, approximately, to the *square* of the current (or voltage); in the D'Arsonval type of instrument the deflection is, approximately, *directly* proportional to the current (or voltage).

accurate reading impossible for currents (or voltages) much below the full capacity of the instrument.

94. Wattmeters. The wattmeter is an instrument for measuring the power delivered electrically to a circuit.

With direct currents it is possible to ascertain the power by using an ammeter and a voltmeter, connected as in Fig. 116, in which case the product of volts by amperes gives the watts of power delivered to the circuit across which the voltmeter is connected.* It is often convenient, however, to obtain the power by direct reading of a single instrument, without computation.

In a wattmeter the moving element is similar to that of a voltmeter as described above, and has high resistance; but instead of being placed between the poles of a permanent magnet, it is placed within a fixed coil of wire. To use the instrument, the current in question is passed through the fixed coil, and the voltage is applied to the terminals of the movable coil. A small current then passes through the movable coil, proportional to the volts at its terminals, just as in a voltmeter. Force action then takes place between the currents in the fixed and movable coils, causing the movable coil to turn, under a torque determined by the product of the two currents. But since the current in the movable coil is proportional to the volts, this torque will be determined by the product of line current (equal to the current in the fixed coil) by voltage (proportional to the current in the movable coil); which product is the same as the power. So the deflection of the pointer will be determined by the watts, and the scale is graduated accordingly.

A wattmeter has four terminals: one pair for the movable, or voltage, coil, and one pair for the fixed, or current, coil.† It is important to avoid confusion between the terminals, since

* A slight correction has to be made, because the current registered by the ammeter includes the small current passing through the voltmeter in addition to that flowing through the load. Of course, the true power delivered to the load is the product of the volts across its own terminals by the current in it alone; that portion of the line current which leaks through the voltmeter contributes no power to the load.

If the voltmeter were connected from d to e instead of from c to e, then the ammeter would register the true current through the load, but the voltmeter would register slightly more than the volts at the terminals of the load, since there would be a slight voltage drop in the ammeter.

† Sometimes only three terminals are provided, one of which serves as a common terminal for both coils.

connection of the current coil where the voltage coil should be would cause short circuit and damage the instrument.

95. Watt-hour meters. A watt-hour meter (popularly called simply a *meter*) is an instrument for registering the total amount of energy delivered to a circuit during the whole period of time since the instrument was installed. The energy delivered during any other interval of time can be ascertained by subtracting the amount registered at the beginning of the interval from that registered at the end of the interval.

It is customary to express amounts of energy delivered by electricity in *watt-hours* or *kilowatt-hours*, although any other unit of energy, such as the joule or the foot-pound, could be used just as well if custom permitted.

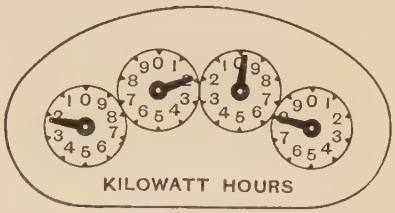


FIG. 117. Watt-hour-meter dial, reading 2198 kilowatt-hours

Courtesy of the Sangamo Electric Company

Most of the direct-current watt-hour meters in use in this country are either of the Thomson type (developed by Elihu Thomson) or of the mercury type; of these two types, the former only will be briefly described here.

In principle the Thomson watt-hour meter is similar to a wattmeter, except that the movable coil is wound like the armature of a dynamo and is provided with commutator and brushes, through which a small current (proportional to the voltage) enters. It is thus able to rotate continuously, the torque acting upon it at any instant being proportional to the power being delivered at the same instant to the circuit connected with the instrument. To the lower end of the spindle of the movable coil is attached a device which opposes the motion with a torque proportional to the speed; the speed accordingly sets itself to such a value that this opposing torque just equals the driving torque. Thus the speed is proportional to the power (or rate of delivery of energy) supplied to the circuit connected with the instrument, and the total number of revolutions is proportional to the total amount of energy delivered to the circuit. The upper end of the spindle engages with a counting mechanism, which, by means of hands on a set of dials, registers the number

of kilowatt-hours that have been delivered. In the meters for ordinary houses dials are provided for units, tens, hundreds, and thousands of kilowatt-hours, each dial having ten divisions round its circumference; for convenience in construction of the counting mechanism, the hands on alternate dials turn in opposite directions. Fig. 117 shows the standard arrangement of dials, the reading being 2198 kilowatt-hours.

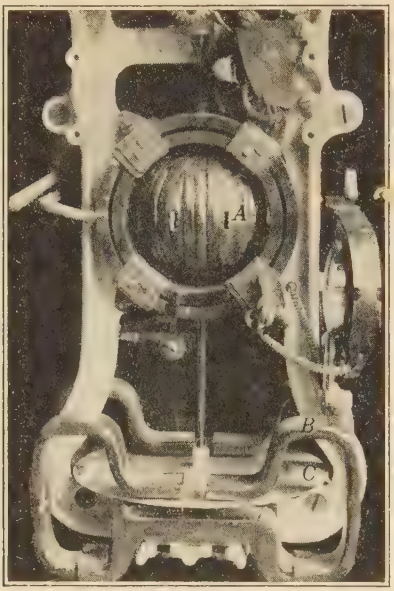


FIG. 118. Thomson watt-hour meter

Partly dismantled, to show some of the essential working parts

The device for opposing the motion with a torque which shall be proportional to the speed is a thin disk of aluminium, the edge of which passes through a narrow air gap between the poles of strong permanent magnets. When the disk turns, eddy currents are set up in it, and the force with which the magnetic field acts on the material carrying them opposes the motion and is proportional to the speed (since the strength of the eddy currents is proportional to the speed, and the magnetic field strength is constant).

The construction of a Thomson watt-hour meter is shown in Fig. 118.

PROBLEMS

1. Find the voltage drop between the terminals of an ammeter whose resistance is 0.003 ohm, when inserted in a line carrying 100 amperes. Also find the power lost in the instrument.

2. Find the current which flows through a voltmeter whose resistance is 5500 ohms, when connected across 220-volt mains. Also find the power lost in the instrument.

3. With the ammeter and voltmeter of Problems 1 and 2 connected as in Fig. 116, find the exact power taken by the load when the voltmeter reads 115 volts and the ammeter reads 10 amperes.

4. To enable a central-station operator to observe the voltage on a line at a point half a mile from the station, a voltmeter in the station is connected with the point by leads of No. 18 copper wire (resistance 6.4 ohms per 1000 feet of wire). The resistance of the voltmeter is 5000 ohms. When the reading of the voltmeter indicates 110 volts between its own terminals, how much is the voltage at the distant point with which it is connected?

5. The ammeter described in Problem 1 was connected in a circuit in which the current is 100 amperes. While so connected, a piece of heavy wire was shunted across the terminals of the ammeter, and its reading then dropped to 20 amperes. What was the resistance of the wire?

6. A heavy conductor whose resistance is known to be 0.0005 ohm is connected in a circuit, and the ammeter of Problem 1 is connected across the terminals of the conductor. If the ammeter then reads 90 amperes, what is the current in the circuit?

7. Two voltmeters like that described in Problem 2 are connected in series with each other across 220-volt mains. The point between the two voltmeters is connected to the ground. Voltmeter *A* is connected to main *A*, and voltmeter *B* to main *B*. State the reading of each of the voltmeters under the following conditions: (*a*) both mains perfectly insulated from the ground, (*b*) main *A* perfectly insulated from the ground, main *B* connected to the ground by a path of negligible resistance, (*c*) main *A* perfectly insulated, main *B* connected to the ground by a path whose resistance is 5500 ohms.

CHAPTER IX

DIRECT-CURRENT MOTORS

96. Types of motors. It has been pointed out that any dynamo can operate either as a generator or as a motor. Accordingly we have the same types of direct-current motors as we have of direct-current generators; namely, separately excited,* shunt-wound, series-wound, and compound-wound.

Although in principle the same machine can be used both as a generator and as a motor, in practice this is very rarely done, because it is so generally necessary to modify various details to fit the special needs of the service to be performed. There is great variety in the service required from motors for different purposes — much greater variety than in the case of generators. Special adaptations are needed for each kind of service, so that a motor intended for a given service is not usually of practical use for a different one.

The differences in the special requirements for different classes of service may be illustrated by a few examples. A street-car motor must be protected against mud, slush, and grit, and be capable of standing severe mechanical shocks; it must be capable of running at any speed within some upper limit and of exerting torque within wide limits; it must be reversible, and its operation is intermittent. A motor for hoisting machinery has similar requirements, except that it does not need protection against mud and slush. A motor for driving the main shaft of a shop should have nearly constant speed, regardless of variations in its load; it need not be reversible, and its operation is continuous. An individual motor for a lathe or a drill press makes similar requirements, except that its speed should be adjustable to several different values. Motors for some other machines, such as punches, planers, or printing-presses, are subject to sudden variation of load during the

* Including as a special case those with permanent magnets, which, however, are not in practical use except as toys.

normal operation of the machine. Motors used in some manufacturing operations require protection against fumes, dust, or grit; in others this is unnecessary.

97. Direction of running. A direct-current motor (unless separately excited) turns in the same direction, whichever way the line voltage is applied to it. If the connections with the line are reversed, the current is reversed in the armature and also in the field winding. The direction of the field is thus reversed at the same time as the current in the armature conductors; so the force upon each armature conductor remains in the same direction as before, and the motor turns in the same direction as before.

To reverse the direction of running, it is necessary to reverse the armature current only (without reversing the field current), or else to reverse the field current only (without reversing the armature current).

98. Relations between speed, torque, and other quantities. In the performance of a direct-current motor the quantities of special interest are four in number: on the electrical side the voltage applied and the current intake (whose product gives the electrical power received), and on the mechanical side the speed and the torque (whose product, multiplied by the proper constant, gives the mechanical power delivered). Of these quantities, the voltage applied (E_t) is fixed by the supply mains, and is usually constant. The torque T is determined by the load put upon the motor, since, for steady running, the torque with which the motor pulls forward must just balance the torque with which the load drags back, — otherwise there would be an unbalanced torque which would either accelerate or decelerate the motor. The speed n and the armature current I_a* are connected with E_t and T by the fundamental equations of the direct-current dynamo, (41) and (44), sect. 71, pp. 89 and 90. Using the abbreviation Z' for $\frac{Zp}{p' \times 10^8}$, we obtain from equation (41)

$$n = \frac{E_t - R_a I_a}{Z'\Phi}. \tag{46}$$

* The total current intake equals I_a in series motors; in shunt or compound motors it exceeds I_a by the current in the shunt field winding. In separately excited motors I_a of course differs from the total current intake.

From equation (44) we have

$$I_a = \frac{T}{1.41\ Z'\Phi}. \tag{47}$$

Also, from equation (41),

$$I_a = \frac{E_t - Z'\Phi n}{R_a}. \tag{48}$$

In these equations R_a and Z' are fixed constants* of the machine in question, and Φ is a function of some of the other quantities.† Accordingly equations (46), (47), and (48) are sufficient for finding the values of n and I_a corresponding to assigned values of E_t and T, provided it is known in what manner Φ depends on the other quantities for the motor in question. This varies with different motors and will be discussed later for different types.

Under ordinary operating conditions E_t is constant, T is the independent variable, changeable at the will of the operator, and n and I are functions of T.

99. Behavior of speed with change of load. $Z'\Phi n$ (see sections 38 and 71) gives the counter-electromotive force generated in the armature. In a motor connected to constant-voltage mains the counter-electromotive force always assumes such a value as to let through just sufficient current to supply the torque required to carry the load, the speed adjusting itself to such value that the proper counter-electromotive force is generated. If the load on the motor is lightened, this makes (momentarily) the torque opposing the motion less than the torque supplied by the motor; accordingly acceleration of the armature takes place. But with increased speed a greater counter-electromotive force is generated;‡ accordingly less current gets through the armature, and less torque is exerted. Eventually a speed is reached such that just the proper current gets through to furnish sufficient torque to carry the (now lightened) load; then acceleration ceases and the motor runs steadily again, but with altered speed. If the load is increased, corresponding changes take place in the opposite direction.

* Neglecting slight changes in R_a due to changes in temperature.

† It is assumed that the field rheostat, if any, is not readjusted; of course, if this is done, Φ becomes an independent variable.

‡ Provided Φ does not decrease simultaneously in greater proportion than n increases, which is possible but does not occur in ordinary motors.

100. Behavior of speed with change of field flux. If the magnetic field of a not too heavily loaded motor is weakened (without change of load), the speed *increases* — a result which may at first thought seem strange, but which is easily explained. If the magnetic field is weakened, the counter-electromotive force, $Z'\Phi n$, is (momentarily) weakened also; accordingly the armature current increases; this makes the torque supplied by the motor greater than is required to carry the load;* accordingly the armature is accelerated. With increasing speed the counter-electromotive force increases and the current decreases, until a speed is attained at which just sufficient current gets through to supply the proper torque for carrying the load; then the motor runs steadily again, but at increased speed. Since Φ is now smaller, the current required to produce this torque is greater than before ($T = 1.41\ Z'\Phi I_a$).

101. Starting of motors. When a motor is running, its counter-electromotive force approximates the supply voltage, leaving only a small excess of the latter for driving current through the armature resistance, in accordance with equation (48). When the motor is at rest, there is no counter-electromotive force; so that if the full supply voltage should then be connected with its terminals, the armature current would be many times as great as under full-load running conditions and (in most cases) would be sufficient to injure the armature very quickly.† For this reason, in starting a motor a rheostat (often called popularly a "starting box") is connected between the armature and the line, as shown in Fig. 119.‡ The full resistance of the rheostat is sufficient to keep the initial current within a safe value. The torque due to the initial current accelerates the motor armature; as the armature acquires speed, it generates counter-electromotive force, which reduces the current. Some of the resistance can then be cut out, resulting in (momen-

* Provided the load is not too heavy, or the field too greatly weakened. If Φ is decreased in greater proportion than I_a can increase, the motor will, of course, exert *less* torque than before and be retarded.

† Practically, there would not be time for this current to damage the armature if the motor were properly installed with protection by circuit-breakers or fuses, which would act quickly; but in protecting the motor from injury, they, of course, disconnect it from the supply voltage and prevent it from starting.

‡ The figure is drawn for the case of a shunt motor; but obviously the same necessity for a starting rheostat would exist with other types of motors to be operated from constant-voltage mains.

tary) increase in current, with consequent increase of torque and acceleration and further increase of speed and counter-electromotive force, which again reduces the current. Then more resistance can be cut out, and finally all of it, the motor then receiving the full line voltage.

Evidently the purpose of the starting rheostat would be more or less completely defeated if the rheostat handle should be moved along too quickly; it should be allowed to rest a moment on each point, so as to give the motor time to gain in speed before the handle is moved to the next point. In practical operation the sound of the motor (which varies with its speed) guides the operator as to the proper rate for moving the rheostat handle. From the point of view of the operator, starting rheostats are somewhat like clutches: both ought to be thrown on gradually (but may be thrown off suddenly), and in both cases too sudden throwing on causes mechanical shocks and roughness of operation.

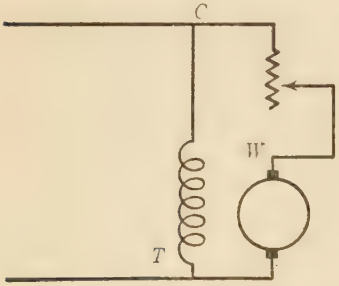


FIG. 119. Connections for motor-starting rheostat

102. Characteristic curves. In studying the behavior of different types of motors it is convenient to make use of curves showing relations between the quantities torque, speed, current intake, and others. In such curves it would seem most logical to regard the torque as the independent variable and represent it by the abscissas, since the torque is determined by the load applied, and this is what the operator varies at will. It is more customary, however, to let the abscissas represent current instead. In this book, curves drawn with the torque as independent variable (abscissas) will be given for each type of motor discussed; those with the current as independent variable will be added in some cases.

In all cases it will be assumed that the voltage across the motor terminals (E_t) is constant. The constant-voltage system of distribution is the only one on which direct-current motors are operated in this country (although sometimes the same motor may be operated on one constant voltage at one time and on a different constant voltage at another time).

THE SHUNT MOTOR*

103. Shunt-motor characteristics. The speed-torque curve of a shunt-wound motor is shown in Fig. 120. With the motor unloaded (torque zero), the speed has the value OP. With increasing torque the speed falls off slightly, the full-load speed (at F) being perhaps 5 per cent less than the no-load speed. The curve would eventually reach the horizontal axis at some point M beyond the limits of the figure. OM would then represent the maximum torque obtainable at all from the machine, with its armature clamped at rest while the full voltage was applied to the terminals. But the portion of the curve beyond F in the figure would correspond to conditions outside the limits of safe operation; its consideration is of interest only for an understanding of the behavior of the motor in case of accidental occurrence of abnormal conditions.

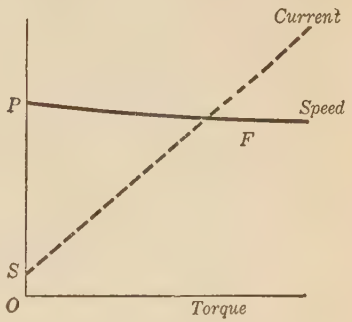


FIG. 120. Speed-torque curve and current-torque curve for a shunt motor

The dotted curve shows the current intake as a function of the torque. The no-load current OS is principally the field current, but a small current has to flow in the armature also for the purpose of maintaining sufficient torque to overcome friction.

Equations (48) and (47), section 98, show that both the curves of Fig. 120 will be straight lines if Φ is constant. Φ is, in fact, nearly constant, but not perfectly so. The field current is constant, since the volts at field terminals are constant; but armature reaction takes place, and slightly weakens the field when the current is heavy. On this account both speed and current are a little greater at full load than they would be in

* No discussion of separately excited motors will be given. Their behavior is altogether similar to that of shunt motors, the only difference being that the field may be excited from a different circuit (perhaps at a different voltage) from that supplying the armature. In both separately excited and shunt-wound motors the field current is uninfluenced by changes in load, and Φ is constant except as affected by armature reaction.

the absence of armature reaction, and both curves are slightly concave upward in the region of ordinary operation.*

104. Starting rheostats for shunt motors. The connections for starting a shunt motor were shown in Fig. 119. It is important that the field be connected with the line outside the rheostat (between T and C in the figure, not between T and W). Then the field reaches its full excitation at once; and when the current starts in the armature, the torque for accelerating it is correspondingly strong, and the motor speeds up promptly.

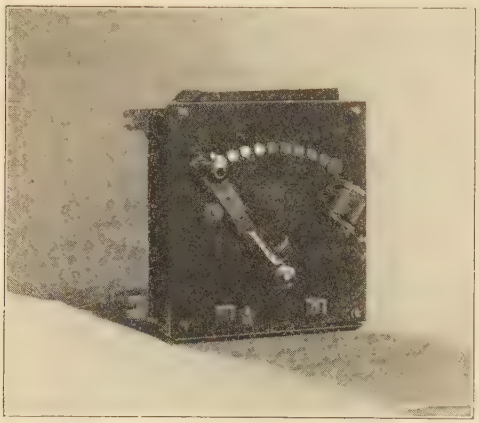


FIG. 121. Shunt-motor-starting rheostat

Courtesy of the Reliance Electric and Engineering Company

In the usual types of shunt-motor-starting rheostats the first position of the handle corresponds to open circuit for the armature (but not for the field), and it is only after moving the handle from its first position that current begins to flow in the armature. When the handle is shifted completely over into its last position, corresponding to zero resistance, a small electromagnet comes into contact with a bit of iron on the handle, and thus holds the handle in this position in opposition to a strong spring which tends to return the handle to the open-circuit position (see Fig. 121). When the motor is disconnected from the mains, or when the supply voltage stops for any reason (such as a shutdown of the central station from which the power is supplied), the electromagnet is no longer excited and consequently releases the handle, which then springs back into the open-circuit position. This arrangement precludes the possibility of applying the line voltage to the armature with the rheostat cut out and the armature at rest, as might otherwise

* After the torque exceeds a certain value (which would correspond to heavy overload in ordinary motors), the speed curve becomes concave downward. See footnote to section 100.

happen through forgetfulness on the part of the operator or in case the supply should be interrupted long enough for the armature to stop, and then be restored. The electromagnet winding should be connected in series with the shunt field winding so that it will release the handle not only when the supply voltage is cut off but also if the field circuit should be broken, since in this event the counter-electromotive force would cease, and excessive current would then traverse the armature.

105. Uses of shunt motors. Since the shunt motor runs at approximately the same speed for all loads, it is well adapted to applications which require (approximately) constant speed; accordingly shunt motors are used for driving main shafts in shops, for motor-generator sets, and as individual motors for woodworking machines, lathes, drill presses, boring mills, and other machine tools in which the load does not vary suddenly during the operation of the machine.

106. Speed control of shunt motors. Sometimes it is desirable to be able to use a motor whose speed can be adjusted at will by the operator, but remains independent of the load while the motor is left to itself. For example, in a lathe different cuts have to be made at different speeds; if the lathe is driven by a motor, it is very desirable to be able to adjust the speed of the motor to the needs of the particular operation to be performed. The shunt motor meets the requirements if provision is made for adjusting its speed.

Equation (46), $n = \frac{E_t - R_a I_a}{Z'\Phi}$, shows that the speed can be adjusted if means can be provided for varying at will either Φ or E_t. A third possibility would be to insert resistance in series with the armature (which is equivalent to increasing R_a), but this method is not often practically useful, since it introduces unnecessary loss of power ($R_a I_a^2$) and since the speed ceases to be approximately independent of the load when R_a is large. Methods of speed control by variation of Φ or of E_t are free from these objections and are used in practice.

The simplest way to vary the speed by varying Φ is to insert a rheostat in the field circuit of the motor, as shown in Fig. 122, and thus vary the field excitation by varying the field current. This method is quite satisfactory when the speed need not be varied over a very wide range of values. For greatly increasing

the speed of a motor it is not suitable, for the reason that at high speeds too much sparking at the commutator takes place; this is partly because the time for commutation of the current in an armature coil is shortened, and partly because the neutral points are displaced more by armature reaction.*

Other methods of varying Φ for speed control depend on varying the length of air gap, or on shifting the armature along the axis so that for higher speeds it is partly outside the field. The latter method is used in the adjustable-speed motor shown in Fig. 123. Motors of this type are so constructed that the highest speed is ten times the lowest. Sparkless commutation at all speeds is attained by a special arrangement of interpoles.

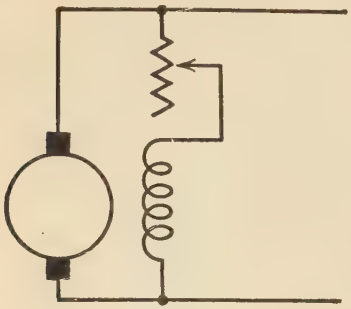


FIG. 122. Diagram of connections of field rheostat for controlling the speed of a shunt motor

A disadvantage (often not practically important) that applies to all methods of speed control by variation of Φ is that the torque ($= 1.41\ Z'\Phi I_a$) falls off in the same proportion that the speed increases, unless the current is increased. With these methods it may sometimes happen that a motor is unable to carry its load at the higher speeds without exceeding safe values of armature current.

Speed control by varying the voltage impressed on the motor terminals requires either some means of varying the supply voltage at will or else a choice of several different supply voltages. The former scheme can be carried out only by using an auxiliary motor-driven generator (whose voltage is controlled by its field rheostat) to supply the voltage to the terminals of the main motor. This method is expensive and cumbersome but extremely satisfactory otherwise, and has found application in the motors for turning the turrets of battleships. The

* For the same armature current the magnetizing action of the armature would be the same; but at high speeds the main field is weaker in this method of control; accordingly the neutral points would be shifted more. But in fact the armature current would be stronger (and thus its magnetizing action greater) at high speeds if the load is the same, since the field is weaker, and thus more current is required to produce the same torque ($= 1.41\ Z'\Phi I_a$).

latter scheme requires several mains at different voltages. For example, Fig. 124 shows four mains, with which it is possible to get a choice of six voltages — 40, 70, 110, 150, 180, or 220 —

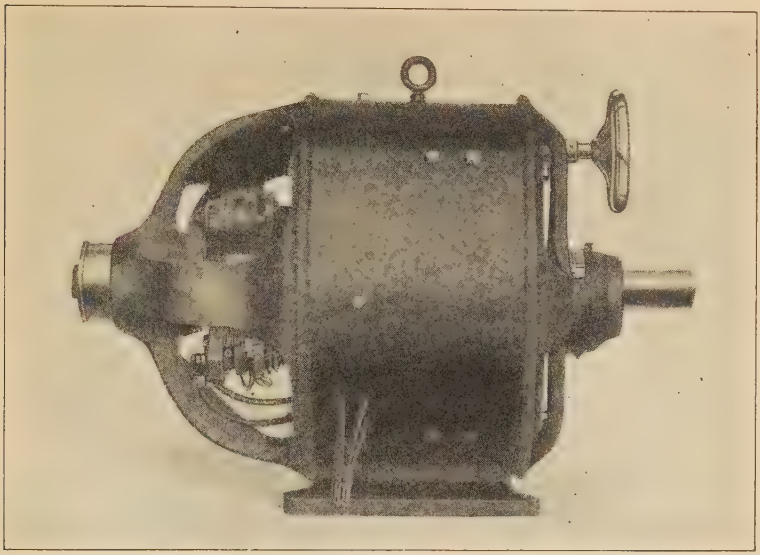


FIG. 123. Adjustable-speed motor

Courtesy of the Reliance Electric and Engineering Company

by connecting *a* to *b*, *c* to *d*, *b* to *c*, *a* to *c*, *b* to *d*, or *a* to *d* respectively. This arrangement then gives a choice of six different speeds; intermediate speeds can be obtained by field-rheostat control. This method is, of course, not available for motors operated from general-distribution mains where only one voltage is available (or two, as in the Edison three-wire system), but it can be used in shops or factories if the necessary mains and generators are installed. Of course rather elaborate switching arrangements are required in order to shift the connections of the motor from one voltage to another. The expense and complexity of the extra generators, wiring, and switching appliances required form a serious disadvantage to this system.

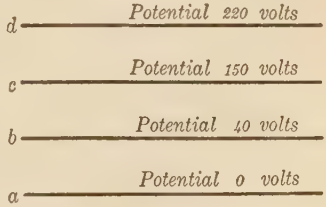


FIG. 124. Multi-voltage mains

THE SERIES MOTOR

107. Series-motor characteristics. The speed-torque curve for a series motor is shown in Fig. 125. Starting with some moderate load (torque OA in the figure), if the load is increased, the armature current $\left(= \frac{T}{1.41\, Z'\Phi}\right)$ increases; but this same current traverses the field winding, so that Φ also increases, and therefore the speed $\left(= \frac{E_t - R_a I_a}{Z'\Phi}\right)$ falls off. The point F corresponds to full load. Beyond F the curve continues to drop and would finally reach the horizontal axis at some point M, corresponding to clamping the armature at rest with full-supply voltage connected to its terminals, and OM would represent the maximum possible torque obtainable from the given motor. But the dotted portion of the curve corresponds to conditions outside the range of ordinary operation and is of interest only for understanding how the motor would behave if abnormal conditions accidentally occurred.

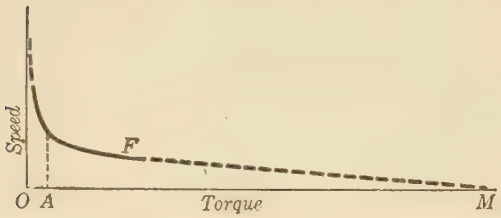


FIG. 125. Speed-torque curve of a series motor

Starting again with the load OA, if the load is decreased, the armature current decreases; this makes Φ smaller, and the motor speeds up. The smaller the load, the weaker the field and the higher the speed, which increases without limit as the load approaches zero. For loads less than a certain lower limit, the speed becomes so great that the motor cannot be operated without danger of flying to pieces by centrifugal force.

Fig. 126 shows the torque and the speed as functions of the current. With increasing current the torque increases in greater proportion. Since the torque is proportional to ΦI_a, and since Φ is roughly proportional to I_a (until the iron of the magnetic circuit is saturated), it follows that the torque is roughly proportional to the square of the current intake, up to saturation of the magnetic circuit; beyond saturation the torque continues to increase with increasing current, but less rapidly.

Summing up the characteristics of the series motor, we see that its speed varies with the load and becomes too high for safety at very light loads; for heavy loads the motor slows down, and the torque increases in greater proportion than the current intake.

108. Uses of series motors. The characteristics of series motors make them suitable for applications in which the speed varies widely; in such applications slow speed is usually permissible when the load is heavy, and high speed is desirable when the load is light. Series motors are not suitable for applications in which the load is ever *entirely* removed, since the motor would then run away. They are used for electric locomotives and street cars and for hoisting machinery and cranes.

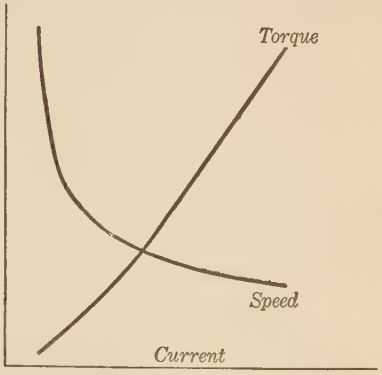


FIG. 126. Curves showing how the torque and the speed vary with the current in a series motor

In operating a street car rapid acceleration after stops is necessary, to prevent too great loss of time at the stops. This requires from the car motors during starting a torque many times greater than is necessary after the car has reached full speed. This requirement agrees exactly with the series-motor characteristic of delivering its greatest torque when the speed is slow. Street-car motors are permanently geared to the car axles, so that there is no possibility that the load can be entirely removed and thus permit the motors to run away.

In cranes and hoisting machinery slow speed is usually not objectionable while a heavy load is being raised; but greatly increased speed is very desirable at other times, such as when cable has to be wound in without load, or when a crane is moving unloaded to the place where it is to pick up a load.

109. Starting of series motors. In starting a series motor resistance must, of course, be placed in series with it until it has gained a certain amount of speed. On account of the intermittent nature of the service required from series motors in

most of their applications, the starting rheostat comes into action much more frequently than is usually the case with shunt motors, which generally remain in operation for a considerable period of time when once started. This requires greater current-carrying capacity in the case of the series-motor-starting rheostat, to avoid overheating. The automatic release of the shunt-motor-starting rheostat is not used with series motors, but protection against excessive current is provided by means of circuit-breakers.

110. Series-parallel control of street-car motors. A street car is usually provided with two motors, each geared to one axle. When the car is started from rest and brought up to full speed,

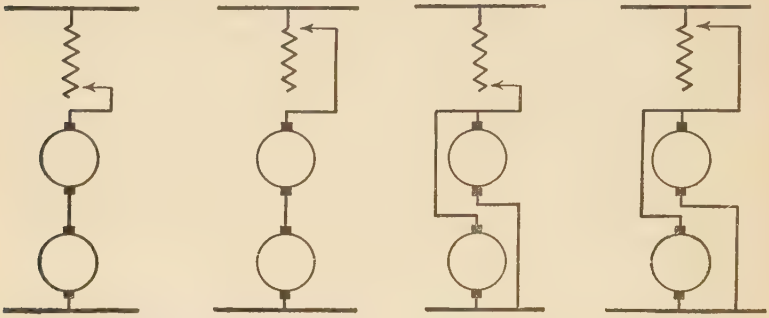


FIG. 127. Diagrams showing sequence of changes of connections in the series-parallel system of control

the motors are connected successively in the ways shown in Fig. 127. At first both motors are in series with the rheostat; resistance is gradually cut out as the motors gain speed, until the two motors are connected directly across the line in series with each other; each motor then receives half the line voltage across its terminals. If left connected in this way the speed will assume a value $\left(n = \frac{E_t - R_a I_a}{Z'\Phi}\right)$ determined by E_t and the road resistances (which determine the torque, and therefore I_a), and the speed will remain constant as long as the road resistance is constant. This speed is approximately half as great as can be attained with the same road resistance when the motors are connected in parallel across the line and each receives the full line voltage across its terminals. To shift from the series con-

nection to the parallel connection, resistance must again be inserted and then gradually removed as the motors gain additional speed.

While the rheostat is in circuit (connections *a* and *c*, Fig. 127), considerable loss of power takes place in it. Accordingly connections *a* and *c* should be used only for transition; but connections *b* and *d* may be held indefinitely, and accordingly represent permissible running conditions.

As compared with controlling each motor separately with a separate starting rheostat for each one, the series-parallel system has the advantage of providing two different running speeds and of permitting the starting of the car with less total loss of energy in rheostats. The latter consideration may be quite important where frequent stops have to be made.

THE COMPOUND MOTOR

111. Compound-motor characteristics. Compound-wound motors may have their field windings so connected that the series field contributes flux in the same direction as that of the shunt field, or in the opposite direction. In the former case the motor is called *cumulative compound*; in the latter, *differential compound.*

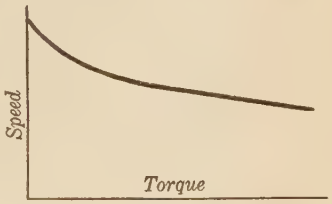


FIG. 128. Speed-torque curve of a cumulative compound motor

The speed-torque characteristic of a cumulative compound motor is shown in Fig. 128. As the load (torque) increases, with consequent increase in armature current, the field becomes stronger, and accordingly the speed decreases. Fig. 129 shows the speed and the torque as functions of the current. When the load is light, the flux from the shunt field predominates; when the load is heavy, the flux from the series field predominates. Accordingly the motor acts at light loads like a shunt motor and at heavy loads like a series motor. Like the series motor, it exerts heavy torque at low speed without proportionate increase in current; but unlike the series motor, it does not run away when the load is thrown off, but instead assumes a definite speed greater than its speed when loaded.

112. Uses of cumulative compound motors. The characteristics of the cumulative compound motor when loaded are so much like those of the series motor that most of the applications for which it would be suitable can be served equally well by the series motor. In cases, however, in which the load may be entirely removed at times, the compound motor has the advantage over the series motor, since it does not race when the load is removed.

Among other applications, cumulative compound motors are used for punches and shears, in which very heavy torque is required while the actual punching or shearing takes place, after which the load suddenly drops to almost nothing; and for pumps, which require heavy starting torque, but from which the load would be entirely removed if a pipe should burst.

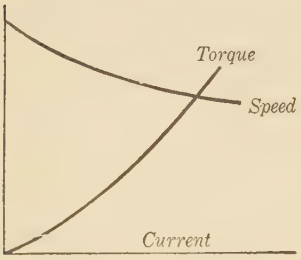


FIG. 129. Curves showing how the torque and the speed vary with the current in a cumulative compound motor

A compound motor can easily be converted into a shunt motor by short-circuiting the series field winding. This weakens the field and causes the motor to speed up (provided it is not too heavily loaded). Compound motors are sometimes used in this way for elevators. At starting, when heavy torque is required for accelerating the elevator, the series field is in circuit; after the full speed with series field has been attained, the series field is cut out. The speed then reaches a higher value than it could have reached if the series field winding had been left in the circuit.

113. The differential compound motor. The differential compound motor may be thought of as a shunt motor provided with a series winding in addition to the shunt winding. Since the series winding opposes the shunt winding, it causes the speed at all loads * to be higher than would be the case in its absence. According to the number of series turns, the motor may be under-compounded, flat-compounded, or over-compounded; that is, with increasing load the speed may fall off (although

* That is, at all loads within a certain limit, which would probably lie beyond the rated full load (See footnote to section 100).

to less extent than in the shunt motor), may remain constant, or may increase.

When under-compounded or flat-compounded, the characteristics of the motor are quite similar to those of the shunt motor, but its speed is more perfectly constant at different loads. For this reason differential compound motors are occasionally used when very perfect constancy of speed is desired (as in the motor for a motor-generator set); but in the great majority of applications shunt motors provide a speed that is nearly enough constant, especially as some adjustment of speed is possible by the use of a field rheostat.

When the motor is over-compounded, its speed increases with increasing torque. This means that such a motor cannot be used for a constant load: either it would break down and stop, or else it would run away. If, in trying to operate it, its torque were a little less than sufficient to carry the load, retardation of the armature would take place, and the torque would then become still less, so that the motor would eventually be brought to rest; if, on the other hand, its torque were initially a little too great for the load, then acceleration of the armature would take place, and the torque would then become still greater, so that the motor would speed up more and more. Thus its operation with constant load is unstable and therefore in practice impossible. It would be possible, however, to operate such a motor with a load that increases with the speed; for instance, in driving the propeller of a boat.

PROBLEMS

1. The small generator of Problems 1, 3, and 4, Chapter IV, is connected to run as a shunt motor across 110-volt mains, and its magnetic flux per pole is now 1,000,000 maxwells. Find its speed at full load (armature current 18 amperes); also the torque developed by the armature at full load.

2. The torque required to overcome friction being 12 pound-inches, find the armature current and speed of the machine in Problem 1 when unloaded.

3. If by accident the armature of the machine in Problem 1 should be connected (when at rest) directly to the 110-volt mains without a starting rheostat, find the armature current, (*a*) assuming that the 110 volts is maintained at the motor terminals and

(*b*) assuming that 110 volts is maintained at a distant point, connected with the motor by mains whose resistance is 0.6 ohm. For assumption (*a*) find the force in pounds tending to strip one side of an armature coil from the core. The coil contains 8 armature conductors (on each side); the flux density may be taken as 10,000 gausses.

4. The two motors of a certain electric car each have the following characteristics: normal running current, 50 amperes; armature resistance, 0.2 ohm; $\Phi Z'$ (when taking 50 amperes), 120 volts for each revolution per second. Find the speed and torque of the motors (*a*) when in series and (*b*) when in parallel across 550-volt mains.

5. With the conditions as in Problem 1, except that the motor is unloaded (that is, the armature current is negligible), the shunt field circuit is accidentally broken while the motor is running, and the flux per pole drops to 100,000 maxwells. What will the speed of the motor become?

6. Under the conditions of the preceding problem, suppose that the motor is now fully loaded (that is, the motor is required to exert the same torque as in Problem 1). (*a*) How much would its armature current become? (*b*) Assuming that it could stand this current, what would be its speed?

CHAPTER X

POWER LOSSES AND EFFICIENCY OF DIRECT-CURRENT DYNAMOS

The discussion in this and the following chapter will be carried out with direct-current dynamos especially in mind, but it applies to alternating-current machinery as well, with obvious slight modifications in a few places.

114. Power losses in dynamos. If the dynamo were a perfect machine, a generator should deliver as much power in electrical form as it receives in mechanical form, and a motor should deliver as much in mechanical form as it receives in electrical form. Practically, it is not possible to prevent some of the power put into a dynamo from escaping in other forms than that of useful output. That is, we meet with so-called "losses" of power or energy in dynamos; "leakages" would be a better name, since the energy is not really lost — in fact, we have to go to some trouble to dispose of it — but merely escapes or leaks from the stream of energy passing through the machine.

It is obviously important to keep these losses as small as practicable, not only in order that the dynamo may properly perform its function as a converter of energy between mechanical and electrical forms but also because the energy concerned is commercially valuable; and again because the "lost" energy takes the form of heat within the machine, which has to be removed. Much study and research have therefore been spent in the effort to produce dynamos which will operate with very small power losses, and such success has been attained that in the larger modern dynamos the losses amount to only 3 or 4 per cent of the output.

The principal losses in dynamos may be classified into three groups: (1) armature loss, (2) field losses, and (3) stray losses.

115. Armature loss. The armature loss is that due to the flow of the current through the armature resistance, and is accord-

ingly equal to $R_a I_a^2$, where R_a is the resistance of the armature between brushes,* and I_a is the current in the armature. The amount of this loss, for any armature current, can easily be computed when the armature resistance is known.

This loss evidently increases as the load (I_a) is increased.

The energy lost all takes the form of heat, appearing initially in the winding of the armature and passing by conduction into the insulation, core, shaft, etc.

116. Field losses. The field losses are those due to the flow of current through the resistance of the field windings, and are given for each winding by $R_f I_f^2$, where R_f is the resistance of the winding (including its field rheostat, if any), and I_f is the current in it.

The loss in a shunt field winding will be the same for all loads if the terminal voltage is the same, whereas the loss in a series field winding will increase with increase of load. In either case the amount of loss can be easily computed when the resistance and current are known.

The energy lost all takes the form of heat, appearing initially in the conductor material of the field winding (and rheostat, if any) and passing by conduction into the insulation, field cores, frame, etc.

The armature and field losses are sometimes grouped together under the name of *copper losses* because they occur in the copper parts of dynamos, although they are in no way peculiar to copper, but would occur just the same if any other known substance should be used as the conductor material for armature or field windings.

117. Stray losses.† The stray losses include all losses other than those due to the flow of current through the resistance of armature and field windings. They come principally from four sources: (1) mechanical friction, (2) air resistance, (3) hysteresis, and (4) eddy currents.

* This, of course, includes losses occurring in the brushes, brush contacts, commutator bars, and leads, along with the loss which occurs in the armature winding. The expression $R_a I_a^2$ is only approximately correct as regards the loss at brush contacts, which do not behave like ordinary resistances; instead, a definite drop in voltage (whose amount depends on the kind of brush but not on the current) occurs at a brush contact.

† As used here (and in many other textbooks of electrical engineering), the term *stray losses* has an entirely different meaning from the term *stray load losses* as used in the standardization rules of the American Institute of Electrical Engineers.

Mechanical friction takes place at the bearings of the shaft and at the brushes. The energy lost, of course, takes the form of heat, appearing at first at the rubbing surfaces and passing by conduction (and convection in the lubricant) to other parts of the machine.

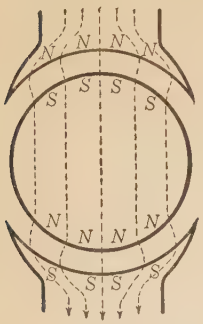


FIG. 130. Location of polar regions on an armature core

Armature at rest

Air resistance, or windage, takes place at the surface of all the moving parts. The heat produced from this loss is largely carried away by the air.

Hysteresis in the iron of the armature core causes loss of power, as may be understood from the following considerations: As the armature rotates, the magnetic flux in its core has to shift continually with respect to the core. Fig. 130 shows the magnetic lines through the armature core of a bipolar dynamo; a north pole (or polar region) exists where the lines pass from iron to air, and a south pole (or polar region) where they pass from air to iron. As the armature rotates, the polar regions on the core remain fixed in space, but shift with respect to the core. Owing to hysteresis, however, the magnetization of the core lags behind the magnetomotive force supplied by the field, with the result that the polar regions of the core persist a little after they pass beyond the field poles. This is shown exaggeratedly in Fig. 131. The forces of attraction between the field poles and the polar regions of the armature core then pull backward against the rotation of the armature. The energy used in maintaining the motion of the armature in opposition to these forces appears in the form of heat in the armature core.

FIG. 131. Displacement (exaggerated) of the polar regions on an armature core in motion, due to hysteresis

Eddy currents occur to some extent within the laminations of armature cores, even when the laminations are very thin. Forces accordingly act between the field and the currents in the armature core, in a direction opposing the motion of the armature. The energy used in maintaining the motion in

opposition to these forces appears in the form of heat in the armature core, just as in the case of hysteresis.

The hysteresis and eddy-current losses are often grouped together under the name of *core losses*, sometimes called also *iron losses*, although they are not peculiar to iron as such, but might occur in any other core material if future research should ever discover one as satisfactory as iron.

The stray losses cannot easily be calculated, but their total can be measured, provided the dynamo can be run as a motor, without load. Under these conditions its power output is zero, and its total power intake is absorbed by the losses. The power intake can be measured by a wattmeter (or by ammeter and voltmeter in direct-current machines); if the armature and field losses (computed from the resistance and currents in the armature and fields) are subtracted, the residue is the total stray loss.

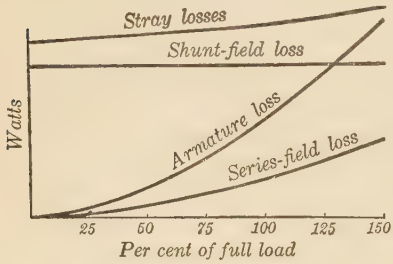


FIG. 132. Curves showing how the losses in a compound-wound generator vary with the load

The total stray loss is nearly independent of the load in constant-voltage machines. It varies with speed and field excitation, being approximately proportional to the voltage generated in the armature.

118. Variation of losses with load. To illustrate how the different power losses in a dynamo vary with its load, curves are shown in Fig. 132. The curves belong to a small flat-compounded direct-current generator rated for 125 volts and 16 amperes, but their general character would be the same for any other dynamo whose terminal voltage is constant.

The slight rising of the stray loss with increasing load is due to the fact that the total induced electromotive force $E\ (= E_t + R_a I_a)$ increases while the terminal voltage E_t remains constant.

119. Flow of energy through a dynamo. It is of interest to note at what parts of the flow of energy through a dynamo the different losses take place. For this purpose the diagrams of Fig. 133, for direct-current dynamos, are convenient. The armature and field losses are lost from the electrical power, but

the stray loss is lost from the mechanical power (even though some parts of the stray loss are not mechanical in origin).

120. Determination of the efficiency of dynamos. By definition, the efficiency of a dynamo is the ratio of power output to power input. In a generator the output is electrical power, and the input is mechanical power. Since it is very much easier to measure electrical power than mechanical power, and since the losses can be measured more easily (and with less gross error) than the mechanical power, it is customary to determine the power input of a generator from the relation

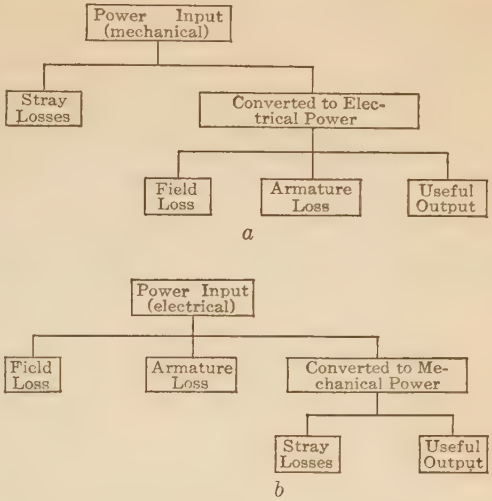


FIG. 133. Diagrams to represent the flow of energy in dynamos

a, generator; *b*, motor

$$\text{Power input} = \text{power output} + \text{losses}$$
$$= P_o + R_a I_a^2 + R_s I_s^2 + R_f I_f^2 + S, \qquad (49)$$

where P_o* is the power output, R_a, R_s, and R_f the resistances of armature, series field, and shunt field respectively, I_a, I_s, and I_f the currents in armature, series field, and shunt field respectively, and S the total stray loss. Then the efficiency is equal to

$$\frac{\text{Power output}}{\text{Power input}} = \frac{P_o}{P_o + R_a I_a^2 + R_s I_s^2 + R_f I_f^2 + S}. \qquad (50)$$

Similarly, in a motor the output of mechanical power may be determined from the relation

$$\text{Power output} = \text{power input} - \text{losses}$$
$$= P_i - R_a I_a^2 - R_s I_s^2 - R_f I_f^2 - S.\dagger \qquad (51)$$

* Equal to $E_t I$ in a direct-current generator.

† In direct-current motors $P_i = E_t I$, where I is the total current input.

Then the efficiency is equal to

$$\frac{\text{Power output}}{\text{Power input}} = \frac{P_i - R_a I_a^2 - R_s I_s^2 - R_f I_f^2 - S}{P_i}. \quad (52)$$

In practice the value obtained for the efficiency by using the relations (50) or (52) does not agree perfectly with that obtained by direct measurement of both input and output, for the reason that under actual running conditions the losses may differ somewhat from the values used for these losses (determined under other conditions). Accordingly the term *conventional efficiency* is used for the value obtained by use of the relations (50) or (52). The value obtained by actual measurement of both input and output of power is called the *directly measured efficiency.*

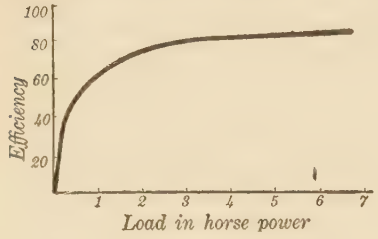


FIG. 134. Variation of efficiency with load in a certain five-horse-power motor

121. Dependence of efficiency upon load. The efficiency of a dynamo will evidently vary with its power output or load. The manner or variation, in the case of a certain 5-horse-power shunt motor, is shown by the curve of Fig. 134.

For other dynamos the corresponding curve varies somewhat in shape, since the value of the different losses varies with different machines. In all cases, however, the efficiency is zero at no load, increases with increasing load up to a maximum value, and beyond that falls off with further increase of load. Subject to these limitations, it is possible for the designer of a dynamo to modify the shape of its efficiency curve somewhat, since the relative magnitude of the different losses can be varied, even though it is not possible to make any one of them disappear entirely. For example, the armature loss can be diminished by increasing the cross section of the armature conductors; or the eddy-current loss, by using thinner laminations in the armature core.

In the choice of a dynamo for some particular purpose the shape of the efficiency curve is of interest, as well as the value of the efficiency at any one load. If it is expected, as in the majority of applications, that the machine will operate at or

near its full load most of the time that it is in use, then it is desirable that the efficiency curve have its maximum at full load; it is comparatively unimportant if the efficiency is low at light loads. But if the machine is likely to run a large part of the time at light load, as happens in many railway motors, then the efficiency curve should have its maximum at less than full load, and it is important that the efficiency be high throughout the range of loading at which the machine will be operated. The efficiency curve of Fig. 134 is suitable to the former case, and that of Fig. 135 to the latter case.

122. Relation between efficiency and size of dynamos. Large dynamos are more efficient than small ones. This is partly because it is easier to construct a large machine for high efficiency than a small one, and partly because the greater value of the energy concerned in a large machine justifies greater effort to obtain efficiency than would be worth while in a small one.

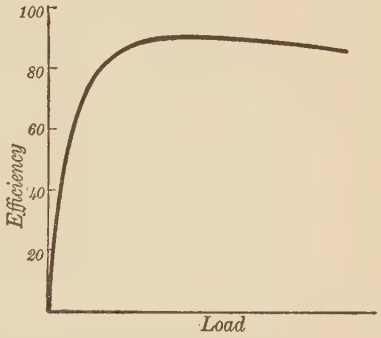


FIG. 135. Variation of efficiency with load in a certain railway motor

For instance, in such a machine as a fan motor, consuming perhaps 50 watts, the total power concerned is insignificant; at 10 cents per kilowatt-hour, such a motor could be run eight hours a day all summer at a cost of $3.60, and if its efficiency were as low as 50 per cent the lost energy would cost only $1.80. On the other hand, in such a machine as the 40,000-kilowatt generator of the Hell Gate Station of the United Electric Light and Power Company, intended for continuous operation at full load, even 1 per cent of the output amounts to 400 kilowatts; and if the electrical energy were as cheap as 1 cent per kilowatt-hour at the switchboard, this would represent $4 an hour, or $96 a day, or about $35,000 a year. Capitalizing this at 10 per cent gives $350,000 as a sum that it would be worth while to spend in order to gain a single per cent of increased full-load efficiency in such a machine.

Approximate values for the full-load efficiency of different sizes of dynamos (as at present constructed) are shown on the following page.

Rated Capacity (Kilowatts or Kilovolt-Amperes)	Full-Load Efficiency (Per Cent)
1	80
5	84
10	86
50	90
100	91
500	93
1000	95
5000	97
10000	97.2

PROBLEMS

1. Compute the efficiency of the small dynamo of Problems 1, 3, and 4, Chapter IV, when operated as a shunt generator delivering its full-load output of 2 kw. at 125 volts. The resistance of the shunt field (with field rheostat) is 60 ohms; the stray power losses are 225 watts. (Do not forget that the armature current differs from the current output.)

2. Find the efficiency of the same machine when operated as a shunt motor taking a total current of 20 amperes from 110-volt mains. Assume that the field rheostat has not been changed; the stray power losses now amount to 200 watts.

3. Find the efficiency of the generator of Problem 1 when delivering 1 kw. at 125 volts. The resistance of shunt field with rheostat is now 62.5 ohms; the stray power losses are 220 watts.

4. To run a certain shop 50 horse power is required for 8 hours a day, 305 days in the year. To supply this power, one of two 50-horse-power motors may be installed: motor *A*, which costs $900 and has full-load efficiency of 88 per cent, or motor *B*, which costs $1300 and has full-load efficiency of 92 per cent. The annual expense for upkeep, depreciation, interest, and taxes amounts to 15 per cent of the cost of the motor in each case, and the power used costs 3 cents per kilowatt-hour. Which motor would be the more economical to install? How much would it save annually compared with the other motor?

5. A certain 500-kilowatt generator runs at full load for about 4 hours a day, but for the remaining 20 hours its load averages only 100 kw., and its efficiency at this loading is 75 per cent. Would it pay to install a 100-kilowatt generator whose full-load efficiency is 92 per cent, to carry the light load during the 20 hours of the day (the 500-kilowatt machine being operated only during the hours of heavy load)? If so, how much would the annual saving be? The 100-kilowatt machine, with its prime-mover, would cost $4000 (which is equivalent to an annual expense of $600 for fixed charges). The power for driving the generators costs 1 cent per kilowatt-hour.

CHAPTER XI

CAPACITY AND RATING OF DIRECT-CURRENT DYNAMOS

123. Heating of dynamos. It has been pointed out how each of the losses in a dynamo results in the appearance of heat within the machine.

If a dynamo, originally at the temperature of its surroundings, is set in operation, this heat will cause its temperature to rise; then as the dynamo becomes warmer than its surroundings, it will begin to transmit heat to them at a rate depending on the difference between its own temperature and that of the surroundings. (The temperature of the surroundings is designated by the term *ambient temperature.*) If the load on the dynamo is kept constant (so that the rate of production of heat will be constant), a condition will finally be reached in which the rate of dissipation of heat into the surroundings just equals the rate of production of heat within the dynamo. In this condition all parts of the dynamo are hotter than the surroundings, but not equally so, since heat is more rapidly produced and less rapidly dissipated in some parts than in others. But the temperature at each point will have a definite value, constant as long as the load on the machine and the ambient temperature are constant. To reach this condition may require some hours (or even days, in very large machines) after starting the dynamo.

Evidently the temperature reached by any assigned point in the machine is determined by the load at which the machine is (continuously) operated, assuming that the ambient temperature remains constant.

124. Capacity of dynamos. The *capacity* of an electrical machine is the greatest load it can carry continuously* without exceeding safe conditions of operation. The operating conditions (speed, terminal voltage, etc.) must of course be specified; they should be the same as those for which the machine is

* Or for a specified time (instead of continuously), in the case of machines intended for intermittent operation.

intended. Capacity is expressed in terms of kilowatts of power output* (kilovolt-amperes in alternating-current machinery).

The capacity of a dynamo is limited principally by the rise of temperature with increasing output; but sparking at the commutator, or mechanical stresses due to heavy torque, may also limit the load that can be safely carried. It is a somewhat arbitrary matter to decide precisely where the limits of safe operation occur in any of these particulars. In order to have some common understanding, the American Institute of Electrical Engineers has drawn up elaborate rules governing the various matters that enter into the determination of capacity of electrical machinery.†

Injury from high temperature usually comes from deterioration of the insulation, which is a gradual process, proceeding more rapidly as the temperature is higher. The temperature limits permitted for different insulating materials by the Institute rules‡ are shown on page 151. These temperatures, of course, refer to those at the hottest spot in the insulation in question.

The metallic parts, if not exposed to the air, can stand without injury any temperature up to their respective melting points. A great increase in the capacity of a dynamo (so far as this is limited by temperature) might then be attained by constructing it with refractory insulation (Class C) throughout. It would not be practically advantageous to do this, however, because (among other reasons) the efficiency would be low on account of the greatly increased RI^2 losses.

125. Importance of facilitating the escape of heat. Since the principal limitation upon the capacity of a dynamo is due to the rise of its temperature during operation, any arrangement that hastens the escape of heat also increases the capacity. If it were possible to remove instantly, just as fast as it is produced, the heat which appears in the various parts of a dynamo, then no rise of temperature would take place at all, and there

* For motors it is more common to express the capacity in horse power; but the American Institute of Electrical Engineers recommends expression in kilowatts for motors as well as for generators.

† Standards of the American Institute of Electrical Engineers (1922 revision); also A. I. E. E. Standards No. 5: Direct-Current Rotating Machines, Generators, and Motors, July, 1925.

‡ A. I. E. E. Standards No. 1: General Principles upon which Temperature Limits are based in the Rating of Electrical Machinery and Apparatus.

TEMPERATURE LIMITS FOR INSULATING MATERIALS

CLASS	DESCRIPTION OF MATERIAL	MAXIMUM TEMPERATURE TO WHICH THE MATERIAL MAY BE SUBJECTED
O	Cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in oil.	90° C.
A	Cotton, silk, paper, and similar organic materials when impregnated or immersed in oil; also enamel as applied to conductors.	105° C.
B	Inorganic materials such as mica and asbestos in built-up form combined with binding substances. If Class A material is used in small quantities in conjunction for structural purposes only, the combined material may be considered as Class B, provided the electrical and mechanical properties of the insulated winding are not impaired by the application of the temperature permitted for Class B material. (The word *impair* is here used in the sense of causing any change which could disqualify the insulating material for continuous service.)	125° C.
C	Inorganic materials such as pure mica, porcelain, quartz, etc.	No limit yet selected.

would be no limit to the capacity of the machine, in so far as the capacity depends on rise of temperature.

The escape of heat from a dynamo takes place first by conduction from the interior to the surface, and then by radiation from the exposed surfaces, or by passage from the surface to the air and flow of the heated air away from the machine.

To facilitate radiation, the exposed surfaces of dynamos are painted black; to facilitate the escape of heat by means of air currents, as good ventilation as possible is provided throughout the machine. The frames of dynamos are so constructed as to allow free circulation of the air;* ventilating ducts through the interior of the armature are provided in most machines above the smallest sizes; sometimes vanes are attached to the moving parts to stir the air in the neighborhood of the machine, and in very large machines forced ventilation is often provided.

The machine should always be located in as well-ventilated and cool a place as possible. So far as concerns limitations due

* Exceptions occur in the case of machines which must be used in dusty or wet surroundings, such as railway motors.

to heating, the capacity depends on the temperature of the surroundings; in cold surroundings it is safe to load a dynamo more heavily than in warm ones.

It is interesting to note that since the heating arises entirely from the losses of power within the dynamo, anything that improves the efficiency (decreases the losses) at the same time increases the capacity, so far as this is limited by rise in temperature.

126. Relation between the size and capacity of a dynamo. It can be shown that the capacity of a direct-current dynamo is related to the size and speed of the machine according to the equation

$$C = \xi d^2 l n, \tag{53}$$

where C is the capacity in kilowatts, d the diameter of the armature, l its length (within the magnetic field), n the speed, and ξ a coefficient (called the *output coefficient*) depending on several features of design which are not absolutely constant in different machines, but vary within fairly narrow limits. If d and l are expressed in inches, and n in revolutions per minute, the value of ξ generally lies between 0.000015 (small machines) and 0.000055 (large machines).

Equation (53) is known as Kapp's output equation. It is often useful in making a rough or preliminary estimate of the size of machine for a given capacity.

127. Relation between speed and capacity. From equation (53) we see that, other things being equal, the capacity increases with the speed; or, for a given capacity, the size of machine required is less as its speed is greater. Accordingly, in selecting generators or motors for installation, it is advantageous to choose machines with the highest speeds suitable to the purpose in question, thus economizing in first cost and in space occupied.

128. Rating of dynamos. It is customary to attach to every dynamo a name plate or rating plate bearing the name of the maker, the type of the machine, the maker's serial number, and certain information about the conditions of operation for which the machine is intended, including capacity, speed, voltage, and current.

The figure for the capacity given on the rating plate is called the *rating* (or *rated output*) of the dynamo; the figures for the other quantities are called the *rated* speed, voltage, and so on.

There is a distinction between capacity and rating. The capacity is the greatest power output consistent with safe operation (as defined by whatever specifications are agreed upon); it is therefore an actual physical quantity belonging to the machine and is determinable by test. The rating, on the other hand, is simply the figure for the output marked on the name plate.

The rating ought to equal (or be a little less than) the capacity under the hottest surroundings in which the machine is likely to operate,* but conservative manufacturers sometimes rate their machines considerably lower.

129. Overloads. An *overload* is a load in excess of the rated output. Moderate overloads *for short periods of time* are entirely permissible in operating a dynamo, provided the temperature of the machine has not already reached the limit of safety.

The possibility of injury from overload comes from (1) increase in torque, (2) sparking at the commutator in direct-current machines, and (3) production of heat. There is always ample mechanical strength to take care of the increased torque, and machines meeting the standards of the American Institute of Electrical Engineers will stand momentary overloads of 50 per cent without injury to the commutator. But the increased production of heat will eventually, though not immediately, raise the temperature of the machine to a value which may exceed the limit of safety.

It is very common to specify for a given dynamo the permissible time for some stated overload. Thus most direct-current generators are intended to be able to stand an overload of 25 per cent for two hours and one of 50 per cent momentarily, the machine having previously been running continuously at rated full load.

In the practical operation of dynamos it is a very common occurrence that demand arises temporarily for a greater load than that for which the machine is rated. To decide just how far it is safe to overload a generator or a motor is a matter calling for good judgment on the part of the operator and involving such considerations as the probable duration of the

* The standard ambient temperature fixed by the Institute rules is 40° C. The rules require that machines destined for use with higher ambient temperatures shall be the subject of special guaranty by the manufacturer.

overload, the temperature of the machine when the overload is first applied, the temperature and ventilation of the surroundings, and the general condition of the machine. It should be borne in mind that the carrying of an overload without actual breakdown is no proof that the machine has not suffered; deterioration of the insulation may have been caused.

130. Overspeeds and overvoltages. Within moderate limits it is permissible to operate a dynamo (continuously) under other conditions than those for which it is rated, although this usually involves some sacrifice of efficiency.

If the speed is above normal (the terminal voltage being normal), there is increase in the tendency to spark at the commutator, in vibrations, and in stresses due to centrifugal force. The standards of the American Institute of Electrical Engineers require ability to withstand an overspeed of 25 per cent without injury.

If the speed is below normal, the field excitation must be increased (in the case of a generator) to maintain the terminal voltage at its rated value. A lower limit of speed is thus fixed by the amount of field excitation available.

If the voltage is above normal, the dielectric stress in the insulation is increased. So large a factor of safety is provided in the insulation* that a very great increase in voltage would be necessary to break down the insulation — much greater than could be attained in a generator at normal speed with its maximum field excitation; but the increased difference of potential between adjacent commutator bars increases the possibility of flash-over between them.

PROBLEMS

1. A 1000-kilowatt dynamo is running at full load. If its efficiency is 95 per cent, find how many calories of heat, due to its losses, it produces per hour. How much coal would have to be burned to produce the same amount of heat, the calorific value of coal being about 8000 calories per gram?

2. What would be the approximate size (volume) of the armature of the machine in Problem 1 if its speed is 300 revolutions per minute?

* The standards of the American Institute of Electrical Engineers require the insulation to be able to stand a voltage equal to twice the normal voltage of the machine, plus 1000 volts.

3. In choosing a motor for driving the main shaft of a shop, a certain motor A is being considered, and a motor B, of the same capacity but twice the speed. Compare the volume of the armature of B with that of A. Assuming that the cost is approximately proportional to the volume, how much cheaper is B than A?

4. Compute the total amount of heat produced in one hour of running the small generator of Problems 1 and 3, Chapter X, (*a*) if the generator runs at full load (output, 2 kw.) the whole hour and (*b*) if it runs at 50 per cent overload (output, 3 kw.) for 15 min. and then at half load (output, 1 kw.) for 45 min. When running at 50 per cent overload, the resistance of the shunt field circuit is 58 ohms and the stray power losses are 240 watts.

CHAPTER XII

ALTERNATING-CURRENT GENERATORS

It has already been pointed out (sect. 57, p. 63) that the electromotive force developed in any dynamo alternates in direction with respect to the armature conductors in which it is generated.* In direct-current dynamos it is only through the operation of a commutator that one terminal is kept continuously positive and the other continuously negative. It is, of course, possible to omit the commutator and provide other means for connecting the armature winding to the outside circuit, and when this is done in a generator the voltage delivered (and any current it may drive) alternates in direction.

131. Advantages and disadvantages of alternating currents. For the majority of applications of electrical energy it is possible to use alternating currents as well as direct currents. This is true of most lighting, heating, and power applications, although in any individual case alternating currents may be less satisfactory than direct currents or may be more satisfactory. But for most applications there are no prohibitive difficulties in the way of using alternating currents, and, broadly speaking, alternating currents and direct currents are about equally satisfactory for the majority of applications.

In addition, the use of alternating currents has the very great advantage that the voltage can be easily and economically transformed to any desired values by the use of stationary transformers. This not only gives flexibility to alternating-current systems by making it possible to use different voltages in different parts of the same system, but it also makes possible the economical transmission of electrical energy over long distances, since the voltage of the generator can be transformed and delivered to a transmission line at very high voltage and then, at distant points of the line, transformed down again to such voltages as may be needed for various applications at

* Except in homopolar dynamos, which are very rarely used.

those points. This ease of transformation is the great outstanding advantage of alternating currents.

On the other hand, alternating currents have the disadvantages that they cannot be used at all for charging storage batteries or for many other electrochemical applications, or for the excitation of the fields of dynamos; they are less satisfactory than direct currents for adjustable-speed motors and for arc lamps, and they are not well adapted for use in systems where the load must be carried at times by a storage battery.

When an electrical system is to be installed to meet the general needs of a community, the one great advantage of alternating currents usually outweighs the disadvantages mentioned. If there are parts of the system where direct currents must be used, it is possible to supply them by means of some form of converter from alternating to direct currents, such as a motor-generator set. Accordingly most of the general distribution of electrical energy is effected by means of alternating currents, although there are many special situations for which direct-current distribution is more suitable.

A minor advantage in favor of alternating currents is that alternating-current apparatus is, as a rule, somewhat simpler in construction and operation than direct-current apparatus. On the other hand, the theory of its action is usually more complicated.

132. Alternators. The alternating-current generators commonly used in present practice operate by the relative motion of an armature and a *constant* magnetic field. Such machines are called *alternators*. The term *alternating-current generator* is more general, including possible types (like the induction generator discussed in section 223) which operate by means of a *varying* magnetic field.

The fundamental principles of action of an alternator are the same as those of a direct-current generator, and the main features of its construction are accordingly much the same.

133. Construction and excitation of field. In revolving-armature alternators the construction of the field frame and field winding is substantially the same as in direct-current generators. Revolving-armature alternators are not often used at present, except for very small machines.

In revolving-field alternators the armature conductors lie on the inside of a frame within which the field structure revolves,

as explained in section 51. Fig. 136 illustrates such an armature. Two types of field structure, or "rotor," are in use: the salient-pole type and the cylindrical type.

In the salient-pole type the poles stick out radially from a central mass of iron (corresponding to the yoke in a stationary field) which commonly has the form of a heavy ring attached by spokes to a hub and shaft, as shown in Fig. 137. This type is suitable for machines that run at the lower speeds.

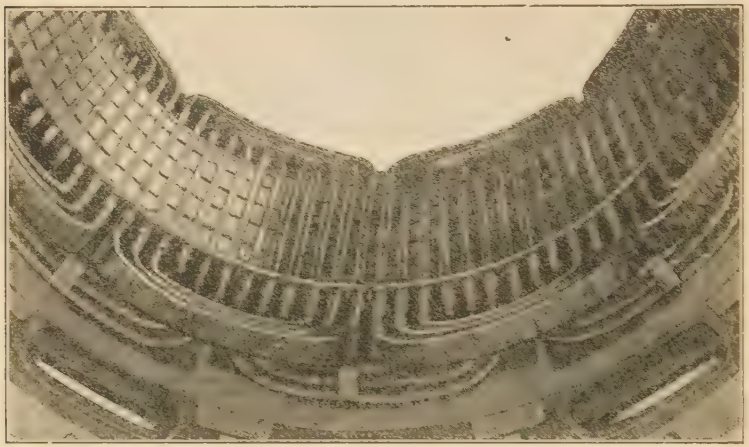


FIG. 136. Part of a stationary armature
Courtesy of the Wagner Electric Manufacturing Company

In the cylindrical type the rotor has a smooth surface without projecting poles, and the winding is placed in slots as shown in Fig. 138, thus producing a set of polar regions on the surface of the rotor. This type is suitable for the highest speeds, since it is mechanically stronger and more free from air friction than the salient-pole type and can be more perfectly balanced. It is used in turbo-alternators, or alternators directly connected with steam turbines.

In both types the direct current for exciting the field is brought to its winding through slip-rings on the rotor shaft.

The advantage of the revolving-field type of alternator over the revolving-armature type is that comparatively small current has to be carried through sliding contacts, the field current being only a small fraction (from 2 to 5 per cent) of the full-load

armature current. If the armature revolves, its whole current has to pass through sliding contacts; if it is stationary, its terminals can be soldered securely to the mains leading from it.

Alternators are always separately excited, since their own output cannot be used for field excitation.* The direct current for field excitation is usually derived from a small auxiliary direct-current generator known as an *exciter*. The exciter may be belted to the alternator, or it may be placed on the same shaft as the alternator, or it may be driven entirely independently. In central stations exciter bus bars are placed behind the switchboard and supplied with direct current from one or more generators; the exciting current for any alternator can then be taken from them.

FIG. 137. Salient-pole rotor, showing some of the poles removed from the spider

Courtesy of the Westinghouse Electric and Manufacturing Company

134. Frequency of alternators. The frequency of an alternating current or voltage is the number of complete cycles per second, a cycle being the complete range of values assumed by the alternating quantity in starting from zero and passing through its positive values, returning to zero and passing through its negative values, and coming back to zero again. When an alternator is in operation it is evident that a cycle must be completed every time *two poles* pass by a point of the armature (in a revolving-field machine), or every time a point of the armature passes by two poles (in a revolving-armature machine). Accordingly the frequency will equal the number of revolutions per second multiplied by half the number of poles, or

$$f = \frac{np}{2}, \tag{54}$$

* Unless converted to direct current, which would require apparatus offering no advantage over the use of an exciter.

where f is the frequency, n the number of revolutions per second, and p the number of poles. Of course the frequency of all the currents and voltages throughout an alternating-current system is the same as that of the generators which supply it.

135. Standard frequencies. In this country the frequency of 60 cycles per second is the standard for lighting and general applications of alternating currents. In some motor applications, especially for traction, a lower frequency has advantages; for such applications 25 cycles per second is the standard.

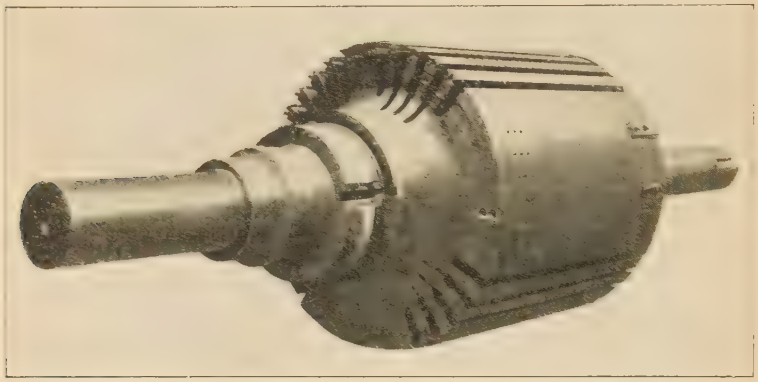


FIG. 138. Cylindrical rotor (without winding)

Courtesy of the Westinghouse Electric and Manufacturing Company

Other frequencies, however, have been used in the past and are still used occasionally; in foreign countries different frequencies are common, especially that of 50 cycles per second.

Unfortunately there is no such easy way to transform from one frequency to another as there is from one voltage to another. Transformation of frequency can be accomplished only by means of moving mechanism, which is comparatively inconvenient and inefficient. With the extension of electrical transmission systems it often becomes desirable to connect two or more systems together, as will be discussed in section 253. Therefore it is especially desirable that the same standard of frequency should be used universally, to facilitate interconnection between systems not now connected, and avoid the necessity for any change of frequency when interconnection is made.

It does not seem practicable, however, to reduce the number of standard frequencies below two (60 and 25). For lighting, it is necessary that the frequency be not too low (not less than about 40); otherwise flickering of the lamps is perceptible. On the other hand, for some motor applications a much lower frequency offers decided advantages (see section 231). An effort was made at one time to compromise on 40 cycles per second as a single standard, but this has now been abandoned.

136. Armature windings of alternators. The winding of the armature of an alternator is usually of the open-coil type; that is, the winding starts from one terminal and ends at another without closing upon itself; it forms an open circuit except as closure may take place outside the machine, between the terminals of the winding. Fig. 139 shows by diagram the usual arrangement of conductors in a single-phase alternating-current armature winding. The conductors lie in bands *a*, *b*, *c*, etc., the center of each band occupying the same position relative to the nearest magnetic pole.

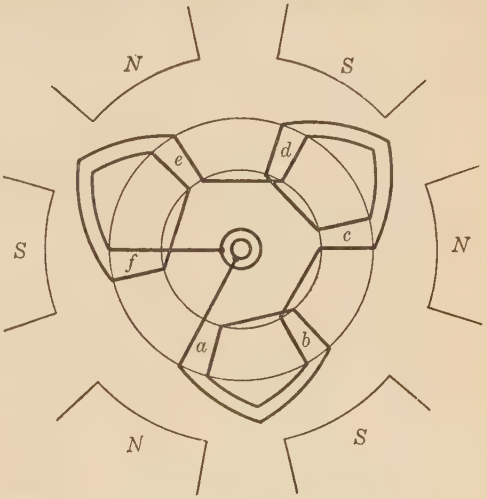


FIG. 139. Diagrammatic representation of an alternator armature winding

The number of conductors in a band is ordinarily larger than that shown in the figure, which is merely diagrammatic; but the width of the bands is usually much less than the distance from one pole to the next, the winding thus not covering the whole armature surface but lying in bands with unfilled spaces between.* The reason for this is that if the bands were widened so as to cover the whole armature surface, as shown in Fig. 140, there would be some pairs of conductors, such as *4* and *5*, or

* Unfilled in a single-phase armature, to which the present discussion refers; filled with other windings in a polyphase armature.

8 and *9*, which occupy nearly the same position but are traversed in the opposite direction in passing through the winding; the electromotive force induced in one conductor of such a pair would accordingly neutralize at all times that induced in the other (very nearly). Thus there would be no advantage in including such a pair of conductors in the winding, as they would not appreciably increase the electromotive force induced in the winding, although they would add to its resistance.

137. Collector-rings and brushes. The electrical connection between the winding of the rotating member (whether it be the armature or the field) and its outside circuit is effected by means of slip-rings or collector-rings. These are rings of bronze or copper mounted on the shaft concentrically with it and insulated from it, and connected with the ends of the winding. The brushes rest against the collector-rings. The construction and mounting of brushes are much the same as in direct-current machines, except that metal brushes are occasionally used and that the brush-holders are supported directly from the frame of the machine and not from a rocker arm, since there is no need to provide means for shifting the brushes.

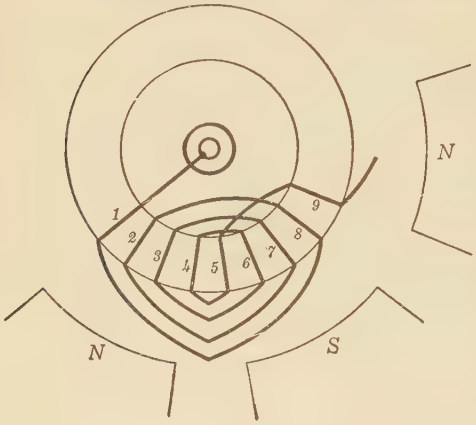


FIG. 140

138. Wave-form of electromotive force from an alternator. Consider first the electromotive force induced in a single armature conductor of an alternator. Its value at each instant is equal to the rate of cutting of magnetic flux at that instant, and will accordingly vary with the time somewhat as shown in the curve of Fig. 141. The shape of the curve in any actual case is determined by the distribution of magnetic flux along the pole face and in the gaps between the poles. The negative half-wave will be exactly similar to the positive half-wave, provided

the distribution of flux is exactly the same (with change of sign) along the two adjacent poles responsible for the two halves of the wave; and all succeeding waves will be exactly similar to the first if the distribution of flux is the same along all the poles. In actual machines the distribution of flux is approximately the same for all poles, but it should be borne in mind that this is not necessarily the case. It is *possible* to have alternators in which the negative half-waves are quite different in their form from the positive half-waves.

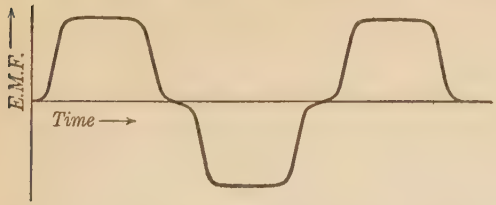


FIG. 141. Curve showing variation of electromotive force with time in an alternator conductor

Nonsinusoidal wave-form, corresponding to a case in which the flux density is nearly constant across the pole faces

The distribution of magnetic flux density along a pole face is influenced by the length of the air gap. By making this different at different parts of the pole face, the wave-form of the electromotive force induced in an armature conductor can be made to differ at the will of the designer of an alternator. For instance, by shaping the pole faces as shown in Fig. 142 the wave will be higher at the center than at the edges. It is desirable that the wave should have the form of a sine curve; accordingly most alternators are constructed with the air gap shorter at the middle of the poles than at the edges. When the wave has the form of a sine curve, the electromotive force (or current) is called *sinusoidal*.

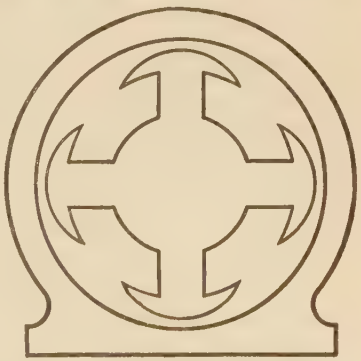


FIG. 142. Showing how the pole faces may be shaped so as to make the flux density greater at the center than at the edges

The electromotive force then has a more peaked wave-form than that of Fig. 141

Whatever be the wave-form of the electromotive force induced in any one conductor, that in every other conductor will

be identically similar; but it will be displaced in time,* as shown in Fig. 143, since different conductors pass corresponding points of the magnetic field at different times.

The electromotive force induced in the whole winding is simply the resultant of that in all the conductors. On account of the displacement in time of the electromotive-force waves in the different conductors, it is evident that the total electromotive force cannot be found merely by taking the product of the number of conductors Z by the electromotive force in one of them. The maximum value of the total electromotive force is less than Z times the maximum for any one conductor, and the wave-form of the total electromotive force differs † from that of the electromotive force of any one conductor.

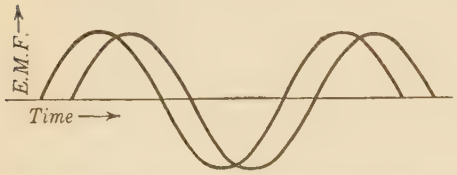


FIG. 143. Electromotive-force curves for two different conductors on the same alternator armature

When mains are led out from the terminals of an alternator, the electromotive force of the machine is of course transmitted to them and to all branches from them, so that the voltage across mains at any point in the system goes through a set of instantaneous values like the electromotive force of the generator; but the wave-form of the voltage across mains is not necessarily the same as that of the electromotive force induced in the generator, nor is it necessarily the same in all parts of the same transmission system. When an alternating electromotive force is applied to a circuit the current that flows alternates, of course, also, but the wave-form of the current is not necessarily the same as that of the voltage that maintains it.‡

* Except in the case of conductors occupying the same slot.

† Except when the wave-forms are sinusoidal.

‡ When an alternating electromotive force is applied to a circuit, the current (and therefore also the voltage drop to any point of the circuit) depends not only upon the resistance of the circuit but also upon its inductance and capacitance, in accordance with the theory developed in Chapter XIV. The effects of inductance and capacitance are different according to the frequency.

When an alternating quantity follows a wave-form which is not sinusoidal, it is equivalent (by Fourier's theorem) to the resultant of components which are sinusoidal, but the frequencies of which differ (being integral multiples of the fundamental frequency). When a nonsinusoidal electromotive force is applied to a circuit, its different components contribute components to the current, of corresponding

139. Effective values of alternating quantities. In very many features of alternating-current behavior the instantaneous values and wave-forms of electromotive forces or currents are not of interest, but it is of interest to be able to state by a single figure how intense the alternating electromotive force or current is. For this purpose the intensity of an alternating electromotive force or current is expressed in terms of *effective* volts or *effective* amperes.* The *effective* value of an alternating quantity is defined as the square root of the average value of the squares of the instantaneous values throughout a cycle.

The principal reason why this comparatively complicated quantity is used rather than the simple average throughout a half-cycle, arises from the behavior of measuring instruments when used with alternating quantities. Every type of ammeter (or voltmeter) which is suitable for use with alternating currents gives a deflection which is (approximately) proportional to the *square* of the current (or voltage). When such an instrument is used with alternating currents, its deflection is accordingly determined by the *average of the square* of the current (or voltage), the moving parts of ordinary instruments being much too slow-acting to follow the instantaneous values of alternating quantities of the usual frequencies.†

Another reason why effective values are convenient is that the heating effect of a current depends on its square. In the case of an alternating current the heating would accordingly depend on the average of the square of the current, which is the same as the square of the effective value. So the formula for the rate of production of heat (or power lost in heat), $H = RI^2$, holds true without change for alternating currents when I is taken as the effective value of the current.

frequencies, but of magnitudes which are *not* proportional to those of the components of the electromotive force (since the effects of inductance and capacitance vary with the frequency). Accordingly the resultant of the components of current will in general have a wave-form differing from that of the electromotive force.

* The term *r.m.s.* value is sometimes used to mean the same as the effective value, r.m.s. being abbreviated from "root-mean-square." In England the term *virtual* value is frequently used instead of effective value.

† Instruments in which the deflection is proportional to the first power of the current (like those of the D'Arsonval type described in section 91) cannot be used with alternating currents, since the direction of deflection reverses with the current. If they are connected with an alternating-current supply, the needle simply vibrates slightly about the zero point: before it has time to deflect appreciably in one direction, the current reverses and tends to deflect it in the other direction.

PROBLEMS

1. Find at what speed the revolving field of Fig. 137 will have to turn to generate electromotive force of frequency 60 cycles per second. At what speed will that of Fig. 138 have to turn to generate the same frequency?

2. Beginning at a point midway between the poles of an alternator and going along the armature past a pole, the magnetic flux density was found to vary as follows: for distance in centimeters from the starting point of 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40 the flux densities in gausses were 0, 2000, 7000, 9000, 10,000, 11,000, 10,000, 9000, 7000, 2000, and 0 respectively. If this field moves past an armature conductor 40 cm. long with speed of 1500 centimeters per second, find the instantaneous voltage generated in the conductor at each of the positions specified, and find the effective voltage.

3. 550 kw. of power is to be delivered at a point 10 mi. from a generating station by means of a transmission line consisting of two No. 3 copper wires (resistance 0.20 ohm and weight 160 pounds, per 1000 feet of wire). The voltage between mains is 11,000 volts at the point of delivery of the power. Find how much power is lost in the line, and find the cost of the copper in the line at 13¢ a pound. If the power were delivered at 550 volts, instead of 11,000, find how much the copper in the line would then cost if sufficiently heavy mains were provided to keep the line loss of power the same as before.

4. In some radio sending stations alternating currents of a frequency of 10,000 cycles per second are obtained from generators designed for high frequency. Assuming that 3600 revolutions per minute is the highest practicable speed for driving the generator, how many poles would it have to have?

CHAPTER XIII

ALTERNATING-CURRENT MEASURING INSTRUMENTS

140. Instruments for observing instantaneous values; the oscillograph. The instantaneous values and wave-form of an alternating electromotive force or current can be observed experimentally by several methods. By far the most convenient is by use of the oscillograph, which is essentially a galvanometer with exceedingly light moving parts — so light that the deflection corresponding to any instantaneous value of an alternating electromotive force or current will be accomplished in an interval of time small in comparison with the period of alternation. Thus, if the frequency of the alternating current is 60 cycles per second, the instrument must be sufficiently quick-acting to assume its deflection in a small fraction of a sixtieth of a second.

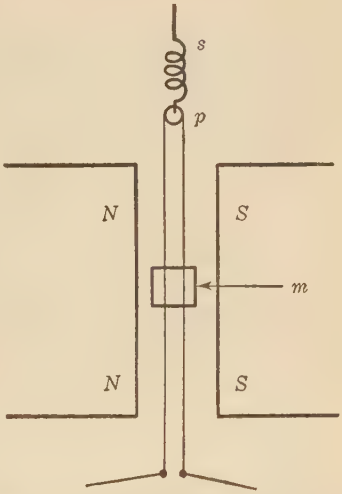


FIG. 144. Diagram to show the principle of action of the oscillograph

Fig. 144 shows diagrammatically the construction of a commonly used type of oscillograph, developed by Duddell. The moving part consists of a single very narrow loop of phosphor-bronze wire placed between the poles of a strong magnet and carrying a small mirror m. The loop passes over a pulley p at the top, suspended by a spring s. When a current flows through the loop, the forces from the magnetic field upon it cause deflection of the mirror (the necessary antagonizing torque being supplied by the spring s). Since the moving system has very small moment of inertia in proportion to the torque acting upon

it, deflection is accomplished very quickly, and any change in current is followed very quickly by change of deflection.* To observe the deflections due to a rapidly varying current, a beam of bright light is reflected from the mirror *m*. If the beam is again reflected from a revolving mirror, the behavior of the varying current can be observed by the eye; or if the beam is allowed to fall on a moving photographic film, a permanent record can be obtained. Fig. 145 shows such a record.

The current-carrying element of an oscillograph is a very delicate appliance and is adapted for carrying only very small currents. When it is to be used for observing larger currents, it is connected in parallel with a (noninductive) shunt, and

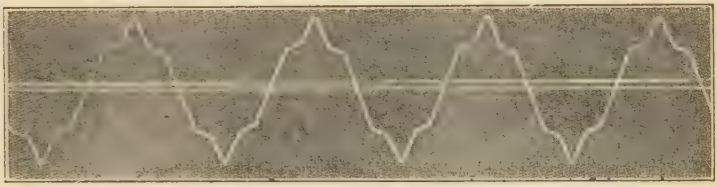


FIG. 145. Oscillograph record

thus carries only a fraction of the current. When it is to be used for observing voltages, it is connected in series with a high (noninductive) resistance.

141. Instruments for observing effective values; ammeters. The principal types of alternating-current ammeters for observing effective rather than instantaneous values are the electrodynamometer type, the induction type, the plunger type, and the hot-wire type.

In the *electrodynamometer type* the current (or some definite fraction of it) passes through a movable coil and a fixed coil, in series. The movable coil is suspended within the fixed coil; and when a current flows, the forces between the two coils cause the movable coil to deflect until the springs attached to the moving system are distorted sufficiently to balance the deflecting torque. A pointer is attached to the moving system to show the indications of the instrument on a scale. The force between the moving and fixed coils is at each instant propor-

* The period of free vibration of the moving system of an oscillograph is of the order of 0.0002 second.

tional to the square of the current (for any given relative position of the two coils), and remains in the same direction when the current reverses, since it reverses in both coils at the same time. The correctness of the indications is not affected by the frequency of the current,* and the instrument can be used for direct currents as well as for alternating currents.

In the *induction type* the current energizes a system of fixed coils so as to produce a rotating magnetic field in the space within them and thus cause a torque to act on a cup-shaped conductor suspended by springs within the coils, the action being of the same nature as that in an induction motor, explained in section 214. The suspended conductor carries a pointer for showing the indications of the instrument on a scale. No electrical contact with the moving system is required. The instrument will not respond to direct currents; and with alternating currents its indications are affected by the frequency of the current, so that it reads correctly only for some one particular frequency. In most practical applications this last feature is not particularly objectionable, since the instrument is calibrated for the standard frequency of the system on which it is to be used, in which the actual frequency does not often differ much from its intended standard value.

In the *plunger type* of ammeter the current passes through a fixed coil, producing a magnetic field which acts on a movable plunger of soft iron. The plunger is mounted on pivots, and its shaft is attached to spiral springs and carries a pointer. The torque acting on the plunger at any instant is proportional to the intensity of its magnetization multiplied by the field strength, but the intensity of its magnetization is (approximately) proportional to the field strength. The field strength being proportional to the current, the torque is thus proportional (approximately) to the square of the current. The correctness of the indications is so little affected by the frequency that the instrument can be used equally well on any frequency in common use (though not on radio frequencies) and on direct current. A widely used form for the plunger is that of an

* This statement applies to instruments in which the whole current traverses both coils. If the instrument is arranged so that either or both of the coils carry only some fraction of the current, then the correctness of its indications may be affected by the frequency. Within the range of ordinary frequencies, the effect is usually very small.

inclined vane (developed by Elihu Thomson), as shown in Fig. 146; when current flows in the coil, the vane tends to set itself parallel to the axis of the coil.

In the *hot-wire type* the principle of action is to measure the current by observing the expansion due to rise of temperature of a wire when it is traversed by the current. The rise of temperature is a function of the average square of the current. The expansion of the wire can be made to cause motion of a pointer, movable over a graduated scale. The instrument can be used equally well for any frequency (including radio frequencies) and for direct currents. Hot-wire ammeters can be constructed very cheaply, but it is difficult to prevent the wire from altering its characteristics in consequence of repeated heating and cooling.

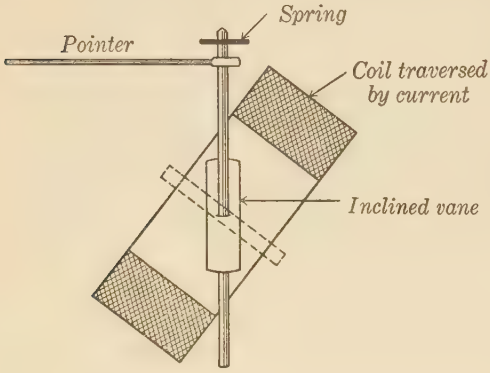


FIG. 146. Diagram to show principle of action of inclined-vane ammeter

In another type of instrument which depends on the heating effect of the current, the heat is made to raise the temperature of a thermo-element and thus cause a weak direct current to flow through an ammeter of the D'Arsonval type. The thermo-element and ammeter coil form a local circuit not connected electrically to the line; but the strength of the line current determines the strength of the current in the local circuit, and the ammeter scale is graduated to read the value of the line current.

142. **Calibration of alternating-current ammeters.** Since the ampere is defined as that current which will deposit silver at the rate of 0.00111800 gram per second, standard ammeters should be calibrated by connecting them in series with cells for the electrolytic deposition of silver and making comparison between ammeter readings and corresponding rates of deposition of silver. Alternating currents cannot be used, of course, since the electrochemical action would reverse with each reversal of current. But electrodynamometers can be calibrated on direct current by means of a silver cell, and then their readings on alternating current will give correct

values of effective amperes. Other types of alternating-current ammeters can then be calibrated by comparison with a standard electrodynamometer (or instrument of the electrodynamometer type).

143. Voltmeters. For each type of alternating-current ammeter there is a corresponding type of voltmeter, differing from the ammeter only in having a high resistance and a scale reading in volts, and in minor details (see section 92, p. 118). Further discussion of these instruments will be unnecessary here.*

144. The spark-gap voltmeter. For the measurement of very high voltages the spark-gap voltmeter, or spark gauge, is widely used. This consists of a spark gap between points whose distance apart is adjustable. The points are connected with the voltage to be measured, so that it is applied across the air between the points. Starting with the points wide apart, we shorten the distance between them gradually, until sparks just begin to pass through the air between them. The maximum distance at which spark-over occurs determines the voltage. This distance is measured, and the corresponding voltage is taken from tables of spark-over voltages which have been prepared. Corrections have to be made for pressure, temperature, and humidity of the air.

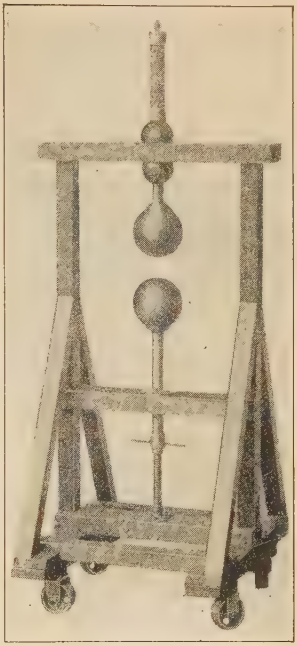


FIG. 147. Spark-gap voltmeter

Courtesy of the Westinghouse Electric and Manufacturing Company

The spark-over is arranged to take place between the surfaces of spheres, as shown in Fig. 147, though it was formerly more common to use needle points. The indications are determined by the maximum voltage during a cycle rather than by

* The indications of the electrodynamometer and plunger types of voltmeter are not independent of the frequency, in contrast with the corresponding types of ammeters. The indications depend on the current traversing the instrument; and, for a given voltage at its terminals, the current will differ with the frequency, in accordance with equation (63), section 159. The hot-wire voltmeter is not affected by differences of frequency.

the effective voltage, but the latter is a constant fraction of the former if the wave-form is sinusoidal.

145. Instrument transformers. An alternating-current ammeter (or voltmeter) can be used very conveniently for measuring currents (or voltages) outside its range by employing a suitable transformer called an *instrument transformer*. In the case of an ammeter the connections are made as shown in Fig. 148, and the transformer (known as a *current transformer*) must be designed with special reference to accuracy of its current ratio. In the case of a voltmeter the connections are made as shown in Fig. 149, and the transformer (known as a *potential transformer*) must be designed with special reference to accuracy of its voltage ratio. The switchboard instruments in central stations, where high-voltage alternating currents are concerned, are always connected by means of instrument transformers. The necessity for bringing the high voltage to the front of the switchboard, with consequent danger to switchboard attendants, is thus avoided.

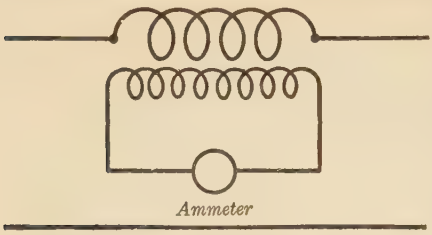


FIG. 148. Diagram of connections for a current transformer and its ammeter

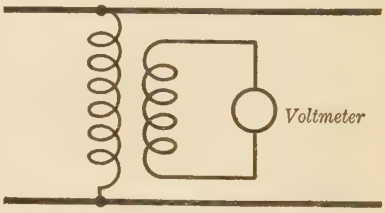


FIG. 149. Diagram of connections for a potential transformer and its voltmeter

146. Wattmeters. The wattmeter, as described in section 94, can be used with alternating currents as well as with direct currents. With each reversal of an alternating voltage, reversal of the current takes place also (though not necessarily at the same instant), so that the deflection of the wattmeter remains in the same direction. The pointer indicates the average power, the instantaneous value of the power being different at different instants during a cycle.

147. Watt-hour meters. The Thomson watt-hour meter, described in section 95, can be made to indicate correctly with alternating currents; but practically it is not used for alter-

nating currents, because a simpler instrument is available in the induction watt-hour meter, developed by Schallenberger. Description of this instrument is postponed to section 229, since it can be better understood after taking up the induction motor.

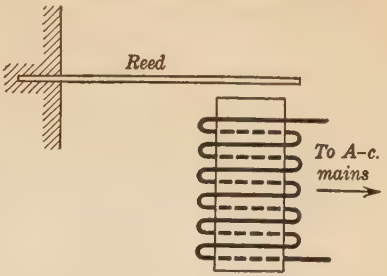


FIG. 150. Diagram to show the principle of action of the vibrating-reed frequency meter

148. Frequency meters. The vibrating-reed type of frequency meter consists of several steel reeds clamped at one end and placed within the field of an electromagnet energized by current from the supply whose frequency is to be determined (see Fig. 150). Each reed has a different frequency of free vibration. That one whose free vibration agrees in frequency with the alternating current vibrates with large amplitude, the other reeds remaining nearly at rest. The free ends of the reeds point toward the front of the instrument and are seen, end on, by the observer, just below (or above) a scale graduated in frequencies.

CHAPTER XIV

ALTERNATING-CURRENT THEORY

149. General statements. All the general laws of electricity, of course, hold good for alternating currents as well as for direct currents; but it does not follow that the *effective values* of voltage or current always stand in the same relations as the voltage or current in statements of the fundamental laws of electricity, which of course apply to instantaneous values.

For example, if a, b, and c are three points in a circuit it is a general truth that the difference of potential from a to c equals that from a to b, plus that from b to c. If an alternating electromotive force is applied between a and c, this truth holds at every instant *for the instantaneous values*. But it does not follow (nor is it in general true) that the *effective* volts between a and c equal the sum of the effective volts from a to b and from b to c.

Owing to the difference between the relations governing effective quantities as observed with alternating-current instruments and those governing the corresponding steady quantities in direct-current circuits, students sometimes obtain an erroneous impression that the behavior of alternating currents presents exceptions to the general laws of electricity. This, of course, is not the case. The instantaneous values of current, voltage, and power conform to the same laws in alternating-current circuits as in direct-current circuits, but the effective values are different quantities and need not follow the same laws. Furthermore, the effects of inductance and capacitance, which seldom come to notice with direct currents, are very prominent with alternating currents.

150. Sinusoidal wave-forms in the alternating-current theory. When the wave-forms of current and voltage are sinusoidal, it is not very difficult to derive the relations governing effective values from the fundamental laws governing instantaneous values.

When the wave-forms are not sinusoidal, the theory is very much more complicated and will not be taken up in this book,

although in some cases the behavior of nonsinusoidal currents or voltages will be touched upon briefly.

The wave-form of the electromotive force supplied by most actual alternators approximates pretty closely to the sinusoidal form, but that of the voltages and currents in circuits supplied by an alternator may be distorted by various causes. Most of the wave-forms met with in practice, however, are at least roughly approximately sinusoidal. The theory based on the assumption of sinusoidal wave-forms can accordingly be successfully applied pretty generally, but it should be kept in mind that it is not always sufficient except for purposes of first approximation.

When the wave-form is sinusoidal, the instantaneous voltage e at the time t is given by the relation

$$e = E_m \sin(\omega t + \theta_1), \tag{55}$$

where E_m is the maximum value reached by e; ω stands for $2\pi f$, where f is the frequency; and θ_1 is a quantity determined by the instant from which time is counted, and has nothing to do with the amplitude or frequency of the voltage curve but only with the instant that the curve crosses the t axis. The instantaneous value of the current is given by the exactly similar expression

$$i = I_m \sin(\omega t + \theta_2). \tag{56}$$

The quantity $(\omega t + \theta_1)$ in equation (55) may be considered as defining how far e has advanced through its cycle of values. It is then convenient to divide the cycle into 360 divisions called *electrical degrees* (since $(\omega t + \theta_1)$ would naturally be expressed in degrees*) and to express fractions of a cycle in terms of degrees. Thus 90° means a quarter of a cycle.

151. Relation between maximum, average, and effective values. When the wave-form is sinusoidal, the effective volts (or amperes) equal $\frac{1}{2}\sqrt{2}$ (or about 0.707) times the maximum; the average value throughout a half-cycle equals $\frac{2}{\pi}$ times the maximum, or $\frac{2\sqrt{2}}{\pi}$ times the effective value.

* When ω is defined as $2\pi f$, as above, ωt is in *radians*. It is necessary to define ω in this way in order to obtain correct results in equations (60), (62), (63), and others. To express ωt in degrees, ω would have to be taken as $360\,f$.

Proof. Let E stand for the effective value of a voltage whose instantaneous values are given by equation (55). Then, by definition,

$$E^2 = \text{ave. } e^2 = \text{ave. } [E_m \sin(\omega t + \theta)]^2$$
$$= E_m^2 \cdot \text{ave. } \sin^2(\omega t + \theta), \tag{57}$$

where the symbol *ave.* stands for *average* throughout a cycle. From the trigonometrical identity

$$\sin^2(\omega t + \theta) + \cos^2(\omega t + \theta) = 1,$$

it follows that

$$\text{ave. } \sin^2(\omega t + \theta) + \text{ave. } \cos^2(\omega t + \theta) = 1. \tag{58}$$

But for each angle A for which the sine has a certain value, there is another angle $(90° - A)$ in the same quadrant whose cosine has the same value. It follows that

$$\text{ave. } \sin^2(\omega t + \theta) = \text{ave. } \cos^2(\omega t + \theta).$$

Whence, from equation (58),

$$2 \text{ ave. } \sin^2(\omega t + \theta) = 1,$$

and

$$\text{ave. } \sin^2(\omega t + \theta) = \tfrac{1}{2}.$$

Substituting in equation (57), we obtain

$$E^2 = \tfrac{1}{2} E_m^2,$$

and

$$E = \tfrac{1}{2}\sqrt{2} \cdot E_m.$$

The average value throughout a positive half-cycle is obtained by calculus as

$$E_{av} = \frac{\int_{\omega t + \theta = 0}^{\omega t + \theta = \pi} E_m \sin(\omega t + \theta)\, d(\omega t + \theta)}{\pi}$$
$$= \frac{E_m[-\cos(\omega t + \theta)]_0^{\pi}}{\pi} = E_m \cdot \frac{2}{\pi}$$
$$= E \cdot \frac{2\sqrt{2}}{\pi}.$$

Hereafter the symbols e and i will be used for instantaneous values of voltage and current, E and I for effective values, and E_m and I_m for maximum values.

152. The revolving-vector diagram. In Fig. 151 let the line OE be drawn to scale to represent E_m of equation (55) and in a direction making an angle θ_1 with the horizontal axis. Now imagine this line to revolve about O with velocity of ω radians (or f revolutions) per second, starting when $t = 0$. Evidently

at every instant the projection of OE on the vertical axis will equal the instantaneous value of the voltage at that instant.

Diagrams of this sort are called *revolving-vector diagrams* or simply *vector diagrams*, or sometimes *clock diagrams*. They are very useful in problems that have to do with sinusoidal alternating quantities.

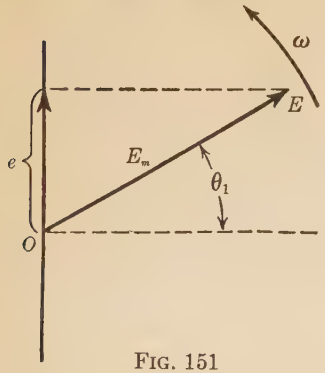


FIG. 151

153. The addition of alternating voltages. The vector diagram furnishes a convenient method for finding the resultant of two or more (sinusoidal) alternating voltages of the same frequency but differing in phase, acting in series. Such arrangement of voltages would occur, for example, when two alternators are connected in series, as in Fig. 152; equality of frequency and constant difference of phase will be assured if the rotors of the two machines are rigidly coupled together (assuming the same number of poles for each machine).

The electromotive force (maximum value) of machine A is denoted by the line A_m in the vector diagram Fig. 153, and that of machine B by the line B_m. When we imagine the lines A_m and B_m to revolve about O, both lines do so with the same speed, so that the angle $(\theta_A - \theta_B)$, or the difference of phase between the two electromotive forces, remains constant, as it should. If we draw the line B_m from the end of A_m outward, instead of from O outward, but in the same direction, as in Fig. 154, and imagine the figure to revolve about O as before, then the projection of B_m on the vertical axis will still represent at each instant the corresponding instantaneous value b of the electromotive force from machine B.

FIG. 152

The instantaneous value r of the resultant of the two electromotive forces in series will at every instant equal the (algebraic) sum of the corresponding instantaneous values of the components a and b, or $r = a + b$. It is plain from Fig. 154 that at every instant $a + b$ is equal to the projection on the vertical axis of a line drawn from O to the

outer extremity of B_m. If, then, such a line, R_m, is drawn, its length will represent the maximum value of the resultant of the two electromotive forces in series, its direction on the diagram will give the phase difference of the resultant from A and B, and the instantaneous values of the resultant will follow the equation

$$r = R_m \sin (\omega t + \theta_R).$$

It appears, then, that the resultant of two voltages in series, of assigned maximum values and assigned difference of phase, can be found graphically by the well-known process of vector

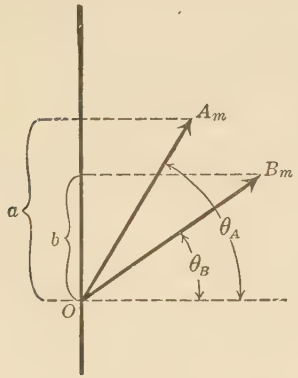


FIG. 153. Vector diagram for the voltages in Fig. 152

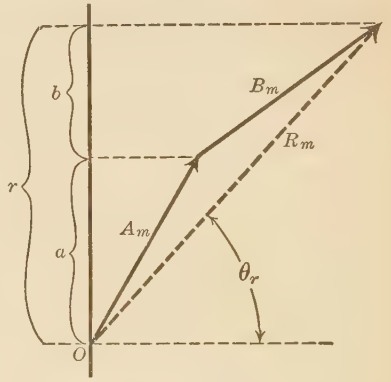


FIG. 154. Vector diagram for finding the resultant of the voltages in Fig. 152

addition, by the same construction as that by which the resultant of two forces of assigned magnitudes and assigned difference of direction can be found.

Since the effective voltage stands in the constant ratio of $\frac{1}{2}\sqrt{2}$ to the maximum, it is possible to draw the lines in the vector diagram proportional to the effective values of the various voltages rather than to their maximum values; this will make no difference in the shape of the diagram, but only in its scale,* and the effective value of the resultant can be scaled directly from it. It is usually more convenient to draw the lines in vector diagrams to represent effective values rather than maximum values.

* But of course when lines representing voltages are projected on the vertical axis, the projections will no longer represent instantaneous values to the same scale.

154. The addition of currents. When two or more (sinusoidal) alternating currents of the same frequency, but differing in phase, come together at a branch point, the resultant current can evidently be found by the use of a vector diagram in exactly the same manner as that just discussed for finding the resultant of voltages in series.

155. The relation between voltage and current in alternating-current circuits. When a voltage, varying in any assigned manner, is applied to a circuit, the current in that circuit is determined by its resistance, inductance, and capacitance.

In direct-current applications the effects of inductance or capacitance are only occasionally of importance, coming to attention only while the current is undergoing a rapid change of value; but in alternating-current applications the effects of inductance and capacitance are of great importance, since the rate of change of the current is always large (at the frequencies in common use).

For instance, in an alternating current whose maximum value is 1 ampere and whose frequency is 60 cycles per second, the rate of change of the current is given by

$$\frac{di}{dt} = \frac{d}{dt}[I_m \sin(\omega t + \theta)] = \omega I_m \cos(\omega t + \theta)$$
$$= 2\pi \cdot 60 \cdot 1 \cdot \cos(\omega t + \theta) = 377 \cos(\omega t + \theta),$$

or the current varies at the rate of 377 amperes per second when $\cos(\omega t + \theta) = 1$, and at lesser rates at other parts of the cycle. To effect any rate of change $\left(\frac{di}{dt}\right)$ in a current flowing through an inductance L, there is required an electromotive force $e = L\frac{di}{dt}$ (over and above any electromotive force required to drive the current through the resistance of the circuit); in this example, then, a very appreciable voltage would be required to maintain the alternating current through even a small inductance.

To study the relations between alternating voltages and currents, it is convenient to consider circuits containing resistance only, inductance only, or capacitance only, respectively. Although in strictness no such circuits exist, nevertheless it is easy to realize actual circuits in which the effects of any two of the quantities named are negligible in comparison with those of the third (at the frequencies in common use).

156. Circuits containing resistance only. (Effects of inductance and capacitance negligible.) In a circuit containing resistance only, at every instant the whole of the electromotive force impressed on the circuit is applied toward maintaining the flow of current through the resistance. By Ohm's law $e = Ri$; and since this holds true for the instantaneous values of voltage and current at each instant, the same relation holds true for the effective values, and $E = RI$. That is, for circuits containing resistance only, Ohm's law can be used for effective volts and effective amperes, just as for the corresponding steady quantities in direct-current applications.

This holds true, whatever the wave-form may be. In such circuits the wave-form of the current will be the same as that of the voltage in all cases, and will cross the axis at the same instant.

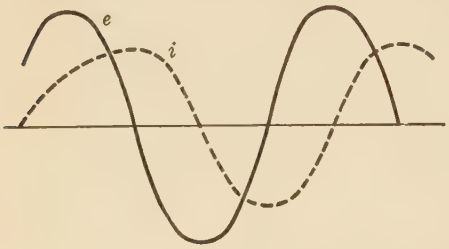


FIG. 155. Curves to show lag of current wave behind electromotive-force wave in a circuit containing inductance

Examples of actual circuits that contain resistance only (very approximately) are incandescent lamps and most rheostats.

157. Circuits containing inductance only. (Effects of resistance and capacitance negligible.) In a circuit containing inductance only, at every instant the whole of the impressed electromotive force must be applied to balancing the electromotive force of self-inductance, $L\frac{di}{dt}$, where L stands for the inductance of the circuit.

When an alternating electromotive force, of any wave-form whatever, is impressed on such a circuit, the effect of inductance is to prevent the current from ever reaching as high a value as it would in the absence of inductance, and to make the current wave lag behind (cross the axis later than) the electromotive-force wave.* Suppose the electromotive force whose wave-form is given by curve e, Fig. 155, is connected with a

* These statements hold true also when resistance is present as well as inductance; but if capacitance is present too, then it is not in general true that inductance prevents the current from reaching as high a value as it would in the absence of inductance (see section 162).

circuit that contains inductance only, at an instant $t = 0$ when e is passing through zero. As long as e remains positive, the current i will keep increasing, since $L \cdot \frac{di}{dt} = e$. But as soon as e reverses and becomes negative, it begins to act in opposition to whatever current has been built up. The current then starts to decrease, is eventually brought to zero, and (the electromotive force being still negative) is then built up in the negative direction and keeps increasing in this direction as long as e remains negative. As soon as the electromotive force reverses back to positive again, it begins opposing the negative current, eventually reducing it to zero and reversing it to positive; after which the cycle of changes just described repeats itself. Evidently the wave-form of the current will in general be different from that of the electromotive force.

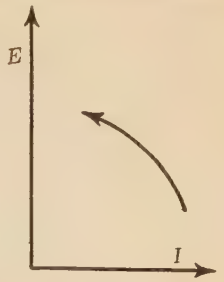


FIG. 156. Vector diagram showing relation of electromotive force and current in a circuit containing inductance only

When the wave-form of the current is sinusoidal, it can be shown that the voltage must also be sinusoidal, and simple relations between current and voltage can be found. In this case $i = I_m \sin(\omega t + \theta)$. By differentiation, at every instant

$$\frac{di}{dt} = I_m\omega \cos(\omega t + \theta) = I_m\omega \sin(\omega t + \theta + 90°), \qquad (59)$$

and the instantaneous values of the voltage are given by

$$e = L\frac{di}{dt} = \omega L I_m \sin(\omega t + \theta + 90°). \qquad (60)$$

That is, to maintain a sinusoidal alternating current in a circuit containing inductance only, there is required a sinusoidal electromotive force 90° ahead of the current in phase, whose maximum value equals $L\omega I_m$.

Since the effective values equal $\frac{1}{2}\sqrt{2}$ times the maximum values, the effective electromotive force E, required to maintain an effective current I through the inductance L, is equal to ωLI. The relation between E and I is shown graphically by vector diagram in Fig. 156.

Examples of actual circuits that contain inductance only (approximately) are electromagnet windings, including the field windings of dynamos.

158. Circuits containing capacitance only. (Effects of resistance and inductance negligible.) If a condenser is inserted in a circuit to which an electromotive force is applied, as in Fig. 157, of course no flow of current can take place *through* the condenser; but current may flow in the circuit containing the condenser, such current increasing or decreasing the charges on the condenser plates. At every instant the voltage across the condenser plates is equal to the charge q divided by the capacitance C. If a constant electromotive force is connected to a circuit containing a condenser, only a momentary flow of current can take place, since the charge on the condenser plates keeps increasing with continuance of the current, and soon becomes so great that the corresponding voltage $\left(\frac{q}{C}\right)$ equals the impressed electromotive force, thus preventing further flow of current. With direct currents, then, a condenser does not act much differently from the dead ends of mains on open circuit. But if an alternating electromotive force is connected with a circuit containing a condenser, alternating current will flow, since the charge brought to the condenser during a positive half-wave can flow back during the following negative half-wave.

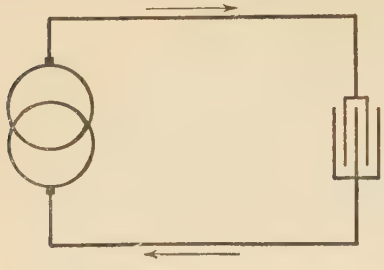


FIG. 157

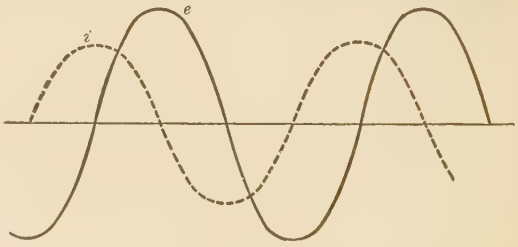


FIG. 158. Curves to show lead of current wave ahead of electromotive-force wave in a circuit containing capacitance

When an alternating current of any wave-form whatever flows in a circuit containing a condenser, the maximum charge

accumulated on the condenser occurs not at the instant of maximum current, but at the instant when the current reverses. The electromotive force $\left(=\frac{q}{C}\right)$ required to maintain the current must then have its maximum later than the maximum of the current, or the electromotive-force wave must lag behind the current wave — just the reverse of what takes place in an inductive circuit. Fig. 158 shows current and electromotive-force waves for a circuit containing capacitance only. Evidently the wave-form of the electromotive force will in general differ from that of the current.

When the wave-form of the current is sinusoidal, it can be shown that the voltage must also be sinusoidal, and simple relations between current and voltage can be found. The value of the charge q on the condenser plates at any instant is given by

$$q = \int_0^t i\,dt = \int_0^t I_m \sin(\omega t + \theta)dt$$

$$= -\frac{I_m}{\omega}\cos(\omega t + \theta) - Q_0$$

$$= \frac{I_m}{\omega}\sin(\omega t + \theta - 90°) - Q_0,$$

where Q_0 stands for the charge when $t = 0$. But Q_0 must be zero, since the average charge throughout both halves of a cycle must be zero, and we accordingly have

$$q = \frac{I_m}{\omega}\sin(\omega t + \theta - 90°). \tag{61}$$

Then

$$e = \frac{q}{C} = \frac{I_m}{\omega C}\sin(\omega t + \theta - 90°), \tag{62}$$

or the required voltage is sinusoidal, 90° behind i in phase, and has its maximum value equal to $\frac{I_m}{\omega C}$.

Since the effective values equal $\frac{1}{2}\sqrt{2}$ times the maximum values, the effective electromotive force E required to maintain an effective current I through the circuit containing capacitance C is equal to $\frac{1}{\omega C} \cdot I$. The relation between E and I is shown graphically by vector diagram in Fig. 159.

Examples of actual circuits that contain capacitance only (approximately) do not often occur in ordinary alternating-current practice, but can easily be realized in the laboratory by connecting a condenser with a source of alternating voltage, using wires of low resistance.

FIG. 159. Vector diagram showing relation of electromotive force and current in a circuit containing capacitance only

It should be noted that the capacitance in a continuous circuit which does not contain a condenser (such as a plain piece of wire) is not zero, but is infinite. Capacitance is the ratio of charge to the difference of potential due to the presence of the charge; and if charges are imparted to the ends of the circuit under consideration, their difference of potential is zero (or becomes zero as soon as time enough has elapsed for the charges to flow along the wire). So the capacitance $\frac{Q}{E}$ is infinite. On the other hand, the capacitive reactance $\frac{1}{\omega C}$ is zero.

159. Circuits containing resistance, inductance, and capacitance. The electromotive force required to maintain a sinusoidal current in a circuit containing resistance, inductance, and capacitance will be the resultant of the electromotive forces required to maintain the same current through the resistance, the inductance, and the capacitance respectively. This can easily be found by vector diagram, as shown in Fig. 160, in which E_R stands for the effective voltage required to maintain the effective current I through the resistance R, and is drawn in phase with the current, and of magnitude RI; E_L stands for the effective voltage required to maintain the current I through the inductance L, and is drawn 90° ahead of I in phase, and of magnitude ωLI; E_C stands for the effective voltage required to maintain the current I through the capacitance C, and is drawn 90° behind I in phase, and of magnitude $\frac{1}{\omega C} I$; and E stands for the resultant voltage.

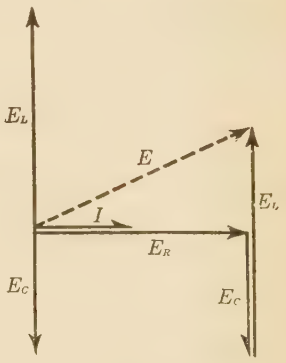


FIG. 160

From the geometry of the figure,

$$E^2 = E_R^2 + (E_L - E_C)^2,$$

or
$$E^2 = (RI)^2 + \left(\omega LI - \frac{1}{\omega C} I\right)^2;$$

whence
$$E = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}. \tag{63}$$

This gives a general expression for the relation between effective current and effective voltage in any circuit (provided the wave-forms are sinusoidal). The form of the expression is similar to Ohm's law, R of Ohm's law being replaced by the quantity $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$.

The phase angle ϕ, by which the voltage leads the current (or by which the current lags behind the voltage), is given by

$$\tan \phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}. \tag{64}$$

160. Impedance. The ratio of effective voltage to effective current in a circuit is called the *impedance* of the circuit. It has the same dimensions as resistance, and is expressed in ohms. For any assigned circuit it has a definite value independent of the intensity of the voltage or current,* but its value is affected by the frequency, and by the wave-forms of voltage and current. It is usually denoted by Z, so that

$$E = ZI, \tag{65}$$

a form similar to Ohm's law.

When the current and voltage are sinusoidal, comparison of equation (65) with equation (63) shows that the impedance has the value

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}. \tag{66}$$

161. Reactance. The quantity $\sqrt{Z^2 - R^2}$ for any circuit is called the *reactance* of the circuit. It has the same dimensions as resistance, and is expressed in ohms. For any assigned cir-

* This is a general statement; but circuits do occur in which the impedance is not independent of the intensity of current. In particular, the impedance of coils wound on iron cores is less with heavy currents than with light ones. This is because, with heavy currents, the iron core is magnetically saturated during those parts of the cycle in which the current has its greatest values, and the inductance of the coil is then less than when the core is unsaturated.

cuit it has a definite value independent of the intensity of the voltage or current,* but its value is affected by the frequency, and by the wave-forms of voltage and current. It is usually denoted by X, so that

$$Z = \sqrt{R^2 + X^2}. \tag{67}$$

When current and voltage are sinusoidal, comparison of equation (67) with equation (66) shows that

$$X = \pm \left(\omega L - \frac{1}{\omega C}\right). \tag{68}$$

In this case the reactance evidently equals the ratio of the effective value of the component of voltage *90° out of phase* with the current, to the effective current; and the resistance equals the ratio of the effective value of the component of voltage *in phase* with the current, to the effective current. The reactance is counted positive or negative, according as the voltage leads the current in phase or lags behind it.

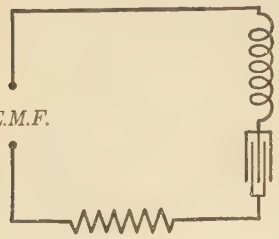


FIG. 161. Circuit capable of electric resonance

162. Electric resonance. The current which flows when an assigned alternating electromotive force is applied to a circuit is determined by the impedance of the circuit. If the frequency changes, the impedance changes, on account of change in the reactance. The inductive reactance ωL increases with increasing frequency, the capacitive reactance $\frac{1}{\omega C}$ decreases, and the total reactance $\omega L - \frac{1}{\omega C}$ may either increase or decrease. $\omega L - \frac{1}{\omega C}$ equals 0 when ω equals $\sqrt{\frac{1}{LC}}$; then the impedance is a minimum and equals R.

If, then, alternating electromotive force of constant effective value but of varying frequency is applied to a circuit like that of Fig. 161, the current will be different for different frequencies and will have a maximum equal to $\frac{E}{R}$ when $\omega = \sqrt{\frac{1}{LC}}$. The voltage across the inductance will then equal ωLI or $E \cdot \frac{\omega L}{R}$, and that across the condenser will equal $E \cdot \frac{1}{\omega CR}$, values which may be much greater than E itself.

* This is a general statement; but exceptions occur (see the footnote to section 160).

As an illustration take R as 2 ohms, L as 1.41 henrys, C as 5 microfarads, and E as 110 volts. The value of ω for maximum current $\left(\sqrt{\frac{1}{LC}}\right)$ is then 377, corresponding to a frequency of 60 cycles per second. For this value the current is 55 amperes; and the voltage across the inductance is $377 \times 1.41 \times 55$, or 29,200, and across the condenser $\frac{55}{377 \times 0.000005}$, or 29,200 likewise. The relation between the component voltages is shown in Fig. 162. If the frequency were 30 cycles per second ($\omega = 188$), the impedance would be

$$\sqrt{2^2 + \left(188 \times 1.41 - \frac{1}{188 \times 0.000005}\right)^2},$$

or 798 ohms; the current would then be only $\frac{110}{798} = 0.138$ ampere; and the volts across the inductance would be 36.7, and those across the condenser 147. Fig. 163 shows the value of the current for different values of the frequency.

FIG. 162. Vector diagram for the voltages in various parts of a circuit in which electric resonance occurs

The phenomena just discussed are known as *electric resonance*. They show how it is possible for the voltage across some portion of a circuit to become much higher than that of the source of the electromotive force, and thus perhaps cause injury. They can occur only when both inductive reactance and capacitive reactance are present; and since capacitive reactance is inappreciable in most ordinary circuits, resonance does not often occur accidentally. However, in long transmission lines the effects of capacitance are appreciable, and resonance might occur in such cases if the lines were not designed so as to avoid the conditions for resonance.*

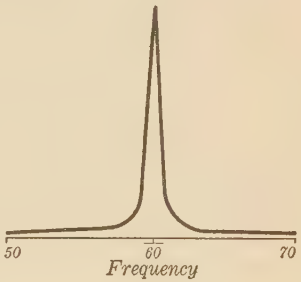


FIG. 163. Curve showing current as a function of frequency in the circuit of Fig. 161

* When the wave-form of the electromotive force is not sinusoidal, it is equivalent, by Fourier's theorem, to the resultant of a sinusoidal fundamental wave and sinusoidal partials whose frequencies are integral multiples of that of the fundamental. It is possible for resonance to occur for one of the partials when it does not for the fundamental. This is one reason why nonsinusoidal wave-forms are objectionable.

163. Power in alternating-current circuits. When energy is delivered to a circuit by an alternating current, the power, or rate of flow of energy, is not steady, but fluctuates with a frequency twice that of the current. This is shown graphically by the curves of Fig. 164 for instantaneous values of voltage e, current i, and power p $(= ei)$. When the current is not in phase with the voltage (as in a circuit containing inductance or capacitance), there are parts of the cycle in which the instantaneous power is negative; in these parts of the cycle the circuit instead of receiving energy returns it, the returned energy being that which has been stored during other parts of the cycle in the magnetic or electrostatic fields associated with the circuit.

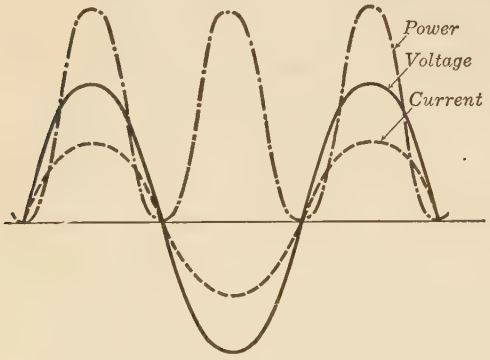


FIG. 164. Curves of instantaneous power, instantaneous voltage, and instantaneous current

Noninductive circuit

In most applications the instantaneous values of the power are not of interest, but rather the average power P. In the particular case of a sinusoidal current in phase with a sinusoidal voltage the average power equals the product of effective volts by effective amperes, or $P = EI$ (this follows from the proof at the end of section 164); but when the current is not in phase with the voltage, or the wave-forms are not sinusoidal, P will differ from EI. However, a general expression for P can be obtained by writing

$$P = EI \cdot F, \tag{69}$$

where F is a factor determined by the difference of phase between current and voltage and their wave-forms.

164. Power factor. The factor F is called the *power factor*. Accordingly the power factor is the ratio of the watts to the volt-amperes, the term *volt-amperes* being used for the product EI of effective volts by effective amperes. The power factor is a pure number, and its value cannot exceed unity. For an assigned circuit it has a definite value, which, however, depends

on the frequency and wave-form of the current (or voltage). It can be found experimentally by taking simultaneous readings of wattmeter, voltmeter, and ammeter and computing the ratio $\frac{P}{EI}$.

The power factor is a quantity of great practical importance. Suppose that some appliance requires an amount of power denoted by P. This will equal $EI \cdot F$, in which E is fixed by the voltage of transmission. Then, for an assigned amount of power, I will have to be larger as F is smaller. Thus the smaller the power factor, the larger will be the RI^2 losses in transmitting the same amount of power over the same line, and the larger the generating capacity required; hence economical operation requires that the power factor of all appliances be as high as possible.

When the wave-forms of current and voltage are sinusoidal, the value of the power factor is $\cos \phi$, where ϕ is the angle of phase difference between current and voltage.

Proof. The instantaneous value of the power is given by $p = ei$. Substituting in this the values of e and i from equations (55) and (56), we have

$$p = E_m \sin (\omega t + \theta_1) \cdot I_m \sin (\omega t + \theta_2).$$

If time is counted from the instant that the voltage passes through zero (from negative to positive), which can be done without any loss of generality, this becomes

$$\begin{aligned} p &= E_m \sin \omega t \cdot I_m \sin (\omega t \pm \phi) \\ &= E_m I_m \sin \omega t \cdot (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi). \end{aligned}$$

Whence

$$P = \text{ave. } p = E_m I_m (\cos \phi \cdot \text{ave. } \sin \omega t \pm \sin \phi \cdot \text{ave. } \sin \omega t \cos \omega t).$$

But ave. $\sin \omega t = \frac{1}{2}$ (see section 151), and ave. $\sin \omega t \cos \omega t = 0$, because for every angle whose sine has a certain value, there is in the same half-cycle another whose sine has the same value but whose cosine equals minus the cosine of the first angle. Therefore

$$P = \tfrac{1}{2} E_m I_m \cos \phi = EI \cos \phi. \tag{70}$$

Comparing with equation (69), we have $F = \cos \phi$, which was to be proved.

ϕ is commonly called the *angle of lag*, though it is possible for the current to *lead* the voltage, in which case ϕ becomes

negative. It has a definite value for an assigned circuit (assuming the frequency to be fixed), determined by the relation (64)

$$\tan \phi = \frac{X}{R},$$

or by the relation

$$\cos \phi = F = \frac{R}{Z}. \tag{71}$$

165. Power in reactive circuits. When the resistance of a circuit is negligible in comparison with its inductive reactance, the current lags 90° behind the voltage, as shown in section 157. Then, by equation (71), the power factor is zero, and accordingly the (average) power is zero, even though the effective current and voltage may be large. In this case the relations between the instantaneous current, voltage, and power are as shown in Fig. 165. The instantaneous power is positive for half the time (during which the magnetic field linking with the circuit is built up) and negative for the other half (during which the magnetic field gives back the energy that had been stored in it).

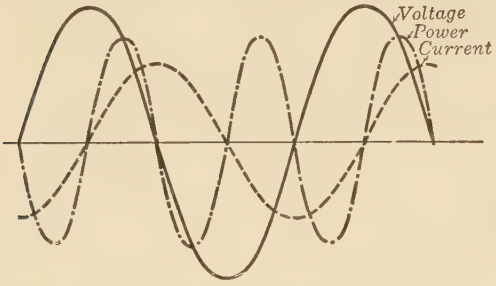


FIG. 165. Curves of instantaneous power, instantaneous voltage, and instantaneous current

Highly inductive circuit

Similar conditions exist when a circuit contains capacitive reactance without appreciable resistance; but in this case the current leads the voltage by 90°, and the energy delivered to the circuit during the positive half-wave of the power goes into building up an electrostatic field.

166. Reactance coils. In a circuit of high inductive reactance and low resistance the impedance will be high, but the loss of power (RI^2) when a current passes through will be small. Such a circuit or coil may be inserted in an alternating-current circuit in order to limit the amount of current that can pass through, in the same manner that resistors, or rheostats, are used in direct-current circuits.

A coil designed to be used in this way is called a *reactance coil* or *reactor*, or a *choke coil*. It has the advantage over a resistor that little energy is wasted in its use and that little heat has to be disposed of.

For example, suppose that it is desired to provide some arrangement by which the current delivered to a low-resistance circuit from 110-volt mains shall be limited to 2 amperes. If the arrangement used is a rheostat, its resistance will have to be 55 ohms, and (if the resistance of the rest of the circuit is negligible) it will consume power at the rate of 220 watts. If, on the other hand, a reactance coil is used having reactance of 55 ohms (at 60 cycles per second this would require about 0.145 henry of inductance), this will limit the current to 2 amperes just as well as does the rheostat. The resistance of such a coil can easily be made as small as 1 ohm, and then it would consume only 4 watts of power when carrying the maximum current it is intended for.

167. Active and reactive components of an alternating current. In dealing with reactive circuits it is often convenient to consider the current as made up of two components, one of which contributes all the (average) power, and the other contributes none. The former is called the *active component*, and the latter the *reactive component*.* The active component must evidently reverse at the same time as the voltage, or be in phase with the voltage, so that the instantaneous values of the power due to it will always be positive. The reactive component must be so far out of phase with the voltage that the instantaneous values of the power due to it will be equally divided between positive and negative values.

Assuming sinusoidal wave-forms, equation (70) shows that the active component must equal $I \cdot \cos \phi$ (effective value). The reactive component must be 90° out of phase with the voltage and therefore 90° out of phase with the active component also. Its value must accordingly be $I \sin \phi$.

Sometimes it is more convenient to resolve the voltage, instead of the current, into an active and a reactive component. Their values will evidently be $E \cos \phi$ and $E \sin \phi$ respectively.

PROBLEMS

1. Two six-pole alternators, A and B, have their shafts coupled rigidly together, with the rotor of machine A 20° ahead of that of machine B. With this displacement between the rotors, the two machines carry out identical motion. (*a*) What is the difference in phase between the voltages generated? (*b*) If A generates 1000 volts

* Formerly called also the *power component* and *wattless component* respectively.

(effective) and B 1200 volts, and the armatures of the two machines are connected in series, find the resultant voltage by vector diagram.

2. A lamp bank and a magnet coil are connected in parallel across 110-volt mains. The lamp bank takes 8 amperes (effective), the magnet coil takes 6, and the current in the mains supplying both is 10 amperes. Find, by vector diagram, the difference in phase between the parallel currents.

3. The frequency being 60 cycles per second, and the resistance of the coil in Problem 2 being negligibly small, find its impedance, reactance, and inductance.

4. What is the maximum voltage when the effective voltage is 110 and the wave-form is sinusoidal?

5. Find the current taken by a 100-watt 110-volt incandescent lamp (a) when it is connected directly across 110-volt 60-cycle mains and (b) when it is connected in series with a choke coil across the same mains, the inductance of the coil being 0.1 henry and its resistance negligible.

6. Under the conditions of Problem 5, (b), find the voltage across the terminals of the lamp and that across the terminals of the coil; also the power taken by the lamp and that taken by the coil.

7. A certain induction motor when lightly loaded takes 5 amperes from 220-volt 60-cycle mains, and a wattmeter shows that it is then taking 550 watts. What is the power factor? By what fraction of a cycle does the current lag behind the voltage (assuming sinusoidal wave-forms)?

8. There are connected in series across 110-volt 60-cycle mains a resistance of 2 ohms, an inductance of 0.5 henry, and a capacitance of 15 microfarads. Find the current, and find the voltage across the terminals of the capacitance.

CHAPTER XV

POLYPHASE SYSTEMS

In many applications of alternating currents two or more voltages are used differing in phase but identical in other respects. Systems employing such voltages are called *polyphase* systems and are designated as *two-phase*, *three-phase*, and so on, according to the number of voltages differing in phase that are employed.

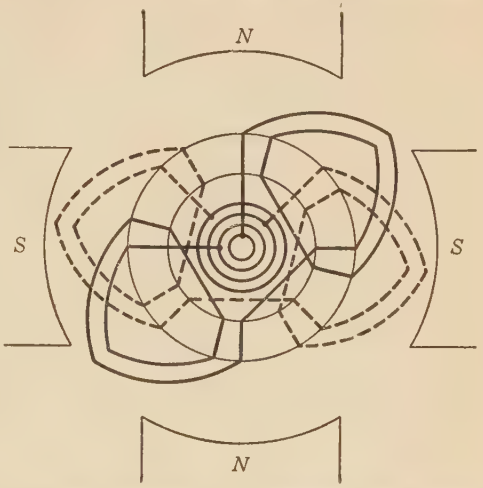


FIG. 166. Diagram to illustrate the relative position of the two armature windings in a two-phase armature

168. Polyphase alternators. It is very easy to produce such sets of voltages from a single alternator by placing on its armature two or more distinct windings, displaced with respect to each other but identical in other respects. This is shown diagrammatically in Fig. 166 for the case of two windings. The electromotive force induced in one of the windings will be identical with that induced in the other, except that each instantaneous value of electromotive force in one winding will occur later than in the other.* The electromotive-force waves for two such

* It cannot be stated *which* one occurs later, unless an understanding is had as to which direction is considered the forward direction in each winding. For instance, if we traverse a winding from terminal 1 to terminal 2, a positive maximum of electromotive force referred to this direction is the same as a negative maximum referred to the direction from terminal 2 to terminal 1. In dealing with polyphase circuits, it is important to have an understanding as to which direction in each circuit is assumed as the forward, or reference, direction. In the diagrams of connections, this will be indicated by arrows.

windings are shown by curves in Fig. 167 and by vector diagram in Fig. 168. These figures are drawn with the phase difference of 90° between the two electromotive forces, this being the difference in phase always used in actual two-phase systems.

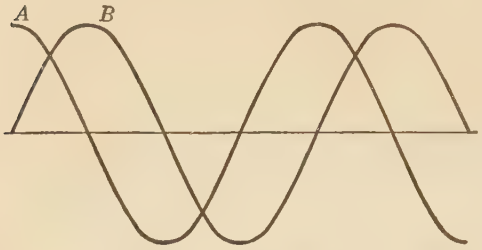


FIG. 167. Curves to show phase displacement between the two electromotive forces in a two-phase alternator

In diagrams to show connections of polyphase armature windings it is customary to represent the windings by lines like those of Fig. 169, which represents the same arrangement as Fig. 166. The small circles at the ends of a winding represent its terminals; they suggest the collector-rings of a revolving armature, though, of course, the diagrams apply equally well to stationary armatures.

169. Two-phase systems. Fig. 169 shows a separate pair of mains leading from each of the two windings. This arrangement is called the two-phase four-wire system.

It is also possible to connect one end of both windings to the same terminal, as shown in Fig. 170; the number of mains is then reduced to three. This arrangement is called the two-phase three-wire system. The voltages between mains *1* and *2* and between mains *2* and *3* are equal (except for difference in phase) to each other and to the voltages induced in winding *A* and in winding *B*; the voltage between mains *1* and *3* is the resultant of voltage *1–2* and voltage *2–3*, and can be found by vector diagram as shown in Fig. 171. The diagram shows that voltage *1–3* is 45° out of phase with both *A* and *B*, and its effective value is equal to $\sqrt{2}$ times that of *A* or *B*.

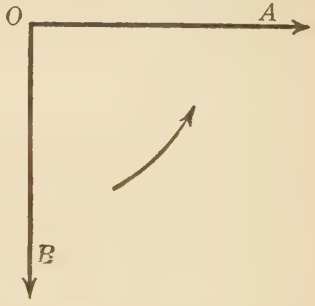


FIG. 168. Vector diagram to show phase relation between the voltages induced in a two-phase alternator

170. Three-phase systems. When three distinct windings are placed on an alternator armature, we have a three-phase alternator. If the three windings are symmetrically placed (as they always are in practice), each occupying about a third of the space from one pole to the next, the electromotive force induced in the second winding will lag one third of a half-period (or 60°) behind that in the first, and that in the third will lag 60° behind that in the second, provided electromotive forces are counted in the same direction in each winding.* This is shown in Fig. 172 and the accompanying vector diagram (Fig. 173), using the dotted lines for voltage B. It is customary, however, to count voltage in the reverse direction in the second winding, as shown by the full lines in Figs. 172 and 173; this gives a symmetrical arrangement in which each of the three electromotive forces differs in phase by 120° from each of the others.

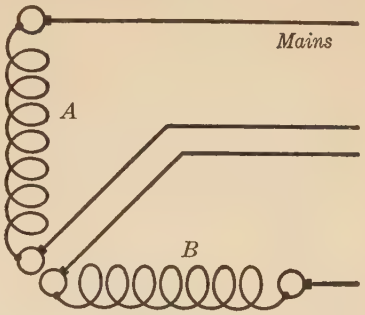


FIG. 169. Diagrammatic representation of the armature windings for each of the phases of a two-phase alternator, with connections to four mains

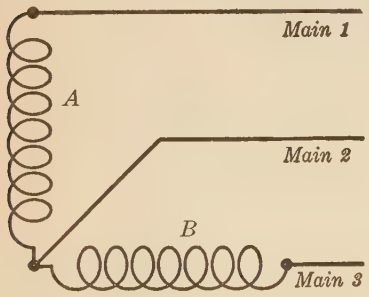


FIG. 170. Diagram of connections for two-phase three-wire system

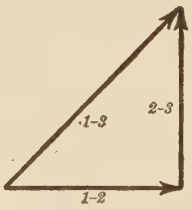


FIG. 171. Vector diagram for voltages in two-phase three-wire system

Each winding can be connected with a separate terminal and main at each end, as shown in Fig. 174, giving a three-phase six-wire system, but in practice the windings are almost always connected together in such a way that only three terminals are

* For example, assuming as the forward direction through each winding, the direction from front to back of the armature along the first conductor on the side of the winding toward which rotation takes place.

used and only three mains required. There are two methods of doing this: the Y (or star) system and the delta (or mesh) system.

171. Y (or star) connections. In the Y system one end of each winding is brought to an armature terminal, and the other ends of the three windings are connected together, as shown in Fig. 175.

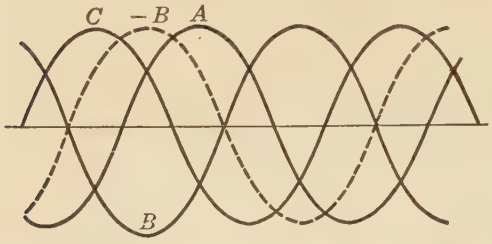


FIG. 172. Curves to show phase relations between voltages generated in the three windings of a three-phase alternator

It is not immaterial which ends are connected together, but the connections must be so made that the forward directions through the windings, counted as explained in section 170 (p. 195), are all outward from, or else all inward to, the common junction.

In this arrangement the current in any main must be the same as that in the winding with which it is connected. But the voltage from one main to another is not the same as that induced in one winding: it is the resultant of the voltage in two of the windings, one of them being traversed in a direction opposite to the reference direction; and its value can be found by vector diagram, as shown in Fig. 176. For example, to find the effective voltage between main *1* and main *2* (the direction from *1* to *2* being counted as forward), we should combine the *reverse* of voltage *A* with voltage *B*. In the triangle *OPQ*, Fig. 176, whose sides represent voltage *A*, voltage *B*, and voltage *1–2* respectively, the angle *OPQ* is 120°, and *A* equals *B*; accordingly angle *QOP* = angle *OQP* = 30°. Then $\frac{1}{2}$ (voltage *1–2*) $= A \cdot \cos 30° = \frac{1}{2}\sqrt{3} \cdot A$, and voltage *1–2* $= \sqrt{3}\ A$. By symmetry, the voltages *2–3* and *3–1* must also each equal $\sqrt{3}\ A$ in magnitude, so that the effective voltage between any two mains is $\sqrt{3}$ times that induced in one of the armature windings. Voltages *1–2*, *2–3*, and *3–1* differ in phase from each other by 120°.

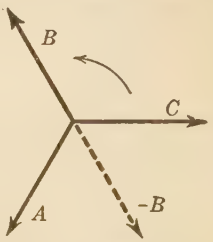


FIG. 173. Vector diagram to show phase relations between voltages in the windings of a three-phase alternator

Sometimes the common junction of the three windings, or "neutral point," is connected with a terminal, from which a main (main *4*) can be led out, giving a three-phase four-wire

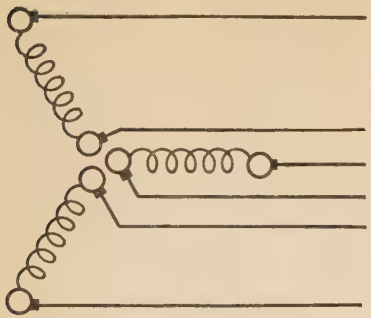


FIG. 174. Diagram showing armature windings and mains in a three-phase six-wire system

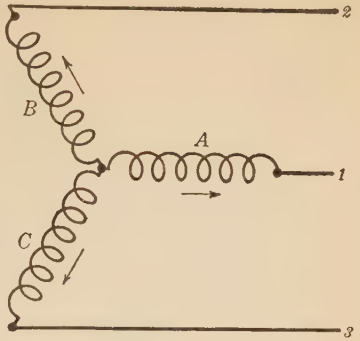


FIG. 175. Y system of connection for windings in a three-phase alternator

system. In this case the voltage between main *4* and any one of the other mains is evidently equal to that induced in one of the armature windings.

172. Delta (or mesh) connections. In the delta system each of the three terminals is connected with the end of one winding and with the beginning of another, as shown in Fig. 177.

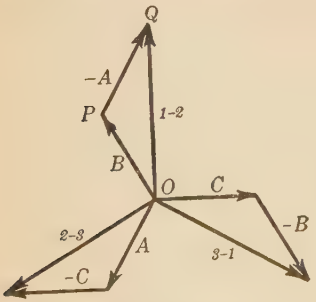


FIG. 176. Vector diagram for voltages in Y-connected alternator

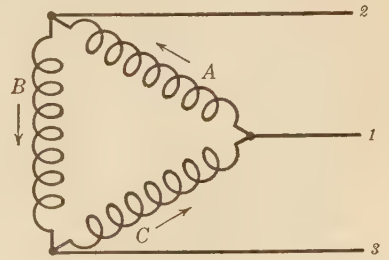


FIG. 177. Delta system of connection for windings in a three-phase alternator

It is not immaterial which end of a given winding is connected with a given terminal, but connections must be so made that the forward directions through the windings, counted as explained in section 170 (p. 195), are all in the same direction

round the "delta," or circuit composed of the three windings in series. If the windings are so connected, the volts induced in any one of them will at every instant be equal and opposite to the resultant of those induced in the other two windings, with reference to a circuit round the delta. But if one of the windings should be connected in the reverse direction, its volts would act in the same direction round the delta as those of the other two windings, so that the three windings would then be short-circuited through one another.

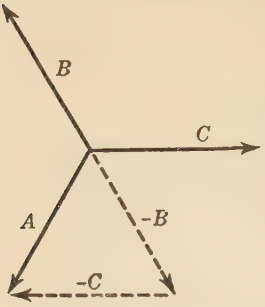


FIG. 178. Vector diagram for voltages in a delta-connected alternator

The voltage across any pair of mains — for instance, voltage *1–2*, Fig. 177 — can be thought of as due to winding *A*, or as due to windings *C* and *B* in series with each other: the value is the same in either case at every instant, as appears from the vector diagram, Fig. 178. Accordingly, if one of the windings *A*, *B*, or *C* should be broken, the voltages across mains *1–2*, *2–3*, and *3–1* would remain unchanged.*

The current in one of the mains is not in general equal to that in one of the armature windings, but the current in main *1* is equal to the resultant of that in winding *C* and that in winding *A* reversed. In the particular case that the load is "balanced" — that is, so arranged that the currents in the three mains are equal — the current in any main equals $\sqrt{3}$ times the current in any armature winding, as shown by the vector diagram, Fig. 179.

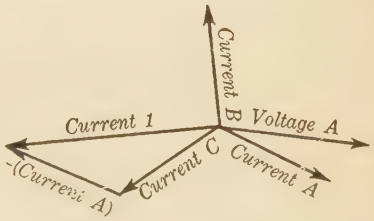


FIG. 179. Vector diagram for currents in the windings and in one of the mains for a delta-connected alternator with balanced load

173. Connections of load in three-phase systems. The load on three-phase mains can be connected either according to the Y system or according to the delta system, as shown in Fig. 180. The Y system can, of course, be used only for a load having three branches,† such as the

* Neglecting effects from voltage drop in the armature.

† Unless there is a neutral main, as in the three-phase four-wire system.

three windings of a three-phase motor. In the delta system the load need have only one branch, such as *a*, Fig. 180; the other branches, *b* and *c*, can be left out without affecting current or voltage in branch *a*. When a load is connected in this manner, the system reduces to a single-phase system, one of the mains and two of the phases of the three-phase system being idle, so far as load *a* is concerned; but it is possible to connect loads in the *b* and *c* phases at other parts of the mains distant from the load *a*.

With a Y-connected load the current in any branch of the load is evidently equal to the current in the main to which that

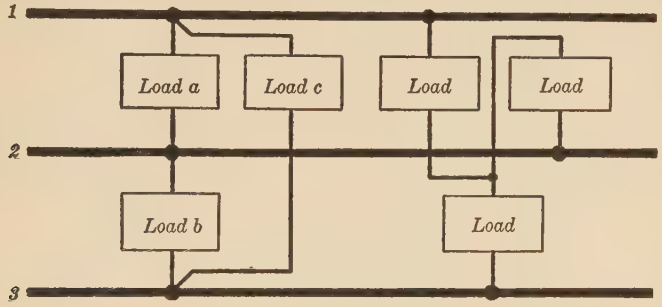


FIG. 180. Diagram to show connections of load to mains in a three-phase system

Delta-connected loads at left, Y-connected loads at right

branch is connected, and the voltage across terminals of one branch of a balanced load equals $\frac{1}{\sqrt{3}}$ times the voltage across mains. With a delta-connected load the voltage across terminals of any branch equals the voltage between mains, and the current in one branch of a balanced load equals $\frac{1}{\sqrt{3}}$ times the current in one of the mains.

In actual three-phase distribution systems most of the load is connected on the delta system, but is distributed all along the mains. Thus, in a city three mains are run along a street, and connections are led off at the various points of consumption of electrical energy, some connections being made between mains *1* and *2*, some between mains *2* and *3*, and some between mains *3* and *1*, as shown in Fig. 181. Sometimes only two of the three

mains are carried to some section of the system, giving that section a single-phase supply instead of a three-phase supply.

For the sake of uniform utilization of equipment and other advantages it is evidently desirable to have the load balanced. Less capacity of generators and line is required for a balanced load than for the same load when not balanced, and the RI^2 losses of power in generator and line are less for a balanced load than for the same load when not balanced. If the load is balanced, the voltage drop is the same in each of the phases. In practical distribution systems the load is rarely perfectly

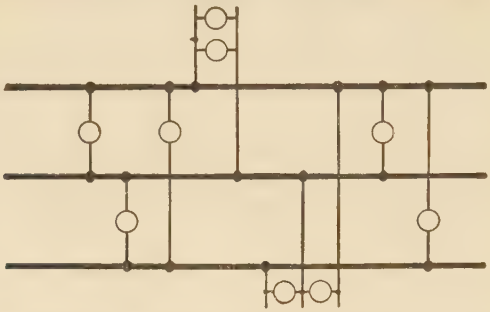


FIG. 181. Diagram to illustrate arrangement of loads in a three-phase distribution system

balanced, but it is usually possible to have it approximately so by using proper care in connecting different consumers among the different phases.

174. Advantages of polyphase systems. In comparison with single-phase systems, polyphase systems have the following advantages:

1. A given amount of power at an assigned voltage can be transmitted more economically. That is, with an assigned allowable loss of power in the line, less copper is needed for a polyphase transmission line than for a single-phase line; or, with an assigned amount of line copper, the power loss is less in a polyphase line than in a single-phase line.

2. Polyphase induction motors are inherently self-starting, whereas single-phase induction motors require special auxiliary arrangements for starting.

3. Polyphase alternators and synchronous motors can be constructed more compactly and cheaply than single-phase motors

of the same capacity, since winding can be placed over the whole surface of the armature (see section 136, p. 162).

4. The torque of polyphase generators and motors is steady, whereas that of single-phase generators and motors pulsates.*

With loads that are not balanced these advantages, with the exception of the second, are only imperfectly attained. They would evidently be lost entirely if the system were so far unbalanced that all the load were carried by one of its phases.

The second and fourth advantages apply equally to all polyphase systems, whatever the number of phases. The first and third are more fully attained as the number of phases is increased. For this reason three-phase systems are much more widely used than two-phase systems. To use a very large number of phases would introduce complications in the wiring and would offer comparatively little gain in the first and third advantages over the use of three phases.

The three-phase system may be considered the standard method for generation and distribution of electrical energy.

PROBLEMS

1. One hundred and fifty 100-watt 110-volt lamps are connected in groups of 50 each across 110-volt three-phase mains. Find the current in each main, and compare this with the current that would have to flow in the mains if the same lamps were connected with 110-volt single-phase mains.

2. In Problem 1 each of the three mains has a resistance of 0.06 ohm. Find the total power lost in transmission, and compare this with the power lost in transmission if the load had been supplied by the single-phase system, using two mains that contained the same amount of copper as the three mains of the three-phase system.

3. In Problem 1 suppose that the lamps were grouped as follows: 25 lamps connected between main A and main B, 50 lamps connected between main B and main C, and 75 lamps connected between main C and main A. Find, by vector diagram, the current in each main and the total power lost in transmission.

* At any instant the force on an armature conductor is lBi; in a single-phase dynamo i reaches its maximum in all conductors at the same instant and passes through zero in all conductors at the same instant; the torque must accordingly vary in value. In a polyphase dynamo, when the current is zero in one phase, it is not zero in others. It can be shown by reasoning which will not be given here that the torque from all phases combined does not vary with the time, provided the load is balanced.

CHAPTER XVI

BEHAVIOR OF ALTERNATING CURRENTS IN TRANSMISSION LINES

175. Skin effect. For alternating currents the resistance of a wire is somewhat greater than for direct currents, owing to the fact that alternating currents tend to flow more at the surface than in the interior of the wire, whereas direct currents flow with uniform density throughout the cross section of the wire.* This is known as the *skin effect*. In small wires at ordinary frequencies it is so small that it can be neglected for most purposes; but in large wires it is quite appreciable at ordinary frequencies, and at very high frequencies it is very pronounced in small wires as well as in large ones.

For example, at 60 cycles per second the skin effect makes the resistance of a No. 0 copper wire (B. & S. gage) about 0.1 per cent greater for alternating currents than for direct currents, but it increases the resistance of a 1,000,000-circular-mil copper cable by about 9 per cent (this applies to a solid cable, not to a stranded cable).

To lessen the skin effect in large cables it is desirable to build them up about cores of nonconducting material.

176. Inductive reactance in transmission lines. Since a pair of mains in a transmission line forms a circuit of only one turn and without iron core, one might suppose that the inductance would be inappreciable; but on account of the large area of the loop that the mains inclose, a considerable amount of magnetic flux links with the circuit when current flows in it. The inductance is accordingly appreciable, and the reactance due to it has to be taken into account. It will evidently be influenced by the spacing between mains, and the size of wire also affects its

* This is an effect of inductance. A stream line of current at the center of a wire links with more magnetic flux than a stream line at the surface. Accordingly the stream line at the center has somewhat greater inductance, and current will change more slowly in it than in a stream line at the surface, the same influences acting on both.

value somewhat. It will evidently be proportional to the length of a line, provided the size of mains and their spacing are the same throughout the line.

To give some idea of its magnitude, a few values are tabulated below.

RESISTANCE AND REACTANCE PER 1000 FEET OF ONE MAIN (COPPER) AT 60 CYCLES PER SECOND

SIZE OF MAINS B. & S. GAGE	RESISTANCE (OHMS) AT 20 C.	REACTANCE (OHMS) With Distance between Centers of Mains			
		18 in.	24 in.	36 in.	60 in.
0000	0.0489	0.1057	0.1123	0.1216	0.1333
0	0.0981	0.1136	0.1202	0.1295	0.1412
4	0.248	0.1243	0.1309	0.1402	0.1519
10	0.997	0.1402	0.1468	0.1561	0.1678

177. Capacitive reactance in transmission lines. Since a pair of mains is a continuous conductor, there is no capacitance *in series* with the circuit of which it forms a part, but the two mains act toward each other like the two plates of a condenser. This means that there is a distributed capacitance along the line, much as if a large number of condensers, each of small capacitance, were connected across the mains *in parallel* with the circuit of which they form a part, as shown in Fig. 182. With a given size and spacing of mains, this capacitance increases with the length of the line.

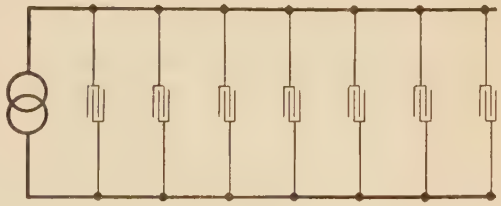


FIG. 182

The effect of this capacitance is that when the mains are connected with an alternating electromotive force, a *charging current* flows back and forth along the mains, even if they are on open circuit. The charging current is not the same at all points along the mains, but decreases with increasing distance from the source of electromotive force. When current is delivered through the mains to appliances on the line, the charging current is superposed on such current. In long lines the

charging current may be quite large (for example, in a 100,000-volt transmission line 100 miles long it might be of the order of 25 amperes); in shorter lines, such as the distribution system of a single city, it is small enough to be negligible for most purposes.

178. Voltage drop in alternating-current transmission lines. The RI drop in voltage between mains has been discussed in section 76 for direct-current distribution systems. RI drop in voltage takes place also with alternating currents and, of course, alternates with the same frequency as the current and in phase with it. In addition, a drop in voltage between mains takes place on account of the reactance of the line; this drop must have magnitude of XI and phase 90° ahead of the current, since it must equal the voltage required to maintain the current through the reactance of the line.

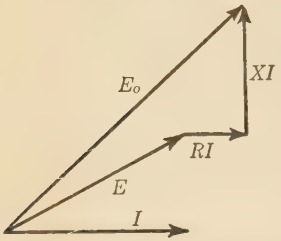


FIG. 183. Vector diagram for the voltage drop in an alternating-current transmission line

The relations between the voltage E_o at the beginning and the voltage E at the end of a pair of mains, and the RI and XI drops, are shown in Fig. 183. The figure corresponds to the case of a single concentrated load at the end of the line. I is shown lagging behind E, as is usually the case in practice; for the sake of clearness the values of RI and XI are shown much larger in comparison with E than would be likely to occur in actual lines. Evidently the difference between voltage at the beginning and at the end of the line ($E_o - E$) depends not only on the constants of the line and on the current flowing but also on the power factor ($= \cos \phi$) of the load.

179. Alternating-current distribution systems. The constant-voltage system is always used for alternating-current distribution systems, except in the case of arc-light circuits (for which the constant-current system is used). The distribution mains are maintained at comparatively high voltage (such as 2300). At various points transformers are connected with the distribution mains, which transform the voltage to some lower value (such as 115). and supply service mains at this lower voltage; the lamps, motors, and other appliances are then connected

directly with the service mains. The service mains belonging to any transformer do not extend very far from it (perhaps a few hundred feet); if electrical energy is needed at some point more distant from the transformer, the distribution mains are led to it, and an additional transformer is installed there.

The distribution mains in a city form a connected system, or network, throughout the whole territory served; the service mains make a large number of distinct systems, or networks, for the most part disconnected from one another. However, when the service mains belonging to one transformer adjoin those belonging to another, it is advantageous to connect them together; then, if one system should be heavily loaded at a time when the other is lightly loaded, the transformer at the lightly loaded end would be able to help the other transformer to carry the load. Or if one transformer should break down, the other might temporarily carry its load until it could be replaced or repaired.*

The service mains belonging to one transformer may, of course, have an entirely different voltage from those belonging to another.

Each transformer should be located as near as possible to the center of load of the territory it serves. A large motor, or other large concentrated load, is usually supplied from a transformer of its own.

The Edison three-wire system is often used for the service mains.

180. Standard voltages. In this country the following standard voltages are in common use for the transmission and distribution of electrical energy in alternating-current systems: for incandescent lamps and household appliances, 110 volts; for motors, 110, 220, 440, 550, and 2200 volts; for distribution mains in cities, 2300 volts; for transmission lines, 4600, 6600, 11,000, 13,200, 22,000, 33,000, 44,000, and 66,000 volts. Many transmission systems use higher voltages (the highest, at this writing, being 220,000), but these higher voltages have not yet been fully standardized.

* If, between the two transformers, the distribution mains are broken, their voltage will still be maintained beyond the break, since the transformer on the outer side of the break will be kept alive from the service mains and will act as a step-up transformer. To disconnect the distribution mains under these conditions, special arrangements have to be made for the switches or protective devices.

PROBLEMS

1. A single-phase transmission line consists of two No. 0 copper mains spaced 36 in. apart. It delivers 1000 kw. of power at 11,000 volts, 10 mi. from the generating station. If the power factor is unity, find the RI drop, the XI drop, and the voltage at the generating station.

2. If in the preceding problem the 1000 kw. were delivered at a power factor of 0.866, lagging, find the RI and XI drops, and the voltage at the generating station. Also find the same quantities if the 1000 kw. were delivered at a power factor of 0.866, leading.

CHAPTER XVII

OPERATING CHARACTERISTICS OF ALTERNATORS

181. Electromotive force generated in an alternator armature winding. In section 71 it is shown that for a single armature conductor

$$e_{av} = p\Phi n, \tag{38}$$

where e_{av} means the average electromotive force in a conductor while passing one pole (or while completing one half-cycle), p is the number of poles, Φ is the magnetic flux per pole, and n is the number of revolutions per second. The effective value E_1 of the electromotive force induced in one armature conductor will then be

$$E_1 = a \cdot p\Phi n \text{ (abvolts)}, \tag{72}$$

where a stands for the ratio of the effective value to the average value over a half-cycle; a has a definite calculable value for any particular wave-form, but differs for different wave-forms; it is called the *form factor*, and its value for a sinusoidal wave-form is $\frac{\pi}{2\sqrt{2}}$ (or approximately 1.11), according to the proof in section 151 (p. 176).

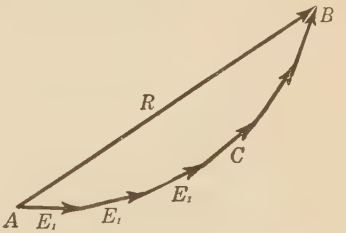


FIG. 184. Vector diagram showing the total electromotive force induced in an armature winding as the resultant of the electromotive forces induced in the individual armature conductors

The total electromotive force in an armature winding will be the resultant of the electromotive forces induced in the individual conductors. These will in general differ in phase, but be identical in other respects. Assuming sinusoidal wave-form, the resultant can be found by vector diagram, as shown in Fig. 184. If, as is usual in alternating-current armature windings, the conductors lie in bands, as shown in Fig. 139, the vector diagram takes the form of part of a regular polygon with very many

sides. The sides represent the electromotive forces in the separate conductors, so that the distance ACB along the perimeter of the polygon is $Z \cdot E_1$, where Z is the number of conductors. The angle between the extreme sides at A and B (which equals the angle subtended by ACB at the center of the polygon) is equal to the phase difference d between the electromotive forces of the conductors corresponding to A and B. But this equals the number of electrical degrees covered by the band of conductors on the armature (two poles corresponding to 360 electrical degrees).

When the number of conductors is large, as is usually the case, the polygon of Fig. 184 approximates closely to a circle. Then the resultant electromotive force E (= ARB in the figure) stands to ZE_1 in the same ratio as the chord to the arc subtending the angle d at the center of the circle. Denoting this ratio by b,

$$E = bZE_1 = ab \cdot Zp\Phi n = C \cdot Zp\Phi n \text{ (abvolts)}$$
$$= C \cdot Zp\Phi n \times 10^{-8} \text{ (volts)}, \qquad (73)$$

where C is a constant belonging to the machine in question. When the band of conductors covers one third of the space from one pole to the next, as is usual in three-phase alternators, $d = 60°$, $b = 0.955$, and C (assuming sinusoidal wave-form) $= 1.06$, so that the effective electromotive force induced in one winding of a three-phase alternator is

$$E = 1.06\ Zp\Phi n \times 10^{-8} \text{ (volts)} \qquad (74)$$

(subject to the assumptions made).

182. Effect of load upon terminal voltage. When an alternator is unloaded, its terminal voltage will equal the electromotive force induced in the winding connecting its terminals; but when it is loaded, the terminal voltage is affected by resistance and reactance drop in the armature and by armature reaction. Under ordinary conditions of practical operation the terminal voltage decreases as the armature current increases, though the reverse may take place if the current leads the voltage (which does not usually happen). The terminal voltage with a given armature current depends not only on the strength of the current (as in direct-current generators) but also on the phase difference between current and voltage. The behavior of the terminal voltage of an alternator with varying current output

therefore cannot be represented by a single characteristic curve such as is used for direct-current generators, but a separate curve is required for each phase difference (or power factor of the load).

To understand the behavior of the terminal voltage, it is convenient to consider first how the alternator would behave if its terminals were short-circuited, so that the terminal voltage would be zero while a current I flowed in the armature (or in each of its windings, if it is polyphase). (To prevent the current from reaching an excessive value the excitation may be reduced.) Let R_a and X_a be the resistance and reactance * of the armature (or of each of its windings, if it is polyphase). To maintain the current, there must be a voltage R_aI in phase with the current and a voltage X_aI 90° ahead of the current, and the resultant of these must equal the voltage impressed on the circuit, which in this case is the total electromotive force induced in the winding. The relation is shown by vector diagram in Fig. 185. In actual alternators X_a is larger than R_a, in much greater proportion than is shown in this figure.

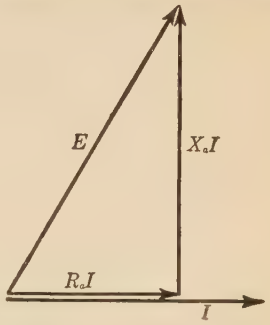


FIG. 185. Vector diagram for voltages in a short-circuited alternator armature

If the machine is not short-circuited, but is loaded, there will still be required voltages R_aI and X_aI in the same phase relations to the current as before, to maintain the current in the armature; but their resultant will not now be equal to the total induced electromotive force, but merely to the vector difference between the induced electromotive force E and the terminal

* On account of inductance, the armature winding has reactance equal to ωL, whether or not the rotor of the alternator is in motion. When the rotor is in motion, there is an additional reactance due to armature reaction; the armature currents cause a magnetic field which (in a polyphase machine with balanced load) occupies a fixed position relative to the main field, notwithstanding the alternations of the armature currents. The cutting of this field by the armature conductors induces in them an electromotive force in phase with the electromotive force of self-inductance. The effect of armature reaction is accordingly equivalent to an increase in the reactance of the armature winding. X_a is meant to include this effect.

In single-phase machines, the effects of armature reaction are more complicated, and are only approximately equivalent to an increase in the reactance of the armature winding.

voltage E_t, as shown in Fig. 186. If the machine is polyphase, it is assumed that the load is balanced and E and E_t refer to total induced electromotive force and terminal voltage for each of the armature windings.

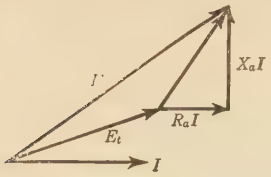


FIG. 186. Vector diagram for voltages in an alternator supplying a current that lags behind the voltage

Fig. 186 is drawn for the case in which the current lags behind the voltage, as usually happens in practice. If the current leads the voltage, the diagram becomes like that of Fig. 187, and the terminal voltage may exceed the total induced voltage.

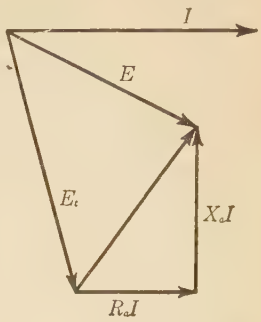


FIG. 187. Vector diagram for voltages in an alternator supplying a current that leads the voltage

Curves showing the terminal voltage of an alternator as a function of its current output, at different power factors, may be seen in Fig. 188. These curves correspond to the characteristic curve of a direct-current generator.

183. Voltage regulation of alternators. In the practical operation of alternators it is important to keep the terminal voltage approximately constant for all loads. This requires variation of the total induced voltage by varying the field flux of the alternator, which is usually accomplished by means of a rheostat in the exciter field circuit, as shown in Fig. 189. This rheostat may be adjusted manually from time to time, as changes in the load require; or the exciter voltage may be adjusted automatically by a voltage regulator responding to a relay controlled by the terminal voltage of the alternator. The construction of automatic voltage regulators will not be described here; the appearance of a widely used type of regulator is shown in Fig. 190, together with curves showing the behavior of the terminal voltage with and without the regulator.

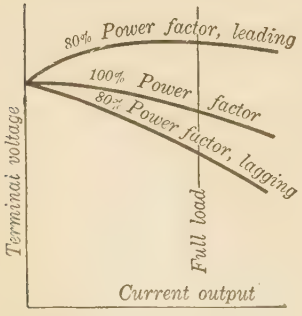


FIG. 188. Regulation curves of an alternator

184. Operation of alternators in parallel. In central stations it is usually desirable (for the reasons explained in section 250) to be able to operate two or more alternators in parallel. This can be done without difficulty, but the behavior of alternators in parallel differs considerably from that of direct-current generators.

If two alternators (*A* and *B*, Fig. 191) are to operate in parallel, it is evident that each one must generate electromotive force at the same frequency as the other. This requires that the two machines run at the same speed if they have the same number of poles; or, if one has a different number of poles

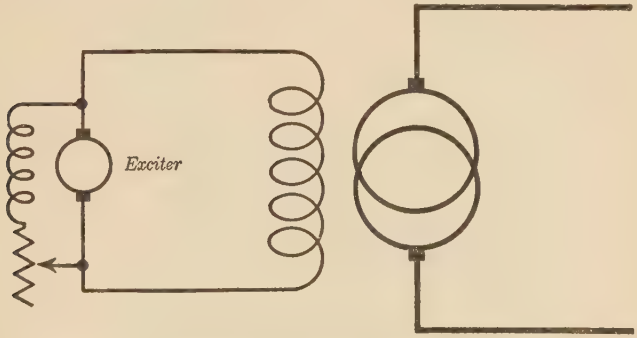


FIG. 189. Diagram of connections to show how the voltage of an alternator may be controlled by a rheostat in the exciter field circuit

from the other, then, by (54), their speeds n_A and n_B must be so related that $n_A p_A = n_B p_B$. When the speeds of two machines are so related as to correspond to the same frequency, they are said to be in *synchronism*. Two alternators which are started in synchronism and connected in parallel tend to remain in synchronism; if there is a difference in phase between the electromotive forces they generate, the leading machine takes an increased share of the load, and the lagging machine a decreased share. This tends to slow down the leading machine and to give the lagging machine a chance to catch up; thus any tendency to lack of synchronism between the speeds at which the machines are driven is automatically corrected.

This action may be explained by the vector diagrams *a* and *b* of Fig. 192, which refer to the same machine. E and E_t have the same values in the two diagrams, but different phase relation. In

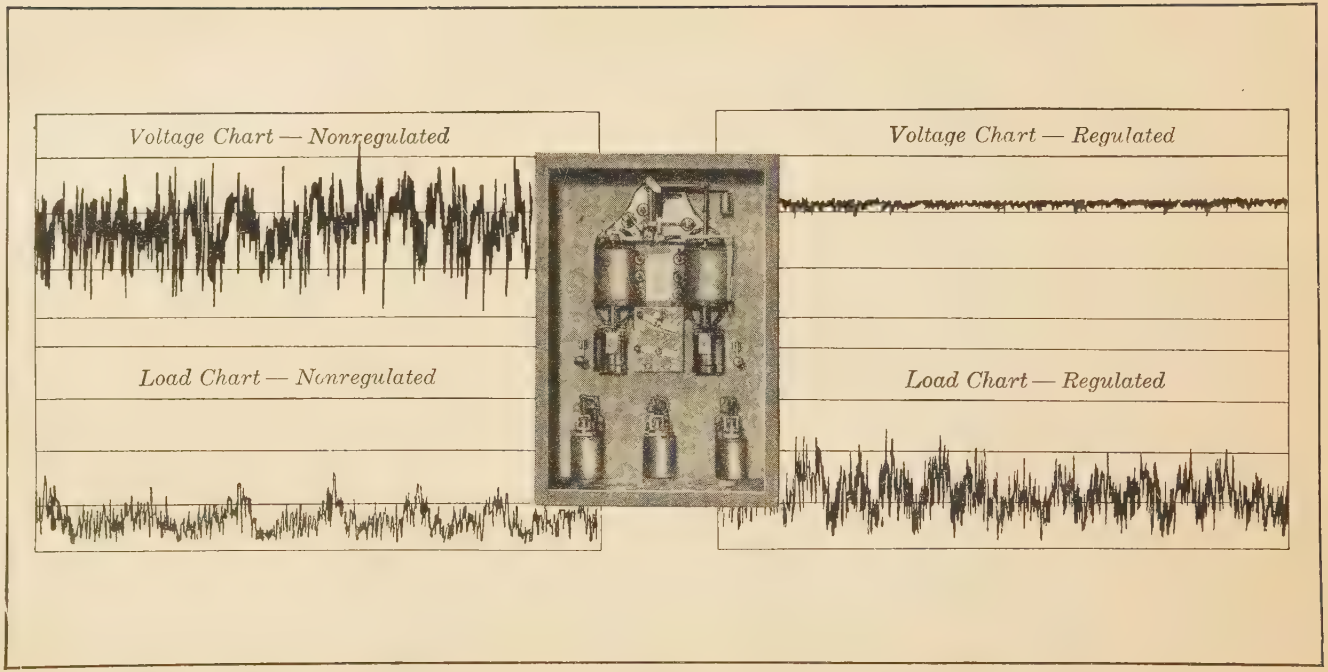


FIG. 190. Automatic generator voltage regulator

a, E is leading E_t by a greater angle than in b, so that the line ZI is longer, and therefore I is greater. The value of I can be obtained from the trigonometric relation $Z^2I^2 = E^2 + E_t^2 - 2\,EE_t \cos\theta$, where θ is the angle of phase difference between E and E_t. The power taken by the machine in case a is $EI_1 \cos\theta_1$, and in case b it is $EI_2 \cos(\theta_2 + \phi)$. Since $I_1 > I_2$ and $\cos\theta_1 > \cos(\theta_2 + \phi)$, the power taken to drive the machine is greater as its induced electromotive force leads its terminal voltage more (within limits).

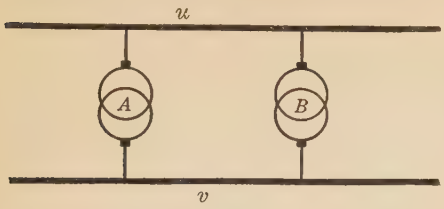


FIG. 191. Alternators in parallel

It is desirable, but not absolutely necessary, that the electromotive force generated by one of the alternators should equal the electromotive force generated by the other. The terminal voltage will, of course, be the same for both machines; but the induced electromotive force differs from the terminal voltage, as is shown in Fig. 186, and it is quite possible to have the induced electromotive forces of the two machines different while the terminal voltage is the same. But when this

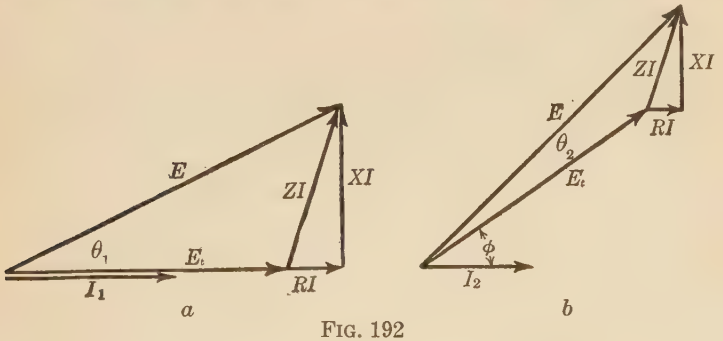


FIG. 192

condition exists, current circulates between the two machines, round the local circuit $AuBv$ (Fig. 191), and wastes energy.

185. The starting of alternators in parallel operation. In Fig. 193 let A be an alternator connected with the bus bars u, v, and already in operation; and let B be a second alternator which is to be put in operation in parallel with A.

The machine B is set in motion, and its field is excited so that its voltage is approximately equal to the voltage across

the bus bars. Before closing the switch S it is necessary to be sure that B is running very nearly in true synchronism with A, and that the voltage of B opposes the voltage of A relative to the local circuit $AuBv$ including the two machines.

A simple way of judging these conditions is obtained by connecting a lamp L round the switch S (connecting the lamp by means of a transformer in case the voltage is too high to be applied directly to the lamp). If B is not perfectly in synchronism with A, its voltage will alternately oppose and assist that of A relative to the local circuit $AuBv$. Thus the lamp will receive a voltage whose value will vary between $A - B$ and $A + B$; the lamp will accordingly flicker, being dimmest when B is opposing A and brightest when B is assisting A. The frequency of the flickering is evidently equal to the difference between the frequencies of A and B, and thus indicates how far B is from synchronism with A. When the flickering is very slow, the switch S is closed at an instant of minimum brightness of the lamp. The connections being once properly established, synchronism is maintained, as explained in the preceding section.

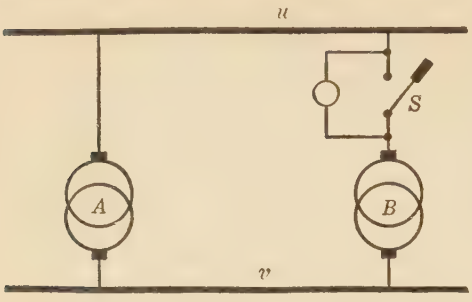


FIG. 193. Diagram of connections to show how a lamp may be used to judge the synchronism and phase relation between alternators

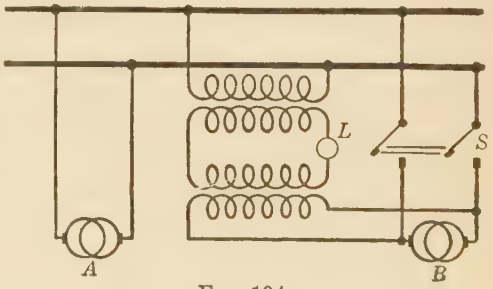


FIG. 194

In Fig. 194 is shown a set of connections which is often used instead of that of Fig. 193. In this arrangement the transformers may be so connected that the switch S has to be thrown at an instant of *maximum* brightness of the lamp, instead of at minimum brightness.

Instead of lamps, the *synchronoscope*, or *synchroscope*, is very often used. This is an instrument which indicates by a pointer the difference in phase between two voltages connected with it. If the two voltages are not synchronous, the pointer rotates. The direction of its rotation determines which of the voltages has the greater frequency — a matter that is not indicated by synchronizing lamps.

186. Sharing of load by alternators in parallel. It has been pointed out (sect. 184) how the power for driving an alternator depends on the difference in phase by which its induced electromotive force leads its terminal voltage, the alternator taking more power as this difference in phase is greater. When two alternators *A* and *B* are operating in parallel, *A* will take a greater share of the load if there is anything to cause *A* to lead *B* in phase; that is, if *A* tends to run faster than in synchronism with *B*. *A* cannot *actually* run faster than in synchronism with *B*, but it will *tend* to run faster if the governor of its prime mover is set so that it takes more power at synchronous speed than *B* does. When this condition exists, the two machines run in synchronism; but *A* keeps ahead of *B* in phase and draws more power from its prime mover, and accordingly delivers to the bus bars a larger share of the total power than *B* does. This holds true even if the induced electromotive force of *A* is less than that of *B*.

When a direct-current generator is operated in parallel with another, it can be made to take an increased share of the load by increasing its field excitation. With alternators in parallel, an increase in the field excitation of one does not increase its share of the load (though it does affect the reactive component of its current). To change the sharing of the load, it is necessary to change the adjustment of the governors of the prime movers that drive the alternators.

187. Capacity and rating of alternators. The discussions of Chapters X and XI on losses, efficiency, capacity, and ratings of dynamos apply (with occasional slight modifications) to alternators as well as to direct-current dynamos, so that but little needs to be added to them here.

The heating of an alternator is determined by its current output. In general this is not proportional to the power output, as would be the case in a direct-current constant-voltage generator. The rate of heating may be just as great with a small power

output ($= EI \cdot F$) as with a large one if the power factor is small in the former case and large in the latter. Accordingly the capacity of an alternator cannot be expressed in kilowatts (unless the power factor is also stated). It is therefore expressed in kilovolt-amperes (kv.-a.).

The rating plate on an alternator gives its frequency and number of phases, in addition to the same items that are given on the rating plates of direct-current generators.

PROBLEMS

1. A certain 32-pole delta-connected alternator is to be so constructed as to generate 2300 volts. The flux from each pole is to be 2,000,000 maxwells, the speed is to be 200 revolutions per minute, the form factor may be taken as 1.11 (corresponding to sinusoidal wave-form), and the phase constant is to be 0.955 (the usual value for three-phase machines). Find how many armature conductors must be provided per phase.

2. A certain three-phase delta-wound generator delivers 300 kw. at a power factor of 0.85, the terminal voltage being 1100 volts. If the load is balanced, find the current in each of the windings of the machine and in each of the mains.

3. A certain delta-connected three-phase alternator is rated for a terminal voltage of 2300, and a current of 220 amperes in the armature winding of each phase. The resistance of the armature is 0.1 ohm for each phase, and X_a is 5 ohms for each phase. Find the total induced voltage at full load if the current is (*a*) in phase with the terminal voltage, (*b*) 30° behind the terminal voltage in phase, and (*c*) 30° ahead of the terminal voltage in phase.

4. Find the full-load efficiency (that is, at an output of 1518 kw.) of the alternator in the preceding problem if the stray power losses amount to 20 kw., and the excitation is obtained by use of an auxiliary direct-current generator which requires 20 kw. to operate it.

CHAPTER XVIII

TRANSFORMERS

188. Essential features of transformers. In essence a transformer consists of two separate electric circuits linking with a single magnetic circuit, such as can be realized by winding two distinct coils of insulated wire on the same iron core, as shown diagrammatically in Fig. 195.

Let either one of the windings be connected with a source of alternating electromotive force E_t', so that a current flows in it and causes an alternating magnetic flux in the core. Then (since this same alternating flux links with the other winding also) an electromotive force E_t'' appears at the terminals of this latter winding, so that a current can be maintained in a circuit connected with it. The winding connected with the source of electromotive force is called the *primary*; the other winding is called the *secondary*.

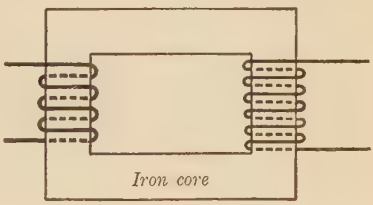


FIG. 195

189. Voltage ratio. It can be shown that the voltage E_t'', which appears at the secondary terminals, stands to the voltage E_t', impressed on the primary, in the same ratio as the number of turns in the secondary N'' to the number of turns in the primary N' (very nearly).

Consider, first, the conditions when the secondary circuit is open. The primary, being a coil of many turns wound on an iron core, has high inductance; accordingly the current that flows through it under the influence of the alternating electromotive force E_t' will be small and will lag behind E_t' in phase by nearly 90° (if the resistance of the primary is small, as is usual). This current is called the magnetizing current and will be denoted by I_M. The magnetic flux ϕ due to it alternates, of course.

It is convenient to fix attention on this alternating magnetic flux and regard it as the fundamental feature of transformer action.

As the flux changes, it induces in any path linked with it an electromotive force equal at each instant to $-\frac{1}{10^8}\frac{d\phi}{dt}$ volts.* That is, each turn of the primary or of the secondary must have induced in it an alternating electromotive force the instantaneous value of which is $-\frac{1}{10^8}\frac{d\phi}{dt}$ volts and which is the same for each turn of each coil (in so far as the whole of the flux links with it without magnetic leakage). The total electromotive force e' induced in the primary is then N' times this amount, and the total electromotive force e'' induced in the secondary is N'' times the same amount, N' and N'' being the numbers of turns in the primary and secondary respectively. Thus we reach the conclusion that, neglecting magnetic leakage, the electromotive force induced in the primary is to that induced in the secondary in the same ratio as N' to N''. This is true for the instantaneous values of the two electromotive forces at every instant and is therefore true for their effective values as well; so

$$\frac{E''}{E'} = \frac{N''}{N'} \text{ (very nearly)} \qquad (75)$$

for both sinusoidal and nonsinusoidal wave-forms.

Now the electromotive force e' induced in the primary must be opposite to and nearly equal to the voltage e_t' impressed on the primary from the mains with which it is connected. In fact, e' is nothing else than the electromotive force of self-induction in the primary.† But while e' is nearly equal to e_t', complete equality cannot exist; there must always be some slight excess of e_t' above e' in order to maintain the magnetizing current through the resistance of the primary.

* The minus sign means merely that the directions of the electromotive force and the flux are related to each other in the same way as the forward directions of the thread and the axis of a *left*-hand screw.

† We may look upon the self-induction, or inductance, of any circuit as due to the fact that when any change occurs in the voltage applied to it, the resulting change of the current is accompanied by a change in magnetic flux which induces in the circuit an electromotive force in a direction opposite to the change in the applied voltage.

Since e' and e_t' are nearly equal, the corresponding effective values E' and E_t' are nearly equal to each other, and we have, when the secondary is on open circuit,

$$\frac{E''}{E_t'} = \frac{N''}{N'} \text{ (very nearly).} \qquad (76)$$

Figs. 196 and 197 show, by curves and by vector diagram, respectively, the relations between E_t', I_M*, Φ, E', and E'', assuming sinusoidal wave-forms throughout.

Suppose, now, that an external circuit or load is connected with the terminals of the secondary: the secondary electromotive force will, of course, maintain a current I'' in it. When

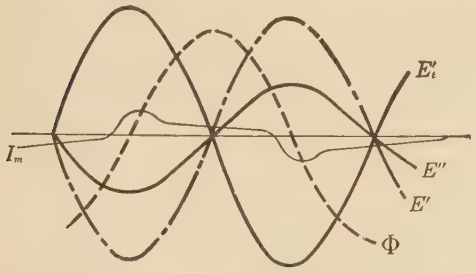


FIG. 196. Curves for various alternating quantities in a transformer

E_t', voltage applied to primary terminals; I_m, magnetizing current; Φ, magnetic flux; E', voltage induced in primary winding; and E'', voltage induced in secondary winding

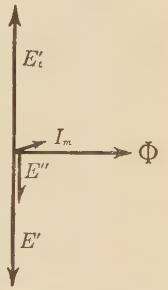


FIG. 197. Vector diagram for various alternating quantities in a transformer

this happens, a current I' (which may be called the *load current*) will flow in the primary in addition to the magnetizing current I_M, the total current I_t' in the primary being the resultant of I' and I_M (as will be explained in the next section). It is then necessary for E_t' to exceed E' more than before, in order to maintain the increased current in the primary; at the same time the terminal voltage E_t'' of the secondary must now become less than E'', in order that current may be maintained

* The lines for the magnetizing current give only an approximate representation of the true magnetizing current. Owing to the peculiar magnetic properties of iron (flux not exactly proportional to ampere-turns), the magnetizing current will not have a sinusoidal wave-form when the flux does; accordingly it cannot be properly represented by a line on a vector diagram, which applies only for sinusoidal wave-forms.

through the resistance of the secondary. But if the resistances of both primary and secondary are small (as is true in actual transformers), E_t' and E_t'' will not differ greatly from E' and E'' respectively; so that

$$\frac{E_t''}{E_t'} = \frac{N''}{N'} \text{ (very nearly)}, \tag{77}$$

which was to be proved.

Equation (75) would be *exactly* true if there were no magnetic leakage; equation (76) would be *exactly* true if there were also no resistance in the primary; and equation (77) would be *exactly* true if, in addition, there were no resistance in the secondary. In most actual transformers, used in accordance with their ratings, equation (77) holds true within about 3 per cent or less.

190. Current ratio. It can be shown that the ratio of the secondary current I'' to the total primary current I_t' is equal to $\frac{N'}{N''}$ (very nearly).

When a current flows in the secondary, it tends to produce in the core an alternating magnetic flux superposed upon that due to the magnetizing current in the primary. Accordingly, if the primary current remained equal to the magnetizing current I_M, the total flux alternating in the core would be changed, and $e'\left(= -\frac{N'}{10^{-8}}\frac{d\phi}{dt}\right)$ would no longer approximately balance e_t'. The result would be that some primary current I', in addition to I_M, would start to flow. The only condition under which equilibrium could be reached would be that in which the magnetizing effect of the additional primary current I' was equal and opposite to the magnetizing effect of the secondary current I''. The magnetizing effect of each winding will be determined by the number of ampere-turns, so that $N'I' = N''I''$, if magnetic leakage can be neglected. Accordingly

$$\frac{I''}{I'} = \frac{N'}{N''} \text{ (very nearly)}. \tag{78}$$

This relation holds true also for the instantaneous values of the currents, but i' and i'' circulate round the core in opposite directions.

Since I_M is small, it is approximately true to put

$$\frac{I''}{I_t'} = \frac{N'}{N''} \text{ (very nearly)}, \tag{79}$$

which was to be proved.

Equation (78) would be *exactly* true if there were no magnetic leakage, and equation (79) would be exactly true if, in addition, the magnetizing current I_M were zero, which would be the case if the magnetic reluctance of the core were zero and no eddy currents occurred in the core. In most actual transformers, used in accordance with their ratings, equation (79) holds true within about 3 per cent or less.

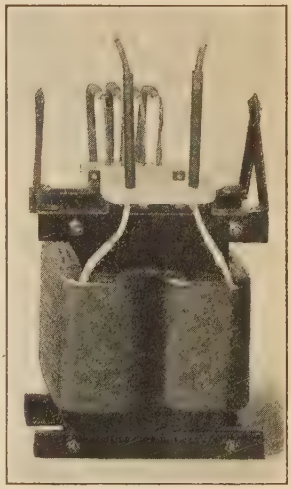


FIG. 198. Core-type transformer

Courtesy of the Wagner Electric Manufacturing Company

191. Construction of actual transformers. *Core-type and shell-type transformers.* In the diagram of Fig. 195 the material is not placed to good advantage. It is better either to extend the windings so as to inclose the core more or less completely, as in Fig. 198; or else to extend the core so as to inclose the windings more or less completely, as in Fig. 199. In the former case the transformer is called a core-type transformer; in the latter, a shell-type transformer.

Construction of windings. For convenience in drawing, the diagram of Fig. 195 shows the primary and secondary placed on different parts of the core; but in actual transformers one winding is placed outside the other, or else is divided into layers which alternate with layers of the other winding. With this arrangement very little magnetic leakage takes place.

Each winding (or each section of it, if it is divided) is constructed as a unit before being linked with the core, insulated copper wire or bar being wound on forms, with sheets of insulating material (mica, fuller board, impregnated paper, or other material) placed between the two coils or between successive layers when the potential difference is great enough to

require it. The arrangement shown in Fig. 200 is typical. The core is built in afterwards, a thick sheet of insulating material being placed within the inner layer of winding before the core is inserted.

Either winding may be used as the primary, but the winding with the larger number of turns always has the higher voltage. It is accordingly called the *high-tension winding*, and the other winding is called the *low-tension winding*, without regard to its use as primary or secondary.

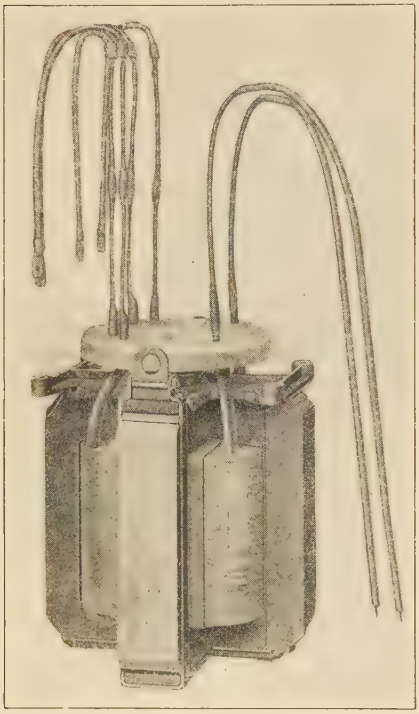


FIG. 199. Shell-type transformer
Courtesy of the General Electric Company

The size of the conductor in each winding is determined by the current it is to carry. That of the low-tension winding accordingly has the larger cross section. The number of turns in each winding is determined by the voltage for which it is intended and by the magnetic flux to be permitted in the core.*

Construction of the core. The core is built up of laminations of silicon steel which are so shaped that they can be fitted round the coils, as shown in Fig. 201. The reason for laminating the core is, of course, to avoid excessive eddy currents. Silicon steel is used principally because the hysteresis loss in this material is less than in other available magnetic materials. It also has the advantage of being

* Let the alternating flux in the core be $\phi = \Phi_m \sin \omega t$, where ϕ is the instantaneous value, and Φ_m the maximum value. Then the instantaneous voltage in one turn is given by $e_1 = \frac{d\phi}{dt} = \omega \Phi_m \cos \omega t$, and the effective voltage by $E_1 = \frac{1}{\sqrt{2}} \omega \Phi_m$. Accordingly, by increasing the number of turns in a coil, it may have the same terminal voltage with a lower value of Φ_m.

"non-ageing"; that is, its magnetic properties are not appreciably affected by long exposure to high temperatures up to the limits ordinarily encountered in the operation of transformers.

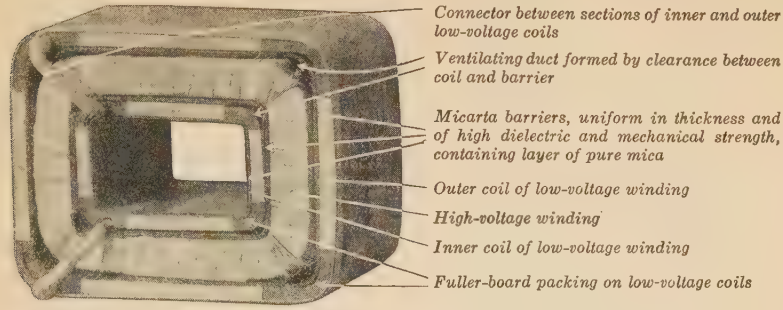

Connector between sections of inner and outer low-voltage coils
Ventilating duct formed by clearance between coil and barrier
Micarta barriers, uniform in thickness and of high dielectric and mechanical strength, containing layer of pure mica
Outer coil of low-voltage winding
High-voltage winding
Inner coil of low-voltage winding
Fuller-board packing on low-voltage coils

FIG. 200. Coil group of small transformer
Courtesy of the Westinghouse Electric and Manufacturing Company

Mounting. Except in certain special types, a transformer is supported inside a casing of sheet iron or cast iron, from which the windings are insulated. The casing is made weather-proof, so that the transformer can be placed out of doors, without any housing. It furnishes protection to the windings, lessens the probability of accident to persons by contact with high-voltage conductors (for which purpose it should be grounded), and serves as a tank for the oil in which the coils are submerged.

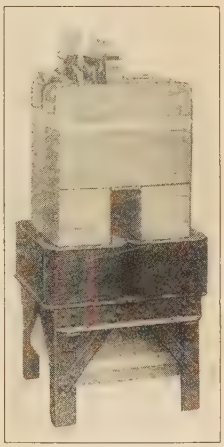


FIG. 201. Shell-type transformer under construction, with part of the core laminations in place

Courtesy of the Westinghouse Electric and Manufacturing Company

Cooling. When a transformer is in use, some means must be provided to facilitate the escape of the heat due to RI^2 losses in the coils and to hysteresis and eddy-current losses in the core. For this purpose the outer casing is filled with a suitable oil ("transformer oil"). The oil (which is also favorable to the insulation) readily absorbs heat from the hot parts of the transformer and carries it by convection currents to the casing. That the casing may dissipate heat more rapidly into the air, it is often corrugated, as shown in Fig. 202, and painted

black. In large transformers a system of piping through which cooling water is driven is sometimes introduced into the oil.

Terminals. The terminals of the transformer windings have to be carried through the casing. This makes very severe requirements for insulation from the casing. When the voltage is comparatively low, the ends of the windings are simply carried through porcelain bushings placed in holes in the

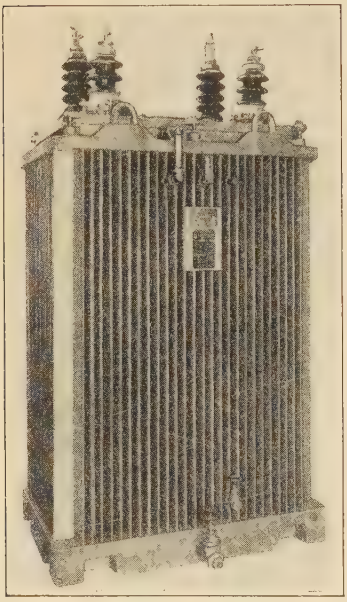


FIG. 202. Transformer in corrugated tank

Courtesy of the Westinghouse Electric and Manufacturing Company

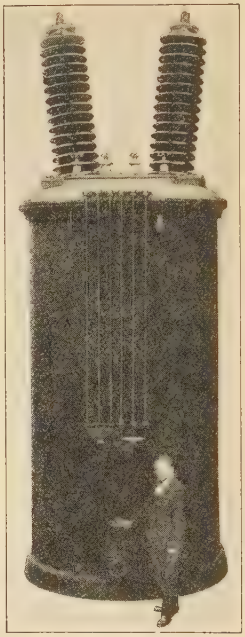


FIG. 203. Transformer for 150,000 volts

Courtesy of the Westinghouse Electric and Manufacturing Company

casing; but for very high voltages, large and elaborate structures have to be provided to carry the terminals of the windings through the casing. Such insulating bushings are prominent in Fig. 203, which shows a 150,000-volt transformer. The low-tension terminals are, of course, placed as far as is practicable from the high-tension terminals.

192. Applications of transformers. The principal applications of transformers are the following: (1) as "power transformers"

in connection with long-distance transmission lines, to step the voltage up at generating stations and down again at distributing substations; (2) as "distribution transformers," to step the voltage down from that of a distribution system to that of the service mains for electrical appliances; (3) as "instrument transformers," to make possible the use of alternating-current instruments for measuring voltages or currents greater than those in the instrument itself.

193. Power transformers. For the economical transmission of electrical energy it is necessary to use very high voltages (say 220,000) on long-distance transmission lines. But it has not yet been found practicable to construct generators for such high voltages; in fact, very few have been constructed for voltages higher than 13,000. Accordingly, transformers of large capacity are used at the generating stations which supply long-distance transmission lines, receiving energy at comparatively low voltage and delivering it to the line at high voltage. A transformer used for delivering energy at a higher voltage than that at which it receives the energy is called a *step-up transformer*. At points where energy from the line is to be utilized, *step-down transformers* are used, taking energy from the line at high voltage and delivering it to local distribution mains at some much lower voltage (say 2300). It would not be feasible to continue the high voltage of the transmission line on the mains of the local distribution system, because the difficulty and cost of high-voltage insulation under these conditions would be prohibitive, and the risk of accident would be too great.

Power transformers are built in large capacities (up to 20,000* kilovolt-amperes or more), but it is not necessary to construct them so large that a single one may transform all the energy required at a given point, since two or more can be made to act in parallel. Power transformers are most often placed out of doors, thus avoiding the necessity of carrying the high voltage of the line into the interior of any building. Since large quantities of power are concerned, high efficiency is very important in power transformers. In the larger sizes full-load efficiency of 99 per cent is usually attained.

* At the time of writing, the largest single-phase transformer in use has the capacity of 20,000 kilovolt-amperes, and the largest polyphase transformer, 25,000 kilovolt-amperes.

194. Distribution transformers. Economical distribution of electrical energy requires the use of rather high voltage for distribution mains or street mains; but many of the appliances by which the energy is to be utilized operate more satisfactorily at much lower voltage, and considerations of safety prohibit the introduction of the higher voltage into ordinary buildings. Accordingly, in any local distribution system, transformers have to be provided, receiving their energy at the comparatively high voltage (usually 2300) of the distribution mains and delivering energy at some comparatively low voltage (say 220 or 110) suited to the conditions under which the energy is to be used. Such transformers are placed at intervals along the mains, as discussed in section 179.

The standard ratings for distribution transformers range from 1.5 up to 200 kilovolt-amperes. With overhead distribution systems the smaller sizes (up to 25 kilovolt-amperes) are arranged for mounting directly on the poles.

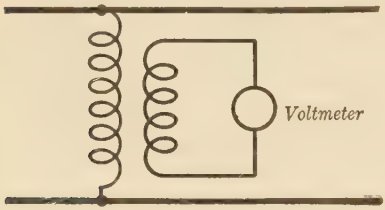


FIG. 204. Diagram of connections for a voltage transformer and its voltmeter

Distribution transformers are left permanently connected with the distribution mains, whether or not any energy is being taken from their secondaries. It is especially important, therefore, that their power losses at no load and at light loads be as small as possible.

195. Instrument transformers. If a voltmeter is connected with a transformer, as shown in Fig. 204, then, according to equation (77), the primary voltage can be obtained from the voltmeter reading (approximately) by multiplying the latter by $\frac{N'}{N''}$. If the transformer is designed with special reference to the accuracy of its voltage ratio, then the primary voltage can be obtained from the voltmeter reading on the secondary with sufficient accuracy for most practical purposes. A transformer used in this manner is called a *voltage transformer* or a *potential transformer*.

If an ammeter is short-circuited across the terminals of the secondary of a transformer whose primary is connected in an

alternating-current line, as in Fig. 205, then, according to equation (79), the line current can be obtained from the ammeter reading (approximately) by multiplying the latter by $\frac{N''}{N'}$. If the transformer is designed with special reference to the accuracy of its current ratio, then the line current can be obtained from the ammeter reading in the secondary with sufficient accuracy for most practical purposes. A transformer used in this manner is called a *current transformer*.

As long as the secondary of a current transformer is short-circuited through the very small resistance of an ammeter, the inductance of the primary is almost completely neutralized, and the primary acts almost like a plain resistance of exceedingly small amount inserted in the line; but if the secondary circuit should be opened, then the primary (consisting of a considerable number of turns linked with an iron core) acts like a high inductance, and, if the line current is maintained through it, the voltage across its terminals rises to a high value. If, as is usually the case, the secondary has more turns than the primary, the voltage at the secondary terminals will then rise to a still higher value. It would accordingly be very dangerous to open the secondary of a current transformer while the primary is carrying a current, and due precautions must be taken against such a possibility when using current transformers.

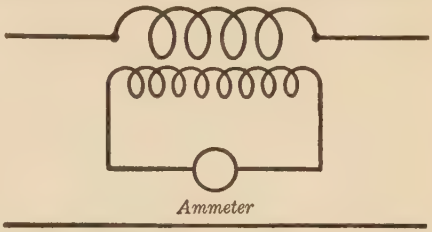


FIG. 205. Diagram of connections for a current transformer and its ammeter

Since the amount of power concerned in instrument transformers is very small, they are of comparatively small size. Matters of efficiency and of disposal of heat, which are so important in power and distribution transformers, are not of importance in instrument transformers.

As mentioned in section 145, instrument transformers are universally used with the measuring instruments on alternating-current switchboards to avoid bringing high-voltage connections to the switchboard. They are also very useful in laboratories and with portable instruments.

196. Transformers with divided secondaries. It is a common practice to wind the secondary of a distribution transformer in two parts, bringing the terminals of each to the outside of the case, as shown in Fig. 206. Each part develops the same

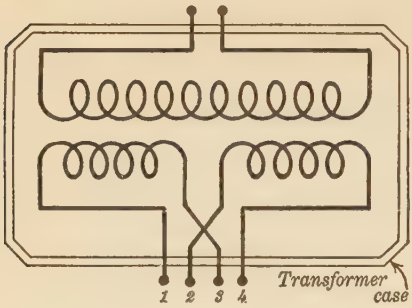


FIG. 206. Transformer with secondary winding divided into two sections

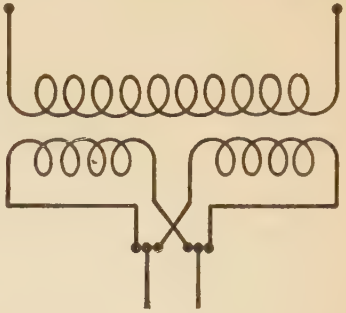


FIG. 207. Transformer with sections of the secondary connected in parallel

voltage (say 110 volts). The two parts can be connected in parallel as in Fig. 207, or in series as in Fig. 208, then giving twice the voltage of one of the parts. In this latter case, mains can be led out on the Edison three-wire system as in Fig. 209.

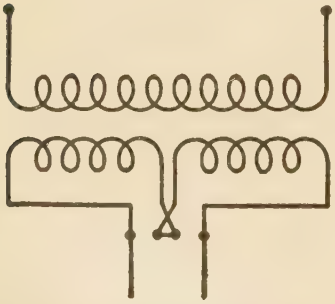


FIG. 208. Transformer with sections of the secondary connected in series

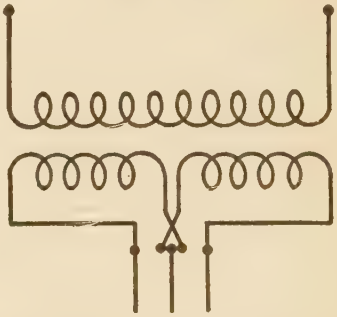


FIG. 209. Transformer with secondary connected for Edison three-wire system of distribution

The reason for crossing wires *2* and *3* inside the case is to avoid letting these wires rest against each other outside the case, where abrasion of the insulation might occur, and to produce an arrangement in which any terminal wire can

make contact with the next one (as might happen by accident) without short-circuiting either part of the secondary.

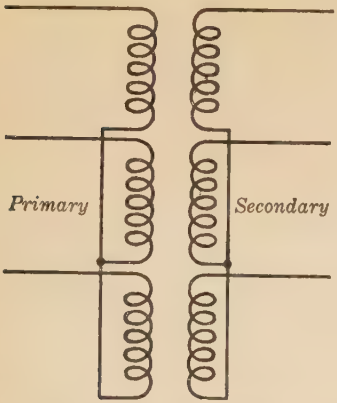


FIG. 210. Three-phase transformation by three single-phase transformers in Y-Y connection

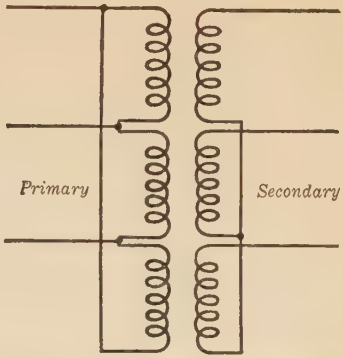


FIG. 211. Three-phase transformation by three single-phase transformers in Y-delta connection

197. Transformers in polyphase systems. Transformation in polyphase systems may be accomplished by using a combination

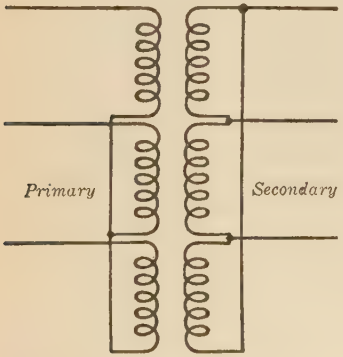


FIG. 212. Three-phase transformation with three single-phase transformers in delta-Y connection

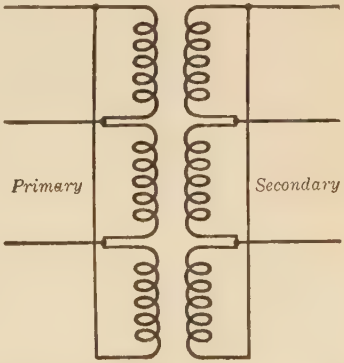


FIG. 213. Three-phase transformation with three single-phase transformers in delta-delta connection

of single-phase transformers or by using a single polyphase transformer with separate windings for each of the phases.

For the case of three phases any of the combinations of single-phase transformers shown in Figs. 210–214 can be used; they

are known as the Y-Y, Y-delta, delta-Y, delta-delta, and open delta (or V) combinations respectively. If the ratio of secondary to primary voltage for one transformer is denoted by R, then the ratio of voltage across secondary mains to voltage across primary mains in these various combinations is R, $\frac{R}{\sqrt{3}}$, $R\sqrt{3}$, R, and R respectively. All these combinations are symmetrical excepting the last (the open delta), in which it is possible for the two transformers to carry different shares of the load, even when the load is balanced as between the three phases. For this reason it is an undesirable arrangement except for temporary or for emergency purposes when it may be necessary to get along with two transformers until a third can be installed.

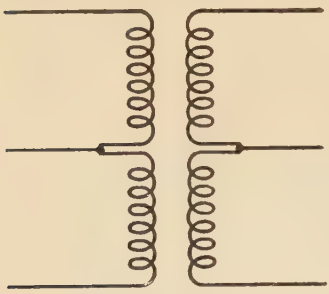


FIG. 214. Three-phase transformation with two single-phase transformers in open delta connection

Fig. 215 shows the appearance of a three-phase transformer (removed from its case). Fig. 216 shows the arrangement of circuits in a three-phase transformer. There is for each phase a primary winding and a secondary winding. The windings may be connected according to any of the schemes described above and shown in Figs. 210–214. The transformer may be thought of as having three cores, A, B, and C, one for each phase. The alternating magnetic fluxes in

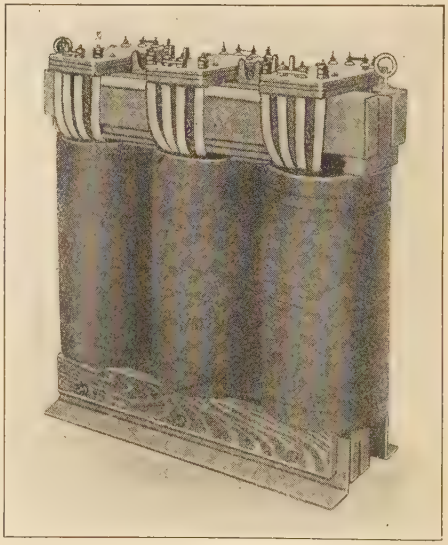


FIG. 215. Three-phase transformer, removed from case

Courtesy of the Westinghouse Electric and Manufacturing Company

these cores are mutually 120° out of phase with one another, and any one of the cores at any instant may be regarded as the return path for the flux from the other two in parallel.

By a proper combination of transformers it is possible to transform from a two-phase system to a three-phase system, or the reverse; or to transform from a system having any number of phases greater than one to a system having any other number of phases greater than one. The nature of these arrangements will not be gone into here. Their practical value is obvious, as they make it possible to interconnect polyphase systems using different numbers of phases (greater than one) and to use polyphase apparatus designed for any number of phases on systems using a different number of phases (greater than one).

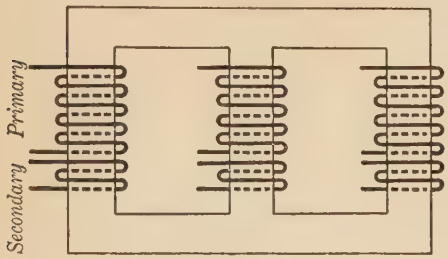


FIG. 216. Diagram to show arrangement of windings in a three-phase transformer

198. The autotransformer. If the secondary of a transformer is connected *in* one of the mains across which the primary is connected, as shown in Fig. 217, then the voltage of the secondary will be added to (or subtracted from) the voltage between the mains supplying the primary. A transformer connected in this manner is called an *autotransformer*. The same effect can be produced by the use of a single continuous winding on a transformer core, as shown in *a* or *b* of Fig. 218 (of course the current flowing in *ps* usually differs from that flowing in *ap* or *as*).

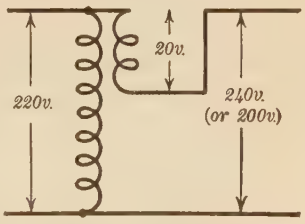


FIG. 217. Autotransformer connections

Evidently any transformation produced by an autotransformer could also be produced by a simple transformer with proper ratio of transformation. But for a given output a smaller transformer can be used when connected as an autotransformer than would be necessary if it were connected as a simple transformer.*

* This statement breaks down if the voltage *as* is less than half the voltage *ap* with the connections of Fig. 218 *a*, or if *as* is more than twice *ap* as shown in Fig. 218 *b*.

This can be brought out best by an example. Suppose it is desired to deliver 10 kilowatts of power at unity power factor at 200 volts, the voltage of the primary supply being 220. If simple transformer connections are used, the primary being connected

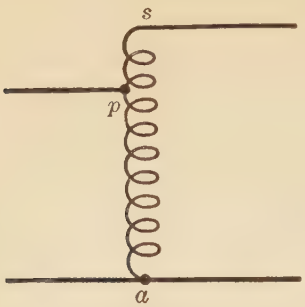


FIG. 218 *a*

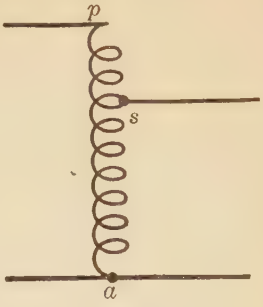


FIG. 218 *b*

to 220-volt mains and the secondary delivering 200 volts, then (neglecting losses) the primary current must be 45.5 amperes, the secondary current 50 amperes, the power received by the primary and transmitted to the secondary 10 kilowatts, and the capacity of the transformer must be 10 kilovolt-amperes. But if autotransformer connections are used (as in Fig. 219), the secondary will need to develop only 20 volts (that is, 220 − 200), while the secondary current is still 50 amperes. Since the currents in primary and secondary are inversely as the voltages developed, the primary current need be only $\frac{20}{220}$ of 50, or 4.55 amperes; the primary accordingly receives and transmits to the secondary only $220 \times 4.55 = 1000$ watts (which is also equal to 20×50 watts, computed from the secondary volts and amperes), and the capacity of transformer required will be only 1 kilovolt-ampere instead of 10 kilovolt-amperes. The remainder of the 10 kilowatts of output passes directly from the primary mains to the secondary mains,

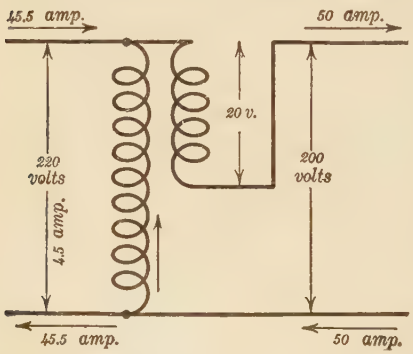


FIG. 219

without transformer action. The arrangement of the currents and voltages is shown in Fig. 219.

199. Voltage regulators. If an autotransformer is provided with a number of terminals, as shown in Fig. 220, then, by using one or another of the secondary terminals, it is possible to alter the secondary voltage through small steps at will. If the various terminals are connected with points with which contact may be made by a movable handle, the arrangement may be used to control the current in the secondary circuit by varying the volts applied to it, thus accomplishing the same purpose as that for which a rheostat is used in direct-current circuits. It has the great advantage over a rheostat that the loss of power in it is very little; furthermore, it can be arranged to let voltages higher as well as lower than that of the supply be taken from it. Voltage regulators acting on this principle have been successfully used, but are not now in common use, since the purpose for which they are used can be more satisfactorily accomplished by means of another device — the induction voltage regulator.

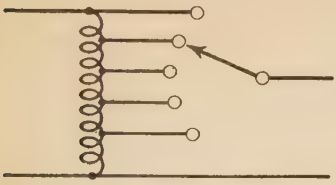


FIG. 220. Diagram of connections of an autotransformer provided with several taps for control of the voltage of the secondary circuit

The induction voltage regulator consists of an autotransformer of which one of the windings is movable and can be rotated with respect to the other. The arrangement is shown diagrammatically in Fig. 221.

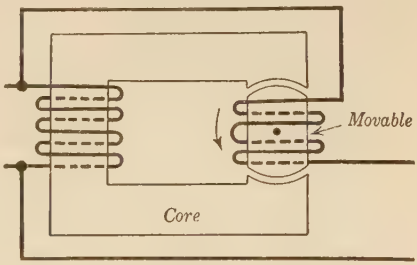


FIG. 221. Diagram to show the principle of action of the induction voltage regulator

In the position shown, all the flux from the primary links with the secondary (neglecting magnetic leakage); but if the movable coil is rotated, then only part of the primary flux links with the secondary, and accordingly less voltage is induced in the secondary than before. If the movable coil is rotated more than 90°, the primary flux then links with the secondary in reversed direction, thus reversing the voltage induced in the

secondary. The regulator can accordingly be used either to raise or to lower the voltage supplied to it.

Induction voltage regulators furnish a very satisfactory means for the independent control of voltage of each line of feeders leaving a central station. For this purpose they are constructed in large capacities, sometimes up to several thousand kilovolt-amperes.

200. Voltage regulation of transformers. In transformers used in the distribution of electrical energy by the constant-voltage system, it is, of course, desirable that the secondary should maintain constant voltage at all loads when the primary is connected to constant-voltage mains. But the RI drop in both primary and secondary increases with increasing load, so that the volts maintained at the terminals of the secondary must fall off with increasing load (the drop in voltage depending on the power factor of the load as well as on its magnitude). In actual transformers designed for such service the "voltage regulation," or percentage difference between no-load and full-load secondary voltage, runs from about 1 to $2\frac{1}{2}$ per cent under ordinary conditions and rarely exceeds 4 per cent under the most unfavorable conditions of practical operation.

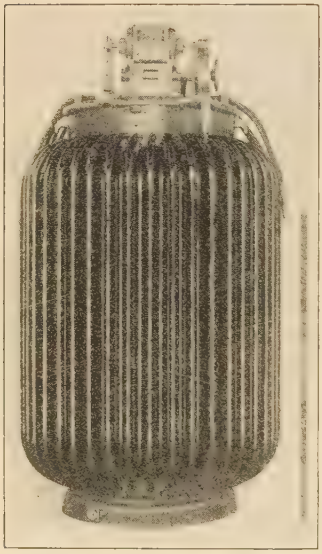


FIG. 222. Motor-operated induction voltage regulator

Courtesy of the General Electric Company

201. Power losses in transformers. The power losses in transformers are eddy-current and hysteresis losses in the core (often called the *iron losses*) and RI^2 losses in the windings (often called the *copper losses*). The RI^2 losses increase with increasing load, and vanish entirely at no load (if we neglect the minute RI^2 loss due to the magnetizing current in the primary). But the eddy-current and hysteresis losses have the same value at all loads, and continue undiminished when the transformer is entirely unloaded, so long as its primary remains connected with the mains.

202. Efficiency of transformers. Evidently the efficiency of a transformer must be zero at no load and increase with increasing load up to a maximum, as shown in Fig. 223. The efficiency depends not only on the magnitude of the load but also on its power factor.

The full-load efficiency of transformers is greater in the larger sizes than in the smaller ones, exceeding 99 per cent in the largest

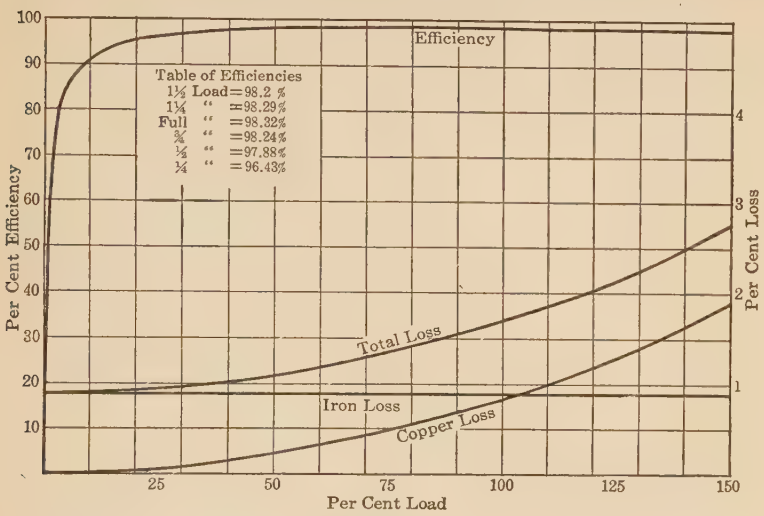


FIG. 223. Curves showing efficiency and losses of a transformer at different loads.

transformers (larger than 1000 kilovolt-amperes). The following table gives the full-load efficiency and the voltage regulation, at unity power factor, of transformers of smaller sizes.

FULL-LOAD EFFICIENCY AND VOLTAGE REGULATION OF 60-CYCLE TRANSFORMERS, VOLTAGES 2200/220–110

CAPACITY IN KILOVOLT-AMPERES	FULL-LOAD EFFICIENCY (PER CENT)	VOLTAGE REGULATION (PER CENT)
1	95.8	2.4
3	97.1	1.9
5	97.5	1.7
10	97.9	1.5
25	98.2	1.3
50	98.5	1.1

Most transformers are kept connected with the source of primary electromotive force at all times, whether or not there is any load on the secondary. The core losses thus continue, even when no output whatever is being delivered. Under these conditions the full-load efficiency of a transformer is often of less interest than its "all-day efficiency," by which is meant the ratio of the total output of *energy* (in kilowatt-hours) in twenty-four hours to the total input of energy in the same period. For instance, a transformer used to supply energy for domestic lighting might deliver 1 kilowatt at an efficiency of 96 per cent for 4 hours and remain unloaded for the rest of the day, with core losses of 25 watts. Its all-day output of energy would then be 4 kilowatt-hours, and its input would be $\frac{4}{0.96} + 20 \times 0.025$, or 4.67 kilowatt-hours, so that its all-day efficiency would be $\frac{4}{4.67}$, or 86 per cent. Evidently the all-day efficiency depends on the manner in which the load varies, as well as on the characteristics of the transformer.

203. Rating of transformers. Under the standards of the American Institute of Electrical Engineers, transformers are rated in kilovolt-amperes available at the output terminals at specified voltage and frequency. The heating of transformers under load (which determines the capacity and rating) follows much the same principles as that of dynamos, as discussed in section 123.

It is customary for the manufacturer to place on each transformer a plate giving its rating, primary and secondary voltages, frequency, number of phases, name of maker, type of transformer, and the manufacturer's serial number.

204. Performance of a transformer at different voltages and frequencies. In the practical operation of distribution systems it sometimes happens that transformers are on hand which are intended for voltages or frequencies different from those in use on the system. So far as its main principle of action is concerned, any transformer ought to work on any frequency and between any pair of values of primary and secondary voltages having the same ratio as the numbers of turns in primary and secondary windings. Actual transformers do, in fact, operate fairly satisfactorily on voltages and frequencies that differ

(within moderate limits) from those for which the transformer is intended, but usually with lowered efficiency and less good voltage regulation.

PROBLEMS

1. A current transformer has its primary connected in a circuit supplied from 110-volt mains and its secondary terminals are connected with an ammeter which reads 5 amperes. How many amperes are flowing in the primary if it has one tenth as many turns as the secondary? If the secondary circuit should now be opened (the primary remaining connected as before), what voltage would be developed at the secondary terminals? (Under these conditions the primary terminals would receive practically 110 volts.)

2. A certain 10-kilovolt-ampere transformer is used for transforming from 2200 volts to 110 volts. Find the current in each of its windings when fully loaded, neglecting losses.

3. Allowing now for losses, compute the full-load efficiency of the transformer in Problem 2 at unity power factor, having given the loss from hysteresis and eddy currents as 150 watts, the resistance of the primary winding as 0.4 ohm, and the resistance of the secondary winding as 0.01 ohm. (The currents may be assumed to be the same as in Problem 2, the secondary voltage the same, and the primary voltage sufficiently increased to provide power for the losses.)

4. If the transformer in Problem 2 is used at full load (unity power factor) for three hours and is idle for the remaining twenty-one hours of the day, what will be its all-day efficiency?

5. In a transformer with divided secondary, as shown in Fig. 206, the primary winding is connected to 110-volt supply mains, and 11 volts is then induced in each half of the secondary winding. Draw diagrams showing how connections could be made to secondary mains so as to obtain each of the following secondary voltages: 11, 22, 88, 99, 121, or 132.

6. In a certain core-type transformer the cross section of the core is 50 cm.^{2}, and the flux density in the core alternates sinusoidally between maximum values of $+ 5000$ and $- 5000$ gausses. Find (*a*) the maximum value of the alternating core flux, (*b*) the maximum value of the rate of change of the core flux if the frequency is 60 cycles per second, (*c*) the maximum value of the electromotive force induced in one turn of the windings, (*d*) the effective value of the electromotive force in one turn if the wave-form is sinusoidal, and (*e*) the number of turns required in a winding for 2300 volts.

CHAPTER XIX

ALTERNATING-CURRENT MOTORS

205. Types of alternating-current motors. Alternating-current motors are of three principal types: synchronous motors, induction motors, and alternating-current commutator motors. These types differ widely in their principles of action and in their characteristics. Before entering upon more complete discussion of any of the types it will be desirable to make a few preliminary statements about each type.

The *synchronous motor*, in principle of action and main features of construction, is simply an alternator reversed; that is, it receives alternating-current electrical energy and delivers mechanical energy instead of receiving mechanical energy and delivering electrical energy. The speed of the motor is rigidly fixed by the frequency of the supply and is independent of the load; the motor cannot operate at all at a different speed.* Synchronous motors may be single-phase or polyphase.

The *induction motor*, in principle of action, consists of a rotor placed within a revolving magnetic field maintained by a stator, or stationary structure whose windings are connected to the supply mains. The rotor has no conductive connection with the supply, but currents flow in its windings as a result of inductive action from the magnetic field, and the forces between these currents and the revolving magnetic field make the rotor tend to follow the field. The no-load speed is determined by the frequency of the supply; with increasing load the speed decreases, but the motor cannot operate efficiently at speeds differing very widely from the no-load, or synchronous, speed. The induction motor is inherently a polyphase appliance, but it can be adapted for use on single-phase supply also if it is provided with special devices for starting.

* Theoretically a synchronous motor should be able to operate also at any speed which is an odd-numbered multiple of the speed just mentioned, but this possibility has no practical importance.

Alternating-current commutator motors are of various types. Broadly speaking, they may be regarded as developed (with considerable modification in some cases) from direct-current motors. Since a self-excited direct-current motor turns in the same direction when the connections with the line are reversed, it can be made to operate on alternating current if it is properly modified to overcome certain difficulties. The speed of commutator motors is not fixed by the frequency of the supply, and some types will run on direct current as well as on alternating current. Alternating-current commutator motors are inherently single-phase appliances, but polyphase types have been developed also.

THE SYNCHRONOUS MOTOR

206. The principle of action of the synchronous motor. In accordance with the general principle of reversibility of dynamo-electric machinery, one would expect that any alternator would be capable of acting as a motor when supplied with alternating current. The facts are in accordance with this expectation, provided the speed is right and the field is excited from some independent source of direct current.

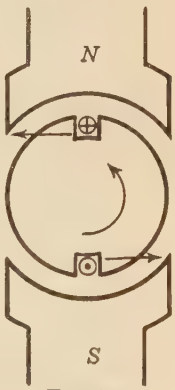


FIG. 224

Let Fig. 224 represent an alternator with revolving armature (for simplicity the winding is shown as consisting of a single turn). At the moment, the current flows inward at the top and outward at the bottom, in accordance with which the forces from the magnetic field on the armature conductors are in the directions shown by the arrows and result in a torque in the direction shown by the curved arrow. If the armature is at rest, this torque will start it turning in the direction shown; but before an appreciable motion is imparted to the armature the current will have reversed (within $\frac{1}{120}$ of a second if 60-cycle supply is used), and this reverses the torque and tends to turn the armature in the opposite direction; hence the only motion produced under these conditions will be a slight vibration of the armature. But the result will be very different if the armature is already in motion with a speed corresponding

to one revolution in the course of one cycle of the current. In this case the armature will have turned through 180° by the time the current has reversed; so that while the current will now come outward from conductor *A* and go inward at conductor *B*, we now have conductor *A* next to the south pole and conductor *B* next to the north pole, and this will give a torque in the same direction as before. Accordingly the torque will tend to maintain the motion under these conditions.

If the machine had more than two poles, of course the speed should then be such that any armature conductor passes two poles in the time that the current completes one cycle. This is called *synchronous speed*. If a revolving-field machine is used, it is obvious that its behavior will be identically the same as that of a revolving-armature machine in the particulars just discussed.

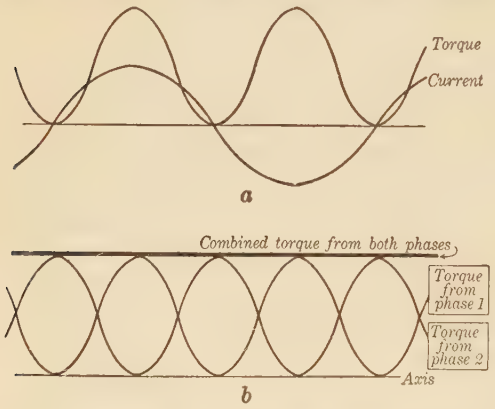


FIG. 225. Curves for the instantaneous value of the torque in a synchronous motor

a, for single-phase motor; *b*, for two-phase motor

From the preceding discussion it is easy to see that in a single-phase machine the torque is pulsating, with a frequency twice that of the current; the torque has a maximum for each positive and each negative maximum of the current, as shown in Fig. 225 *a*. In a polyphase machine the winding for each phase acts in the same manner as that just discussed for a single-phase machine; but at the instants when the torque is small in one phase, it is large in others, and the resultant torque is steady.

207. Starting of synchronous motors. As has already been brought out, synchronous motors are not self-starting; so that when one is used, some independent means has to be provided for bringing it up to synchronous speed. This may be accomplished by providing a small auxiliary motor of some self-starting type. The capacity of such an auxiliary motor need be

only a small fraction (perhaps 5 per cent) of that of the main motor, sufficient to supply the no-load losses of the latter and to accelerate its rotor. For polyphase synchronous motors a simpler method of starting is available and is generally used: certain changes in connections are made which permit the motor during starting to act as an induction motor, as will be explained in section 224.

Before connecting the armature of a synchronous motor to the supply mains it is important to make sure that the alternating electromotive force generated in the armature has very nearly the same frequency as that of the supply voltage and is very nearly opposite in phase to the latter at the instant that the connection is made. The situation is the same as that obtaining when two alternators are to be connected in parallel, as discussed in sections 184 and 185 (pp. 213 and 214), and the methods for synchronizing a motor are the same as described there.

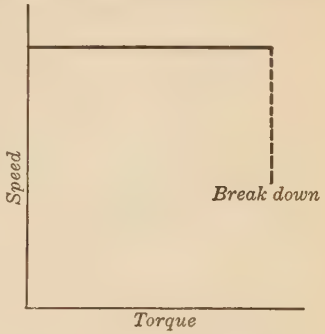


FIG. 226. Speed-torque curve for synchronous motor

208. Speed-torque characteristics of synchronous motors. If a synchronous motor operates at all, its speed must remain absolutely constant if the frequency of the supply is constant (except that slight vibrations in speed above and below the constant mean are possible). The most minute variation from synchronous speed, if continued, would eventually so alter the relative position of armature and field from that which would obtain under synchronous speed that the torque would be reversed; and as soon as this happened, the machine would rapidly be brought to rest.*

When the motor is running, it is found that it can be loaded up to a certain definite maximum, at which it breaks down and comes to rest. Within this maximum its speed is invariable, whatever the load may be. If a speed-torque curve is drawn, it will accordingly have the form shown in Fig. 226 (in the case

* Assuming that the variation from synchronous speed is in the direction of *slower* speed. If it is in the opposite direction, then the reversal of the torque would probably simply slow down the speed back to that of synchronism, when the motor would take hold and run again.

of a single-phase machine, in which the torque pulsates, the figure should be taken to refer to the average torque).

209. How the synchronous motor adjusts itself to variations in load. As the armature rotates relatively to the field, electromotive force is induced in it in the same manner as is the counter-electromotive force of a direct-current motor, although in the synchronous motor this electromotive force alternates, with the same frequency as that of the supply voltage. This counter-electromotive force, however, need not be exactly in opposition to the supply voltage but may have a phase difference from it of 170° or 190° or some other amount. The electromotive force acting to drive current through the motor armature will be the resultant of the line voltage E_l and the counter-electromotive force E_c and is marked E_r in the vector diagram, Fig. 227. The current I through the armature will equal E_r divided by the impedance Z of the armature, and will lag behind E_r by the angle $\cos^{-1}\frac{R}{Z}$, called θ in the figure. The power converted by the motor is $E_c I \cos\phi$; this will usually be negative (as it should be for motor action), because ϕ will usually be more than 90° and less than 270°; if it should come out positive, it would mean that the machine was acting as a generator and not as a motor at all. E_l and E_c are constant; I and ϕ are determined by the resultant of E_l and E_c. Hence the only way that the power ($= E_c I \cos\phi$) received by the motor can change is by a change in the resultant of two constant quantities; and that can be brought about only by a change in the angle between them. So when the load on a synchronous motor is changed, the motor accommodates itself to the altered load by changing the difference in phase between its own electromotive force and that of the line.

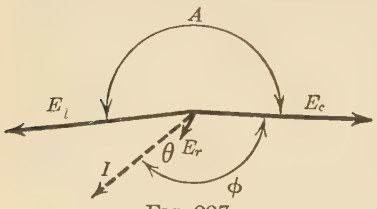


FIG. 227

Figs. 228 and 229 may assist in understanding just how the motor behaves in order to accomplish this. Each figure represents the armature of a motor connected through a line with the armature of a generator which supplies it with energy; for simplicity two-pole revolving-armature machines are assumed,

and each armature is drawn with a winding of just one turn. In each figure both armatures are supposed to be in rapid rotation, but keeping the same angular displacement with respect to each other. This angular displacement (or "coupling angle") corresponds to the angle A of the vector diagram, Fig. 227. The two figures correspond to two different loadings of the motor.

For most values of the coupling angle (or phase difference between E_l and E_c), there will be parts of the cycle in which the torque is reversed (in a single-phase machine or in one phase of a polyphase machine), but the average torque throughout a cycle must have sufficient excess in the right direction to keep the motor going.

FIG. 228. Diagram to represent a synchronous motor running with its voltage 180° out of phase with that of the generator

FIG. 229. Diagram to represent a synchronous motor running with its voltage 210° behind that of the generator

210. "Hunting" in synchronous motors. Unless special means of prevention are provided, synchronous motors tend to exhibit a very objectionable action known as *hunting*, which consists in a vibration in speed above and below the constant synchronous speed, accompanied by vibrations in the torque exerted and in the current. The period of the hunting vibration has no relation to that of the alternating current used, but is many times longer. The cause of the hunting may be understood from the following considerations:

Suppose the motor to be running steadily at some such coupling angle, or angular displacement relative to the generator, as is shown in Fig. 229; then let some additional load be put on the motor. To carry the increased load, the coupling angle must be greater; to attain a greater coupling angle, the motor must momentarily slow down a little, just as one would expect any machine to do upon increasing its load, even though the decrease in speed may be only slight and temporary. When the motor has dropped behind sufficiently to make its coupling angle correspond to the power required to carry the increased

load, its speed is still below synchronous speed, and inertia prevents it from immediately resuming synchronous speed. Accordingly its coupling angle continues to increase *beyond* that required for carrying the increased load; the motor then develops *more* power than is required to carry the load, and its rotor is therefore accelerated. By the time that it has regained synchronous speed its coupling angle still is larger than necessary, with the result that the speed is further increased and goes above synchronous speed, which makes the coupling angle decrease again. But when the coupling angle reaches the value

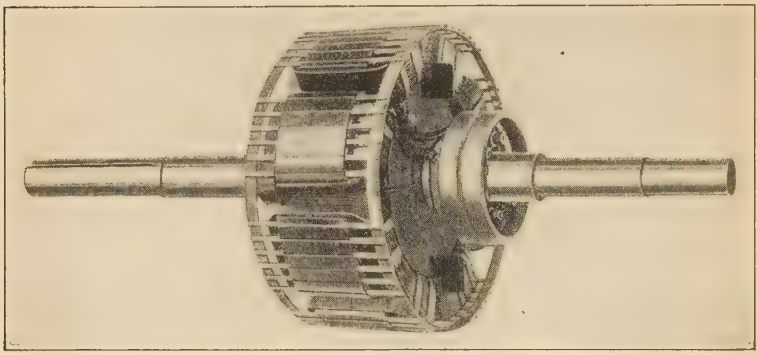


FIG. 230. Damper winding on the pole faces of the revolving field of a synchronous motor

Courtesy of the Westinghouse Electric and Manufacturing Company

corresponding to the load on the motor, the speed is still above synchronism. Thus the speed tends to vibrate above and below synchronous speed. The action may also be looked upon as a vibration of the coupling angle of the rotor superposed upon uniform rotation at synchronous speed. It is quite possible for the amplitude of the vibration to be so large as to cause the motor to break down.

The action is analogous to that which would take place if the machine were driven mechanically by means of an elastic coupling: this would permit it to vibrate relatively to its prime mover, while keeping its average speed constant.

To prevent hunting, — or rather to limit the hunting vibrations to small amplitudes, — synchronous motors are provided with short-circuited windings of a few turns of heavy copper wire

or bars placed on the pole faces, and known as *damper windings* or *amortisseur windings*. If the rotor begins to hunt (that is, to undergo oscillations superposed on its rotation at synchronous speed), then the magnetic field due to the armature currents oscillates backward and forward relatively to the damper winding; currents are accordingly set up in the damper winding, and the forces between them and the armature magnetic field oppose the oscillations of the latter.* One arrangement of a damper winding is shown in Fig. 230.

Damper windings are often provided on alternators as well as on synchronous motors, since alternators also tend to hunt when operated in parallel.

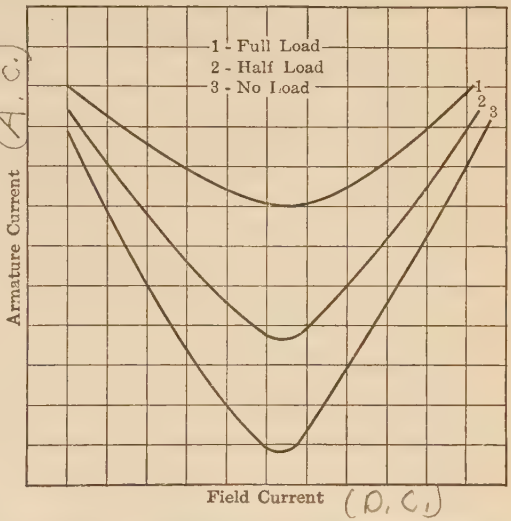


FIG. 231. V curves for a synchronous motor

211. Effect of altering the field excitation of a synchronous motor. The counter-electromotive force generated by a synchronous motor may be altered by changing the field excitation. When this is done, the motor in general continues to run and carry its load, but with a different armature current and at different power factor. The manner in which the armature current varies with the field excitation is shown by the curves of Fig. 231: the load on the motor remains the same for all points of the same curve, but the different curves correspond to different loads. On account of their shape, these curves are called V *curves*.

The V curves show that for any given load there is one particular field excitation at which the motor will carry that load

* As long as the speed is exactly at synchronism, it can be shown by an analysis which will not be given here that the magnetic field due to armature currents retains a constant position relative to the field magnets and damper winding (neglecting short-period vibrations).

with minimum current intake. At this excitation its efficiency will be greater than at any other, and its power factor will be unity. Of course the motor should be operated at this excitation, which is easily determined by trial, by watching an ammeter in the armature circuit while manipulating the field rheostat. If the load changes, the excitation requires readjustment.

The high power factor at which the synchronous motor normally operates makes it a very desirable type of motor from the point of view of the central station (see section 164).

212. The "synchronous condenser." It is an interesting and valuable property of the synchronous motor that it takes current which

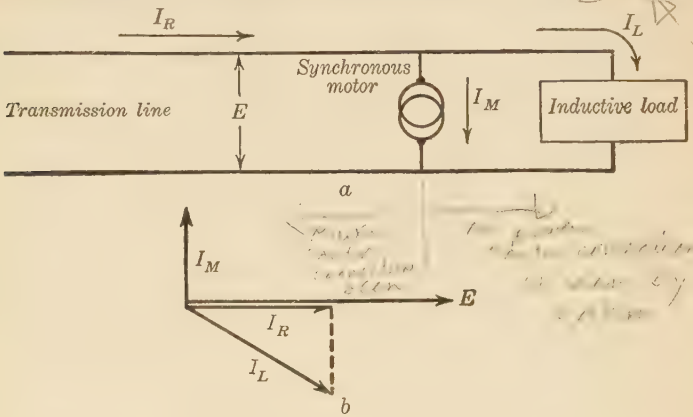


FIG. 232. Use of a synchronous motor for improving the power factor of a line supplying power to an inductive load

a, diagram of connections; *b*, vector diagram

leads the supply voltage in phase when its excitation is raised above that corresponding to minimum current intake (the apex of the V curve). This makes it possible to use synchronous motors to improve the power factor in transmission lines, in which the current usually tends to lag behind the voltage. The action may be made clear by the vector diagram, Fig. 232, in which I_L represents the lagging current taken by an inductive load, I_M the leading current taken by a synchronous motor, and I_R their resultant, which is the current in the line and is in phase with the voltage E, giving unity power factor.

On long-distance transmission lines it is often found desirable to install synchronous motors purely (or primarily) for the purpose of keeping the line current in phase with the voltage and thus

effecting a saving of line losses in accordance with the discussion of section 164 (p. 189). A synchronous motor employed for this purpose is often called a *synchronous condenser*, because its current leads the voltage in phase like the current in a condenser. It may be used to deliver power, or it may simply run idle, in which case, of course, it consumes very little power (merely enough to make good its no-load losses) but may take a large current leading the voltage by practically 90°.

213. Uses of synchronous motors. Despite the disadvantages of not being self-starting and of requiring separate excitation, synchronous motors have some important applications. They are nearly always used in those situations in which perfect constancy of speed is desirable, as in motor-generator sets, or for the motive power in substations. In applications calling for approximate constancy of speed without frequent starting and stopping, such as the driving of pumps or main shafts of shops, synchronous motors are sometimes used instead of the induction motors which have generally occupied this field of applications. The synchronous motor offers the advantage of high power factor at all loads, whereas the induction motor has very low power factor at light loads. The use of synchronous motors to control the power factor on transmission lines has been mentioned in section 212.

THE INDUCTION MOTOR

214. Principle of action of the induction motor. The principle of action of the induction motor depends upon the tendency of a conductor to follow a moving magnetic field. By the method explained in the next section a rotating magnetic field is produced in the cylindrical space inside a stationary structure called the *stator*. If any conductor is placed in this space so that the magnetic flux of the rotating field cuts across it, then electromotive forces are induced in it which in general cause currents to flow, and the forces between the magnetic field and these currents tend to drag the conductor along in the direction in which the magnetic field moves. If the conductor is mounted on an axis coinciding with that of the stator, then the forces yield a torque tending to produce rotation in the same direction as that of the rotating magnetic field. The torque would cease

only if the rotation took place with the same speed as that of the rotating magnetic field, in which case the field would no longer cut through the moving conductor, and accordingly no electromotive force would be induced in it and no current would flow in it.

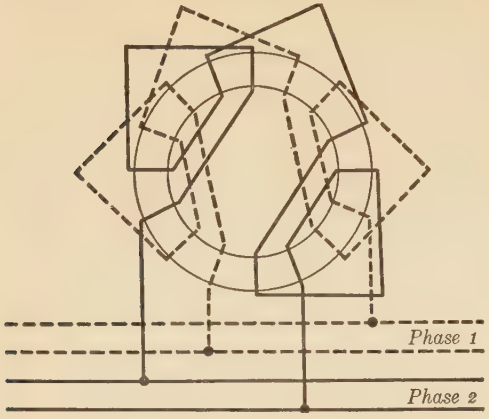


FIG. 233. Diagrammatic representation of the stator winding of a two-phase induction motor

Although the conductor delivers mechanical power to the rotor shaft, it has no conductive connection with the supply mains. The induction motor therefore offers an example of wireless transmission of power — over a very small distance, to be sure: merely the air gap separating stator and rotor.

215. Production of the rotating field. The method of producing the rotating magnetic field may be understood by considering a specific example. Let us suppose that a winding like that of phase *1* in Fig. 233 is placed upon the interior surface of the stator; if current flows in this winding, there will be produced inside the stator a magnetic field having the arrangement shown in Fig. 234. If the current alternates, this field alternates also. Now suppose that a second winding (represented by dotted lines in Fig. 233) is placed upon the stator, exactly like the first winding but displaced with respect to it, so that bands of conductors of the second winding alternate with bands of the first. Let one of

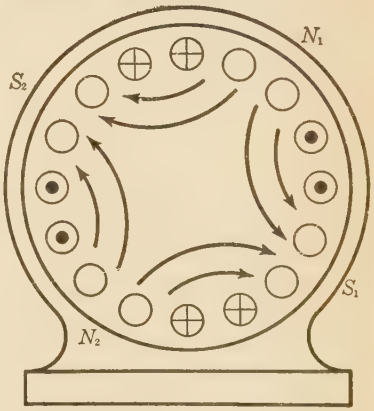


FIG. 234. Diagram to represent the magnetic field within the stator of an induction motor

these windings be connected with one phase, and the other with the other phase, of a two-phase supply. Fig. 234 represents the state of affairs at the instant the current in phase *1* is at a maximum and the current in phase *2* is passing through zero. A quarter of a cycle later, phase *1* current is passing through zero, while phase *2* current has reached a maximum, and the state of affairs is as shown in Fig. 235; the magnetic field is now displaced by one eighth of a revolution from its position in Fig. 234. After another quarter of a cycle, phase *1* current has reached its maximum in the opposite direction to that assumed in Fig. 234, while phase *2* current is passing through zero, and the state of affairs is as shown in Fig. 236, with the magnetic field displaced another eighth of a revolution. Evidently the magnetic field within the stator rotates, completing one revolution for each two cycles of the current.

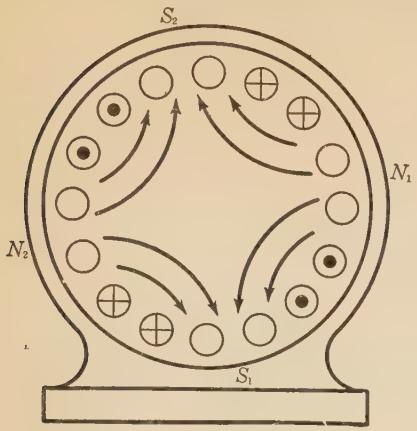


FIG. 235. Diagram to represent the magnetic field within the stator of an induction motor at an instant one quarter of a cycle later than in Fig. 234

In the particular case just considered, the magnetic field has four polar regions. If the windings, instead of being arranged with four bands of conductors for each winding, had been arranged with six for each, there would have been six polar regions. Evidently the stator can be so wound that the field may have any desired number of polar regions. The

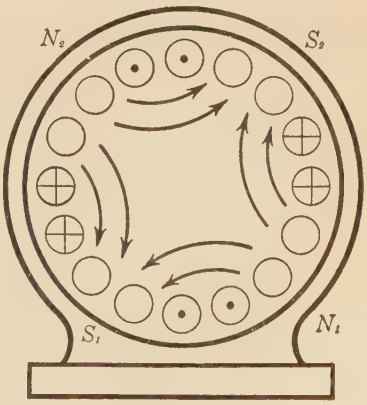


FIG. 236. Diagram to represent the magnetic field within the stator of an induction motor at an instant one half of a cycle later than in Fig. 234

field, in its rotation, advances over the space of two polar regions for each cycle of the current, so that the speed n_s of its rotation, in revolutions per second, is given by

$$n_s = 2\frac{f}{p}, \tag{80}$$

where f is the frequency of the supply, and p the number of polar regions.

It is easy to see how the stator could be wound for three phases, by providing it with three independent windings, instead

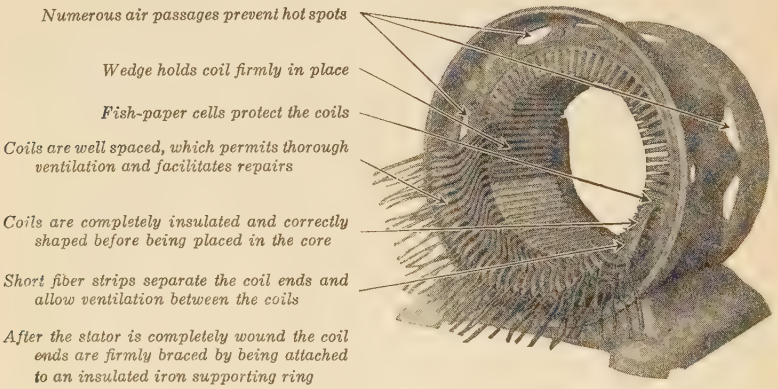

Numerous air passages prevent hot spots
Wedge holds coil firmly in place
Fish-paper cells protect the coils
Coils are well spaced, which permits thorough ventilation and facilitates repairs
Coils are completely insulated and correctly shaped before being placed in the core
Short fiber strips separate the coil ends and allow ventilation between the coils
After the stator is completely wound the coil ends are firmly braced by being attached to an insulated iron supporting ring

FIG. 237. Stator of an induction motor, partly wound

Courtesy of the Westinghouse Electric and Manufacturing Company

of two. The three windings may be connected to three-phase mains according to either the Y system or the delta system.

216. Construction of stator. The stators used in practice are wound in the same manner as shown in Figs. 233 to 236, but with a much larger number of conductors in each winding. The conductors are wound into coils which are inserted in slots in the stator core, the process of construction being similar to that described in section 56 for armature windings. The core of the stator is of iron, laminated so as to diminish eddy currents. The construction is illustrated in Fig. 237, which shows a stator partly wound.

217. Construction of rotor. The rotor, or moving part of the motor, carries the conductors across which the rotating magnetism must cut in order to produce induction-motor action.

These conductors consist of copper wires or bars placed parallel to the axis in slots at the surface of a cylinder or drum which fits closely within the stator, the conductors being connected together at the ends. Theoretically the conductors might have almost any arrangement, since any conductor in general tends to follow a rotating magnetic field; but practically only two arrangements are in ordinary use, that of the "squirrel-cage rotor" and that of the "wound rotor."

In the squirrel-cage rotor all the conductors are short-circuited together at each end by soldering or brazing to heavy copper "end rings," as shown at the ends of the rotor in Fig. 238.

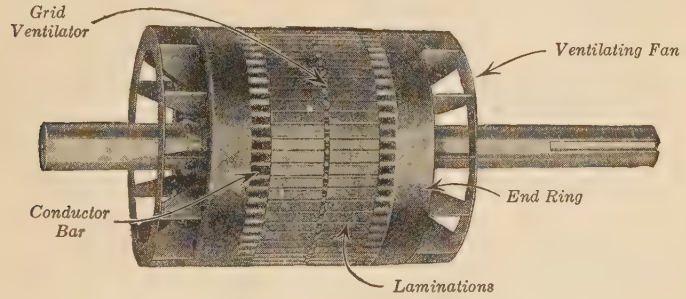


FIG. 238. Squirrel-cage rotor

Courtesy of the Century Electric Company

In the wound rotor the conductors are connected together in windings in the same manner as in the stator windings (though the number of conductors need not be the same as on the stator). Such a rotor accordingly has one winding for each phase. The ends of these windings are brought to slip-rings upon the shaft, and the slip-rings are provided with brushes. By connecting the proper brushes together, the circuit for any winding is completed, either by short-circuit, or by insertion of any desired outside circuit between the brushes.

The squirrel-cage rotor offers the advantages of great simplicity, and the absence of any sliding contacts. The wound rotor is less simple, but on account of the possibility of inserting adjustable resistance (or other circuits) in the rotor windings, it is capable of various adaptations and applications that are not possible with the squirrel-cage rotor.

The cores of rotors are of course made of iron, to complete the magnetic circuit for the flux due to the stator. They are usually laminated, but with comparatively thick laminations, because there is little tendency for eddy currents to flow, owing to the fact that the magnetic flux through the rotor core changes comparatively slowly, since the rotor turns nearly as fast as the rotating field in which it is placed. Such eddy currents as take place are subject to force action from the magnetic field, and thus add to the torque produced — although less effectively than the currents in the rotor conductors.

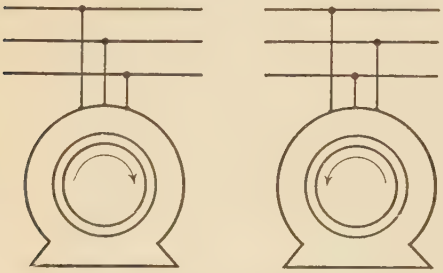


FIG. 239. Diagrams to show reversal in direction of running of a three-phase induction motor by change of connections to two of its terminals

218. Direction of running of induction motors. The direction of running of a two-phase induction motor can evidently be reversed by reversing the connections for one of the phases, since this would cause the magnetic field to rotate in the opposite direction. A three-phase motor can be reversed by interchanging the line connections to any two of its terminals, as shown in Fig. 239.

219. The speed-torque characteristics of the induction motor. Suppose a rotor to be turning at the same speed as the rotating magnetic field in which it is placed; that is, at "synchronous speed." The rotor conductors then keep up with the magnetic field and there is no cutting of magnetic flux, therefore no electromotive force generated, no current in the rotor conductors, and no torque. If the speed n_r of the rotor is less than synchronous speed n_s, then the rate of cutting of magnetic flux is proportional to the difference from synchronous speed $(n_s - n_r)$, and the induced electromotive force, rotor current, and torque are proportional to the same quantity (assuming constancy of magnetic flux and of rotor impedance). The speed-torque curve should accordingly be an inclined straight line, cutting the speed axis at synchronous speed, and sloping to correspond with decreased speed for increased torque. Experiment shows the curve to have very nearly this form, in the neighborhood of synchronous speed.

But if the speed is much below synchronism, then, owing to increasing rotor impedance,* the torque no longer increases in proportion to $n_s - n_r$, but in less proportion; and it is found that a point is reached, beyond which the torque exerted by the motor actually falls off when the speed is further decreased. The form of the speed-torque curve is shown in Fig. 240; from S to A, the torque is practically proportional to $n_s - n_r$; B gives the maximum torque obtainable, and OC is the starting torque. In practice most motors reach rated full load at some such point as F, so that the portion of the curve from F to C does not correspond to conditions that occur in ordinary operation, but is of interest for understanding the behavior of the induction motor under unusual conditions.

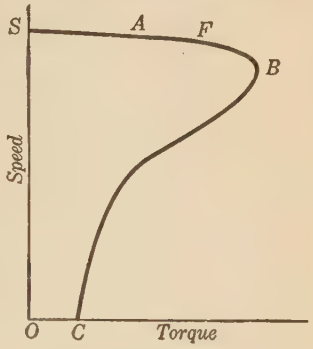


FIG. 240. Speed-torque curve of an induction motor

If the motor is loaded to the point B, it breaks down, since any slight momentary decrease of speed below that corresponding to B would cause the motor to exert less than sufficient torque to carry the load corresponding to B. The motor would accordingly come to rest. If, however, the load is thrown off when the motor is anywhere in the region CB of the curve, it will recover; since the torque it exerts, no longer having to overcome the backward drag from a load, would impart angular acceleration to the rotor. The induction motor is similar to the synchronous motor as regards breaking down when the load reaches a certain maximum; but differs from it in its ability to recover, or to start from rest. The breakdown point of most induction motors does not occur till the load is far in excess of the rated full load; before it is reached, the circuit-breakers would act, and the motor be brought to rest by shutting off the line current.

220. Electrical characteristics of the induction motor. The electrical behavior of an induction motor is very much like that

* The frequency of the rotor currents is proportional to $n_s - n_r$; near synchronism this quantity is very small, and the impedance of the rotor $\left(\sqrt{R^2 + (2\pi fL)^2}\right)$ is determined almost entirely by its resistance. As the frequency increases, the reactance $2\pi fL$ increases in the same proportion, and eventually becomes of more importance than the resistance.

of a transformer, the stator corresponding to the primary of the transformer, and the rotor to the secondary.

In order to explain this analogy, it will be convenient to neglect the effects of eddy currents and hysteresis in the stator and rotor cores, of resistance in the stator windings, and of magnetic leakage between stator and rotor windings. In actual induction motors these effects are quite important; as they are neglected in the discussion below, the conclusions drawn there must be regarded as merely approximate and subject to considerable modification in the case of actual induction motors.

When the rotor is unloaded, it runs at almost synchronous speed, the currents in it are then practically zero, and it does not react appreciably upon the stator in any way. Each stator winding then acts toward the mains across which it is connected simply like a choke coil, permitting the passage of a weak current nearly 90° behind the line voltage in phase. This current corresponds to the magnetizing current in a transformer; its magnitude is merely sufficient to maintain the rotating magnetic field of the induction motor.

When a load comes on the rotor, its speed decreases below synchronism, and currents then flow in the rotor windings. These currents tend to produce a (rotating) magnetic field * superposed upon that due to the stator. But the value of the resultant field must remain unchanged, since this field, in cutting across the stator windings, must induce in each of them a voltage balancing that from the line. To realize this condition, currents must flow in the stator tending to produce a magnetic field equal and opposite to that which the rotor currents tend to produce. These currents are superposed upon the mag-

* The field due to rotor currents rotates in synchronism with the rotating field due to the stator, *regardless of the speed of the rotor*. This may be understood by considering a wound rotor having p polar regions. Let it be loaded so as to run at speed n_r. Let n_f stand for the speed with which the rotor magnetism rotates, and let n_{rel} stand for its relative speed with respect to the rotor. Then $n_f = n_r + n_{rel}$ and we wish to show that $n_f = n_s$, where n_s stands for the speed of the stator magnetism.

By the principle of equation (80), $n_{rel} = 2\frac{f_r}{p}$, where f_r stands for the frequency of the rotor currents. But f_r is such that one cycle is completed in the time that the rotor falls behind the stator's rotating field by two poles. f_r then equals the number of revolutions lost per second relative to the stator's rotating field, times the number of pairs of poles; or $f_r = (n_s - n_r) \cdot \frac{p}{2}$. Accordingly $n_{rel} = 2\frac{f_r}{p} = 2 \cdot \frac{(n_s - n_r)\frac{p}{2}}{p}$ $= n_s - n_r$; and $n_f = n_r + n_{rel} = n_s$, which was to be proved.

netizing current in each winding, and correspond to the load current in a transformer. These currents supply power to the stator; and the load current is accordingly nearly in phase with the voltage, in each stator winding.

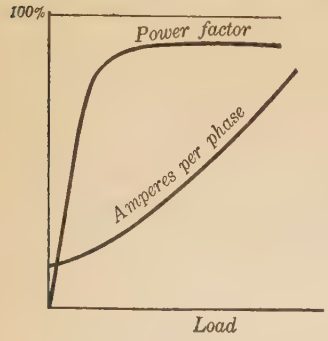


FIG. 241. Curves showing current intake and power factor of a three-phase induction motor at different loads

If the rotor should be clamped at rest, the arrangement would evidently become practically identical with that of a transformer having large magnetic leakage on account of the separation of primary and secondary windings with an air gap between them. With a wound rotor, the secondary might have any kind of load connected across the slip-rings, or might be short-circuited; with a squirrel-cage rotor, the secondary would necessarily be short-circuited.

Fig. 241 shows the stator current per phase, and the power factor, at different loads on an induction motor.

221. Influence of rotor resistance on the action of an induction motor. In a wound rotor it is easy to insert any desired resistances in series with its windings by connecting the resistances across its slip-rings; and the amount of resistance in the rotor windings has an important influence on the behavior of the motor. In a squirrel-cage rotor it is of course not practicable to alter the resistance during operation.

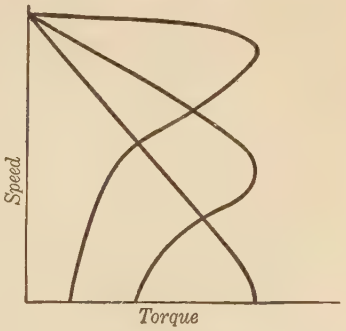


FIG. 242. Speed-torque curves for an induction motor for different values of the rotor resistance

If the rotor resistance is increased, larger electromotive force must be induced in the rotor windings in order to produce the same current; this means that larger difference from synchronous speed (larger value of $n_s - n_r$) is required in order to produce the same torque. In the speed-torque curve the value of $(n_s - n_r)$ for each torque is increased in the same proportion, as shown in Fig. 242.

222. Starting of induction motors. The (polyphase) induction motor is self-starting, as is evident from the speed-torque curve, which shows appreciable torque when the speed is zero. But special arrangements for starting are necessary except in the smaller sizes, in order to prevent injury from the heavy currents that would flow in both stator and rotor, if the motor should be connected directly with the line when at rest. Protection against heavy starting currents can be secured by placing a starting rheostat in series with each stator winding, and cutting out the resistance as the motor gains speed; or by starting the motor on reduced voltage, obtained by the use of proper transformers or autotransformers which are cut out of circuit when the motor has gained sufficient speed. With both of these methods the starting torque is small.

Another method is available for machines with wound rotors, which has the advantage of larger starting torque, and therefore more rapid acceleration of the motor. This consists in connecting rheostats across the rotor slip-rings, resistance being cut out as speed is gained, until the rotor windings are short-circuited for the running condition of the motor. The curves of Fig. 242 show how the starting torque varies when different resistances are inserted in the rotor windings; by proper choice of resistance, the starting torque can be made to have any desired value up to the maximum obtainable from the motor.

223. The induction generator. In accordance with the general principle of reversibility of dynamo-electric machinery, the induction motor can be made to act as a generator by applying mechanical power to the rotor, provided that the rotating magnetic field is maintained by connection of the stator with some independent source of alternating electromotive force. To change from motor action to generator action it is merely necessary to drive the rotor (by outside mechanical power) faster than synchronism. Its action is then quite similar to its action when running as a motor, except that the rotor electromotive forces, currents, and torque are reversed. The torque now opposes the motion, so that the rotor absorbs mechanical energy. With reversal of the rotor currents, the load currents of the stator also reverse, so that the stator now delivers electrical energy. This energy goes to relieve (or to help to drive)

the generator which supplies the electromotive forces for exciting the rotating magnetic field of the stator.

224. Uses of induction motors. Within the range of ordinary operation the speed-torque curve of the induction motor is very much like that of the direct-current shunt motor. Accordingly the induction motor is usually suited for the same applications for which shunt motors are used in direct-current systems, such as for driving main shafts in shops or as individual motors for machine tools, fans, and other applications calling for approximately constant speed. For these applications the squirrel-cage rotor is used.

Induction motors are also sometimes used for applications for which series motors or compound motors are used in direct-current systems, such as for cranes, hoists, electric locomotives, or other applications requiring heavy torque at starting and at low speeds; but single-phase series motors are also used for such applications. When induction motors are used, wound rotors are employed, so that the torque at low speeds may be increased by insertion of resistance in the rotor, in the manner discussed in section 221. For use in electric locomotives, the induction motor offers the advantage of giving reversed torque when forced to run faster than synchronous speed, as discussed in section 223. This means that it serves as a brake in case the train tends to run away in going down grade; but it has the advantage over a friction brake that it acts as a generator under these conditions and returns energy to the line. This kind of braking is called *regenerative braking*; in mountainous regions the amount of energy that may be saved by its use is considerable. No change has to be made in the connections to the motor: it automatically begins to act as a generator when forced to run faster than synchronous speed.*

The induction-motor principle is used for starting polyphase synchronous motors. The armature windings of a polyphase synchronous motor are arranged similarly to the stator windings of an induction motor, and accordingly produce a rotating magnetic field † when connected to their supply mains. The

* Regenerative braking is possible with other types of motors also, but may not take place automatically; special connections and means of control have to be provided.

† That is, rotating relative to the armature; in a stationary armature the field due to its currents rotates at synchronous speed; in a revolving armature the field rotates *relatively to the armature*, with synchronous speed in reversed direction to that of the armature, so that when the armature is running at synchronous speed the field due to its currents is at rest in space.

damper winding placed on the pole faces is similar to the squirrel-cage winding of the rotor of an induction motor. Accordingly induction-motor action takes place, and torque is exerted between the armature and the damper winding (which is attached to the poles); and this torque ceases only when synchronous speed is attained. To start the synchronous motor, then, its field is left unexcited, and its armature is connected to the supply mains (through resistance or at reduced voltage, in order to avoid too heavy armature currents); the motor then picks up speed by induction-motor action; when its speed is near synchronism, the field is excited; synchronous-motor action then takes place and pulls the speed into exact synchronism, whereupon the induction-motor action automatically stops.

The induction motor is the most widely used type of alternating-current motor. It has the great advantage of simplicity of construction. Its principal disadvantages are poor power factor at light loads and difficulty of speed control. There are a number of methods for controlling the speed of an induction motor at will, but all are more or less complicated or inefficient or otherwise unsatisfactory, and will not be discussed here. Furthermore, it is not possible to design induction motors (or synchronous motors, either) for more than a few different and widely separated synchronous speeds (assuming fixed frequency of supply), since the synchronous speed is given by $2\frac{f}{p}$, and it is not practicable to make p very large. For instance, with 60-cycle supply, the possible synchronous speeds (in revolutions per minute) are 3600, 1800, 1200, 900, 720, 600, and so on; 600 revolutions per minute corresponds to 12 poles, the largest number in common use in induction motors of ordinary sizes.

225. Single-phase induction motors. Induction motors are capable of running on single-phase, but are not self-starting on single-phase.

Fig. 243 represents a squirrel-cage rotor placed between the poles of a two-pole stator connected with single-phase alternating-current mains. If the rotor is at rest, the arrangement of currents in stator and rotor will, for some instant during the cycle, be as shown; the direction of the forces from magnetic field on rotor currents is shown by the short arrows. By

symmetry, it is plain that the resultant torque will be zero and will remain zero at all other instants during the cycle.

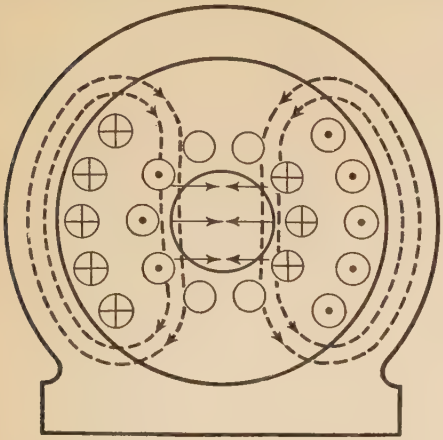


FIG. 243. Diagram to represent arrangement of currents and force action in a single-phase induction motor at rest

But if the rotor is turning, then the conductors next to the poles cut across the alternating magnetism; consequently alternating electromotive forces are induced in them and cause currents to flow in the rotor in addition to those shown in Fig. 243. By reasoning which will not be given here, it can be shown that the alternating magnetic field due to these additional currents is displaced 90 (electrical) degrees from that due to the stator and is 90° out of phase with it. These two alternating magnetic fields accordingly act like those due to the two windings of a two-phase stator. Thus, when the rotor is in motion, a rotating magnetic field exists and acts upon the rotor in much the same manner as in polyphase machines.

226. Characteristics of single-phase induction motors. The speed-torque curve of a single-phase induction motor is shown in Fig. 244. This is similar to the corresponding curve for a polyphase motor in its general character and in showing a breakdown point, but it differs in that the torque drops to zero at zero speed. Except when running far below synchronism, the behavior of single-phase induction motors, both mechanically and electrically, is so similar to that of polyphase induction motors that it does not call for separate discussion in this text.

FIG. 244. Speed-torque curve of a single-phase induction motor

From symmetry it is plain that the motor will run in whichever direction it is started.

227. Starting of single-phase induction motors. To start a single-phase induction motor it is not necessary to bring its speed all the way up to synchronism, as must be done with a synchronous motor. All that is necessary is to impart to the rotor sufficient speed for it to develop a torque greater than that of the (frictional or other) resistances to its motion; it will then take hold and speed up of itself. It is therefore possible to start a single-phase induction motor, like a gasoline engine, by "cranking up." But this method is not used, inasmuch as more convenient ones are available.

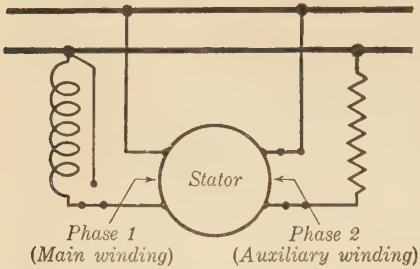


FIG. 245. Diagram to show starting connections for a single-phase induction motor using split-phase starting

One common method is known as *split-phase* starting, the principle of which is shown in Figs. 245 and 246. The stator is provided with an auxiliary winding (starting winding) in addition to its main winding; the main winding and the starting winding are arranged like the two windings of a two-phase stator. For starting, the main winding is connected with the single-phase mains through a reactance coil X, of small resistance; the starting winding is connected with the same mains through a noninductive resistance R. If R_m and R_s stand for the resistances of main winding and starting winding, respectively, and X_m and X_s stand for their reactances, then the current in the main winding will lag behind the supply voltage by a phase difference whose tangent is $\frac{X + X_m}{R_m}$, and the current

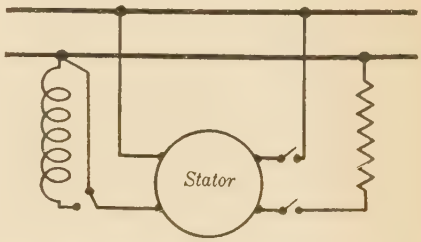


FIG. 246. Diagram to show running connections for a single-phase induction motor using split-phase starting

in the starting winding will lag behind the supply voltage by another phase difference whose tangent is $\frac{X_s}{R + R_s}$. By proper choice of X and R, the currents in the two stator windings can be made to differ in phase by nearly a quarter of a cycle, so that a rotating magnetic field is produced, causing a torque on the rotor, just as in a two-phase motor. The coils X and R at the same time keep the strength of the starting currents within safe limits. After the rotor has gained sufficient speed, the starting winding is disconnected from the mains, and the coil X is removed from the circuit of the main winding, leaving the latter directly across the supply mains. For this scheme of starting, the coils X and R are placed in a "starting box" having an insulating cover on which a double-throw switch is mounted; the connections being properly made, one position of this switch belongs to "starting," and the other belongs to "running."

Another widely used method for starting single-phase induction motors is that of repulsion-motor starting, which will be described in section 236.

228. Uses of single-phase induction motors. Since the running characteristics of single-phase induction motors are so much like those of polyphase induction motors, it follows that in general the uses of the single-phase motor are the same as those of the polyphase motor. But wherever more than one phase is available, the polyphase induction motor should always be preferred to the single-phase induction motor. Since alternating-current distribution systems are usually polyphase, this means that in practice the use of the single-phase induction motor is pretty much limited to cases in which it is not worth while to carry mains to the motor for more than one phase; that is, to the smaller motors used in households, small shops, or temporary locations.

229. The induction watt-hour meter. The watt-hour meters used in alternating-current systems act on much the same principle as induction motors and are therefore called *induction watt-hour meters.*

Like the Thomson watt-hour meter described in section 95, the induction watt-hour meter has a thin disk of aluminium passing between the poles of strong permanent magnets, and thus acted upon by a backward torque proportional to the speed with which it turns. The same disk serves as a rotor when cut across by a rotating (or shifting) magnetic field; such a field is produced by a magnetic structure corresponding to the stator of an induction motor,

and placed at the opposite side of the disk to that occupied by the permanent magnets. The driving torque exerted upon the disk by this field is proportional to the power being delivered to the circuit controlled by the meter.

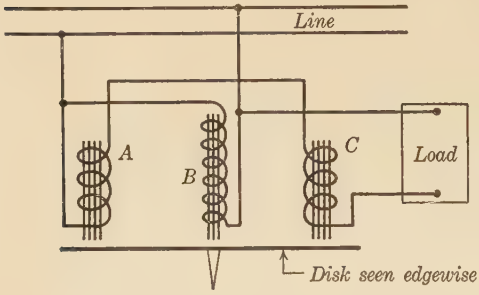


FIG. 247. Diagram for illustrating the principle of action of the induction watt-hour meter

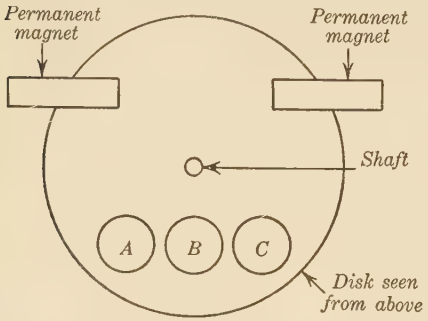


FIG. 248. Diagram for illustrating the principle of action of the induction watt-hour meter

The arrangements for producing the shifting magnetic field vary in details in different meters, but in all cases the principle is the same as that illustrated in Figs. 247 and 248. *A*, *B*, and *C* represent magnet cores. *A* and *C* are wound with a few turns of wire, through which the load current flows; *B* is wound with many turns of wire and is connected across the mains which supply the load. The (alternating) magnetic flux in *A* and *C* is proportional to the load current; that in *B* is proportional to the load voltage. The windings of *A* and *C*, having few turns, are nearly non-inductive, whereas the winding of *B* is highly inductive; thus the flux in *A* and *C* is practically in phase with the load current, whereas that in *B* is practically 90° out of phase with the load voltage. The polarity of the ends of *A*, *B*, and *C* next to the disk then shifts according to the following table (if we assume the load current to be in phase with the voltage):

Time after Starting	Polarity of		
	A	*B*	*C*
0	North	—	South
$\frac{1}{4}$ cycle	—	North	—
$\frac{1}{2}$ cycle	South	—	North
$\frac{3}{4}$ cycle	—	South	—
1 cycle	North	—	South

The disk then tends to follow the shifting magnetic field. The torque acting on the disk from the shifting field can be shown to be proportional to the power delivered to the load ($EI \cos \phi$).

ALTERNATING-CURRENT COMMUTATOR MOTORS

A great many varieties of alternating-current commutator motors have been developed. The present discussion will be limited to two of the most important types: the single-phase series motor and the repulsion motor.

230. Principle of action of the single-phase series motor. In principle the single-phase series motor is the same as the direct-current series motor, but some details of the direct-current motor have to be modified.

As explained in section 97, reversal of the direction of the voltage applied to the terminals of a series motor does not reverse its torque. One would accordingly expect that any series motor would run on alternating current; and so it will, in general; but the motor will spark badly at the commutator and will have very low power factor unless special provision is made for overcoming these defects.

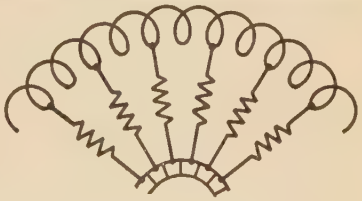


FIG. 249. Diagram to show the insertion of resistance into armature leads in a single-phase series motor

231. Adaptation of series motors for alternating-current operation. The reason for the increased tendency to spark when a direct-current motor is operated on alternating current is because in the latter case the magnetic field is not constant but alternates. An armature coil, while undergoing commutation, links with the field; the variation of the field induces an electromotive force in the coil, and thus tends to produce sparking in the same manner as described in section 66, even though the brushes are at the neutral points and the motor is unloaded. To prevent sparking from this cause, special devices have to be used; the usual method in single-phase series motors is to place resistance in the armature leads, as shown in Fig. 249.

To make the series motor practically useful for alternating-current operation, it is essential to improve its low power factor. This requires that the inductance of both field winding and armature be made as small as possible. Low inductance of the

field winding requires a design of the magnetic circuit with the lowest possible reluctance, so that the field flux may be produced by a small number of turns of field winding, rather than by the larger number that would be required to produce the same flux through a greater reluctance.

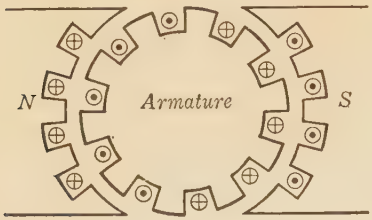


FIG. 250. Diagram to show the direction of currents in armature conductors and in conductors of a compensating winding placed on the pole faces

The inductance of the armature can be neutralized by the use of a "compensating winding," which is a winding placed on the pole faces, as shown in Fig. 250. If this winding is connected in series with the armature so that at each pole face the currents in the compensating winding flow in the opposite direction to those in the armature conductors next to that pole face, then the magnetic field due to armature currents will be neutralized, practically completely, at

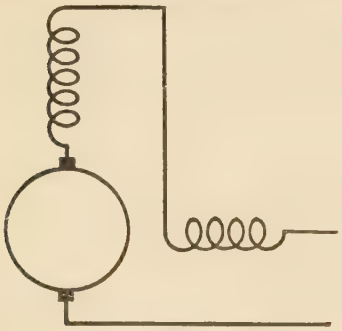


FIG. 251. Diagram of connections of armature, compensating winding, and field winding in a single-phase series motor

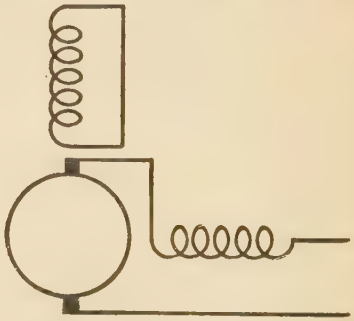


FIG. 252. Diagram of connections for a single-phase series motor showing the compensating winding short-circuited

all times. The inductance of the circuit consisting of armature and compensating winding will then be reduced practically to zero. The connections are shown diagrammatically in Fig. 251;*

* Sometimes the compensating winding is short-circuited, as shown in Fig. 252, instead of connected in series with the armature. In this case it acts toward the armature like the secondary of a transformer, and currents are induced in it which neutralize the magnetizing effect of those in the armature.

the positions in which the field and compensating windings are drawn are intended to show that the latter produces magnetic flux *in* the axis of the flux due to the armature, and that the former produces flux *at right angles* thereto.

The magnetic circuit of the motor has to be laminated throughout to avoid excessive eddy-current losses.

The modifications just described make it practicable to operate the series motor on alternating current without in any way impairing its ability to run on direct current. On alternating current its operation is more satisfactory and efficient as the frequency is lower, since its power factor

$$\left(= \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}\right)$$

is greater as the frequency is lower. For this reason systems on which motors of this type are largely used generally employ a frequency of 25 cycles per second or less, rather than that of 60 cycles per second commonly used for general distribution in this country.

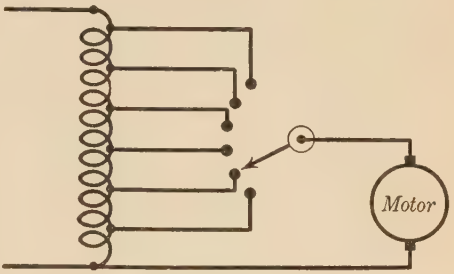


FIG. 253. Diagram showing use of an autotransformer for starting and speed control of a single-phase series motor

232. Characteristics of single-phase series motors. As would be expected, the characteristics of the single-phase series motor are substantially the same as those of the direct-current series motor; that is, the motor runs slowly under heavy loads and speeds up greatly as the load is decreased. The speed has no relation to the frequency of the supply, but is greatly influenced by its voltage.*

233. Starting and speed control of single-phase series motors. The speed of a single-phase series motor, under any assigned load, can be controlled very simply by the use of an adjustable transformer (or autotransformer) for varying the voltage applied to the motor terminals, as shown in Fig. 253. The same arrange-

* Equation (46), section 98, for the speed of a direct-current motor, would give a rough approximation to the speed of a single-phase series motor. The equation so applied would be inaccurate, because the (alternating) counter-electromotive force is not in exactly opposite phase to the supply voltage E_t.

ment can be used in starting the motor, the starting current being kept within safe limits by applying a low voltage at starting, rather than by introducing resistance with consequent loss of energy, as has to be done in starting direct-current motors.

234. Uses of single-phase series motors. Since the characteristics of single-phase series motors are the same as those of direct-current series motors, they are adapted to the same kinds of applications, of which traction is the most important. They are used for driving electric cars and locomotives, the motor being supplied at some moderate voltage (say 550) from the secondary of a transformer carried with the motor, whose primary is connected with a high-voltage line (perhaps at 11,000 volts) by means of a trolley or other form of sliding contact.

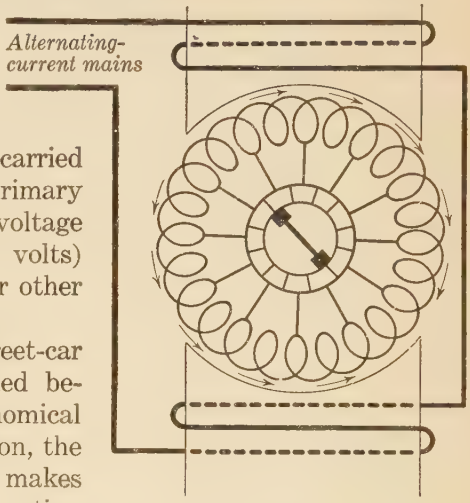


FIG. 254. Diagrammatic representation of repulsion motor

If a direct-current street-car system is to be extended beyond the limits of economical direct-current transmission, the single-phase series motor makes it possible to use alternating currents on the extensions without any changes in the old parts of the system, since cars that are equipped with this motor will run on direct current as well as on alternating current.

The small motors used for driving fans or household appliances are sometimes single-phase series motors. For applications of this kind it is somewhat desirable to use motors which will run on either direct current or alternating current of any frequency (the so-called "universal motors"), since the users are often ignorant of the nature of their supply of electricity.

235. The repulsion motor. In principle the repulsion motor consists of a direct-current motor whose field is excited by alternating current while its armature is short-circuited between brushes. The brushes are displaced from the neutral axis, as shown diagrammatically in Fig. 254.

The action of the motor may be understood by considering that the field is equivalent to the resultant of two fields A and B at right angles to each other, as shown in Fig. 255. The alternating field A induces currents in the armature, on which the field B exerts forces that produce a torque on the armature. Since the armature winding and the field winding A stand in the same relation as secondary and primary of a transformer, the armature current must be proportional to and in phase* with the current in field A, which is identical with that in field B. The motor must accordingly have the characteristics of a series motor. In principle its speed is independent of the frequency of the supply, except as this may affect the transformer action between field A and the armature. Evidently it will not run on direct current (frequency zero).

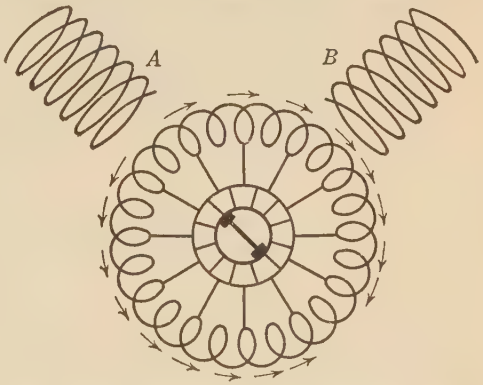


FIG. 255. Diagram for use in explaining the action of the repulsion motor

236. Uses of the repulsion motor. The principal use of the repulsion motor is for starting a type of motor which, after being brought up to speed, runs as a single-phase induction motor. Such a motor is designated as an induction motor, since under normal running conditions it operates as such; but its construction is that of a repulsion motor, as just described. It starts as a repulsion motor; after reaching a certain speed the commutator is short-circuited all round; this transforms the armature into a squirrel-cage rotor, and the motor then runs as an induction motor. The advantages gained by transforming the motor into a single-phase induction motor are that the speed becomes approximately independent of the load and that the efficiency is increased.

* Or 180° out of phase, according to which directions are counted positive for armature winding and field winding A — a matter which does not affect the present argument.

Fig. 256 shows a single-phase induction motor with repulsion-motor starting. In this motor the short-circuiting of the commutator is effected by a ring inside the core of the rotor, which

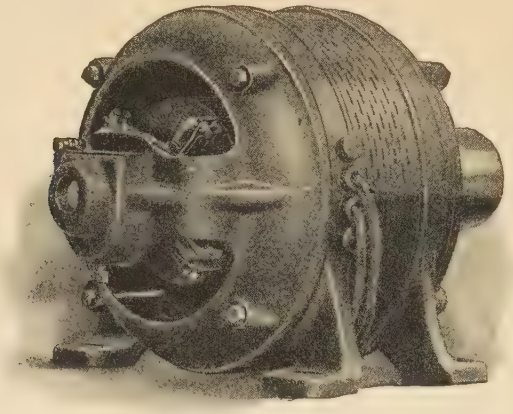


FIG. 256. Single-phase induction motor provided with commutator and short-circuited brushes for starting as a repulsion motor

Courtesy of the Wagner Electric Manufacturing Company

is pressed against the commutator from inside by the centrifugal force of two wedge-shaped weights. At the same time, the brushes are lifted from the commutator, thus saving losses in brush friction.

PROBLEMS

1. A certain 12-pole alternator is to be run as a synchronous motor from 60-cycle mains. In bringing it up to speed, the synchronizing lamps fluctuate once per second just before the switch is closed to connect the motor to the mains. What was its speed just before the switch was closed?

2. A synchronous motor is connected with 1100-volt mains; its armature generates 1000 volts, lagging behind the impressed voltage by 190° when carrying a certain load. Find by vector diagram the resultant voltage available for driving current through the armature; also the current driven through, the armature having a resistance of 0.75 ohm and a reactance of 1 ohm.

3. Draw a sketch showing how to connect a stator winding having sixteen conductors so as to get four polar regions from two phases.

4. What would be the synchronous speed for a rotor used with the stator in Problem 3, if the frequency of the supply is 60 cycles per

second? When the actual rotor speed is 1770 revolutions per minute, find the frequency of the rotor currents.

5. A certain 5-horse-power 220-volt induction motor has a power factor of 85 per cent when running at full load. When delivering 1 horse power, the power factor of this motor is 40 per cent. Compare the current at 1 horse power with that at 5 horse power; also the RI^2 losses. (The efficiency at 5 horse power is 85 per cent; that at 1 horse power is 65 per cent.)

6. Each armature coil of a certain 25-cycle single-phase series motor has four turns. The maximum value of the magnetic flux per pole is 2,000,000 maxwells, and the flux at any instant t is therefore 2,000,000 sin (157 t). The motor has four poles and it is running at 1200 revolutions per minute. Find the greatest voltage induced in an armature coil by reason of cutting through the magnetic field, and the greatest voltage in an armature coil by reason of variation of the field flux. (The maximum flux that can link with an armature coil is the full field flux per pole; the maximum rate of change of this flux is 157 × 2,000,000 maxwells per second. Assume the flux density to be uniform over each pole face. The pole faces occupy 0.7 of the periphery of the armature.)

7. With the conditions stated in Problem 2, find the difference of phase between the current and the voltage generated in the armature of the motor, and find the power developed by the motor.

8. If the induction motor of Problem 4 exerts a torque of 500 pound-feet when running at 1770 revolutions per minute, how many horse power does it develop? At what speed would the rotor have to be driven by outside power to develop the same torque in reversed direction, and how many kilowatts would it then return to the line? (In working this problem all losses may be neglected, although in reality they would affect the results appreciably.)

CHAPTER XX

CONVERSION OF ALTERNATING CURRENT TO DIRECT CURRENT

237. Importance of conversion. As discussed in section 131, there are some very important applications of electricity which require direct current and for which alternating current cannot be used at all. Means for converting alternating-current power to direct-current power are therefore necessary in alternating-current systems, not only as a convenience but rather as an essential, without which the usefulness of the alternating-current system would be very seriously limited.

For conversion on a small scale, a great many devices have been developed; but for large-scale conversion, there are only two kinds of appliances in general use at this time. These are the motor-generator set and the synchronous converter.

238. Motor-generator sets. The motor-generator set consists of an alternating-current motor arranged to drive a direct-current generator. Both machines are usually built on the same bedplate, with the revolving members of both machines on the same shaft, as shown in Fig. 257. Detailed discussion of the behavior of motor-generator sets will be unnecessary, since each of the machines in a set has the same characteristics that it would have if used for any other purpose.

239. Synchronous converters. The synchronous converter is essentially a direct-current generator whose armature is provided with collector-rings in addition to the commutator, as shown in Fig. 258; the connections to the rings are shown in Fig. 259 for the particular case of a two-pole two-ring converter. For simplicity of drawing in this figure and in Figs. 260, 261, 263, and 266, the commutator (which is connected in the same way as in any direct-current dynamo, at the other end of the armature) is omitted, and the armature winding is shown as a ring winding, though actual converters nearly always have drum windings.

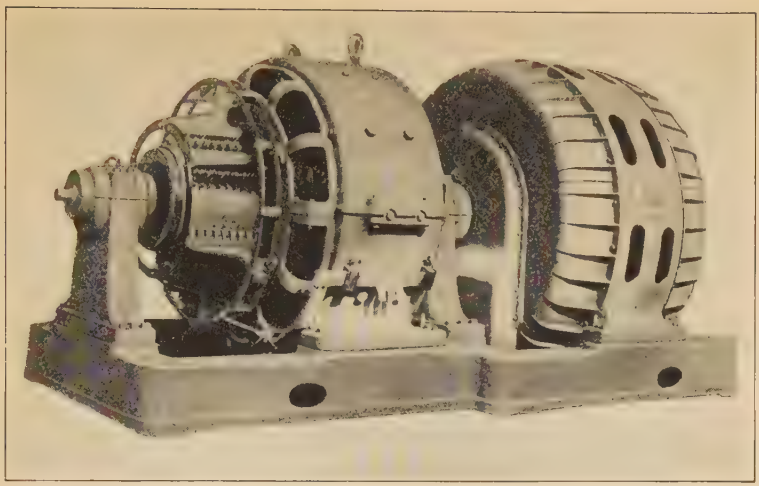


FIG. 257. Motor-generator set (1000 kilowatts)
Courtesy of the Westinghouse Electric and Manufacturing Company

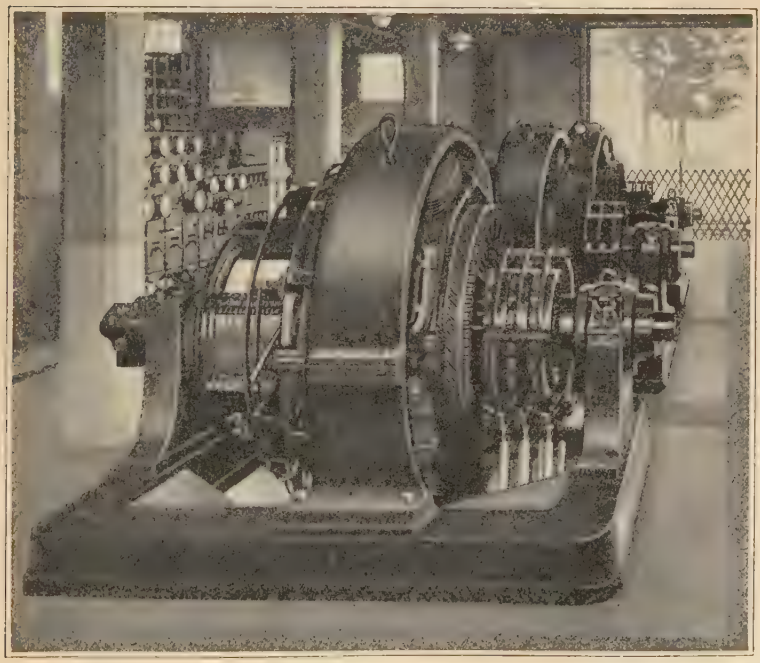


FIG. 258. Synchronous converter (three-phase); commutator at left, collector-rings at right
Courtesy of the Westinghouse Electric and Manufacturing Company

It is evident that when the armature is in rotation an alternating electromotive force will appear between the rings. Thus the machine is able to act as an alternating-current dynamo, either generator or (synchronous) motor; and at the same time direct electromotive force is maintained between the brushes that touch its commutator. Accordingly direct current can be taken from the machine while it is running as a synchronous motor and drawing its power from alternating-current mains. It then acts as a converter. It may also deliver mechanical power at the same time.

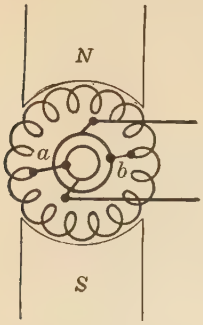


FIG. 259. Diagram to show direct-current armature winding provided with connections to two slip-rings

Two-ring converter

The same machine, if driven by mechanical power, may deliver either direct-current power or alternating-current power, or both simultaneously. If used in this way it is called a *double-current generator*. Or it may be operated as a direct-current motor, and may then deliver alternating-current power (with or without mechanical power at the same time). In this case it is called an *inverted converter*.

240. Converters with more than two rings. Fig. 260 shows a converter with three rings, connected with the armature winding at points separated from one another by one third of the whole winding. When the armature is in rotation, an alternating electromotive force is generated in each of the sections *cd*, *de*, and *ec* of the winding; these three electromotive forces will evidently be identical in all particulars except that each follows the preceding one by a phase difference of one third of a cycle. The voltages between rings are accordingly in the same relation as in the three-phase system, and the machine can run as a three-phase synchronous motor (or as a three-phase generator) while maintaining direct electromotive force between the brushes on its commutator.

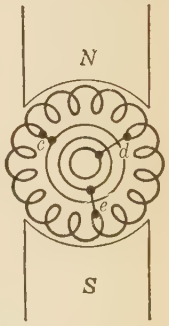


FIG. 260. Direct-current armature winding provided with three slip-rings

Three-ring converter

Fig. 261 shows a converter with four rings, connected with the armature winding at points separated from one another by one fourth of the whole winding. When the armature is in rotation, the alternating electromotive forces in sections *fg*, *gh*, *hi*, and *if* evidently follow one another in phase by one fourth of a cycle, as shown in the vector diagram, Fig. 262. The voltages *fh* and *gi* are equal to each other, and differ in phase by 90°. The machine can accordingly operate as a two-phase motor (or generator) on the two-phase four-wire system, by connecting the mains of one phase to rings *f* and *h* and connecting those of the other phase to rings *g* and *i*.

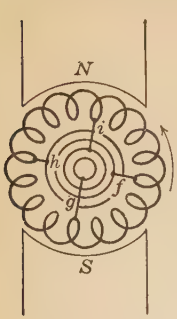


FIG. 261. Direct-current armature winding provided with four slip-rings

Four-ring converter

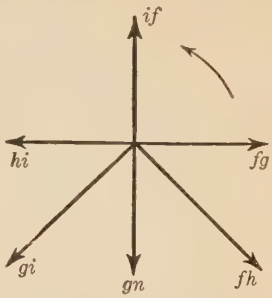


FIG. 262. Vector diagram for voltages in a four-ring converter

Fig. 263 shows the connections for a six-ring converter, and Fig. 264 the vector diagram for its voltages. Six-ring converters are usually run from three-phase systems, one scheme of connections being shown in Fig. 265.

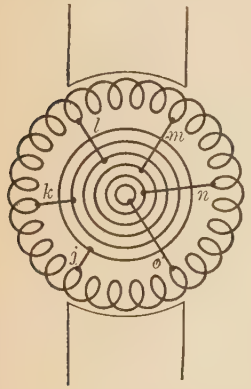


FIG. 263. Direct-current armature provided with six slip-rings

Six-ring converter

In multipolar converters the connections with the armature winding for each ring evidently should be repeated for each pair of poles. For example, the connections for a three-ring four-pole converter are as shown in Fig. 266.

241. Voltage relations in synchronous converters. In a two-ring converter the electromotive force induced between rings reaches its maximum instantaneous value at the instant the points *a*, *b* (Fig. 259) are midway between the poles. This value is evidently equal to the direct electromotive force induced between the brushes at the commutator end, if we assume these brushes to be in the neutral

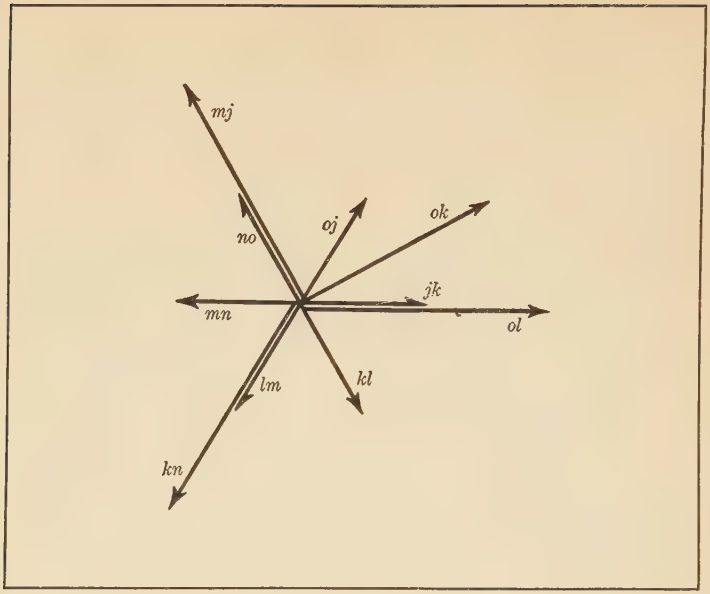


FIG. 264. Vector diagram for voltages in a six-ring converter

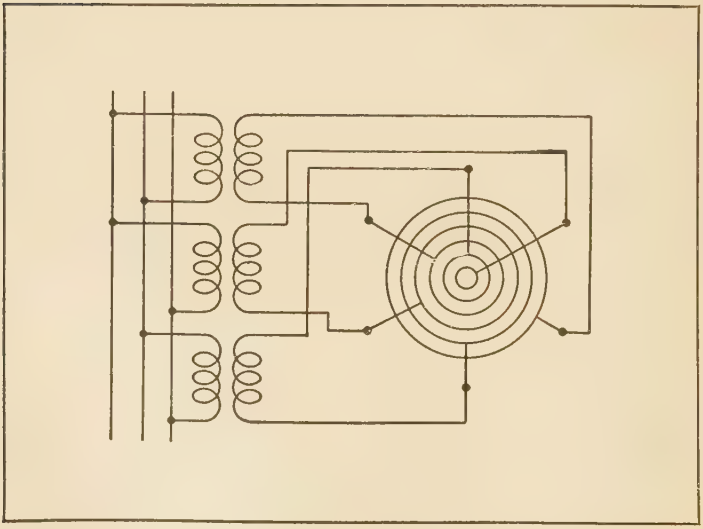


FIG. 265. Diagram of connections for six-ring converter run from three-phase mains

axis. The effective value of the alternating electromotive force equals 0.707 times the maximum value, provided the wave-form is sinusoidal, as is approximately the case in practice. Accordingly, in a two-ring converter, the effective alternating electromotive force between rings is 0.707 times the direct electromotive force between brushes.

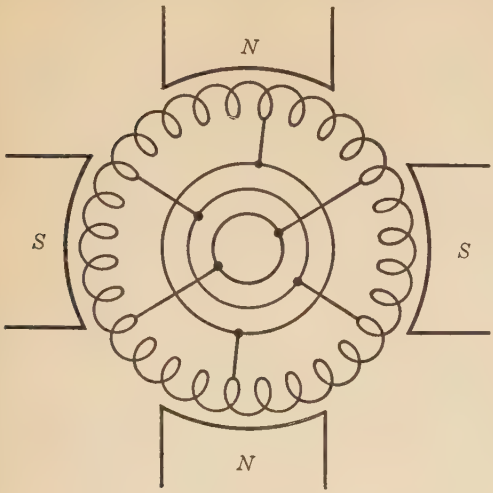


FIG. 266. Four-pole three-ring converter

In general, the alternating electromotive force between any two points of the winding equals the resultant of the electromotive forces induced in the separate armature coils between these points. These electromotive forces are equal in magnitude, but differ progressively in phase, so that the vector diagram for finding their resultant takes the form of part of a many-sided regular polygon, as already discussed in section 181. When the coils are numerous, the polygon may, without appreciable error, be replaced by a circle, of which the complete circumference corresponds to two poles on the machine. On such a diagram (Fig. 267) the electromotive force E_2 between the rings of a two-ring converter will be represented by the diameter of the circle (resultant of coil electromotive forces from one pole to the next). The electromotive forces induced between adjacent rings in the three-ring, four-ring, and six-ring converters will be represented by the lines E_3, E_4, and E_6, respectively, being chords across arcs of 120°, 90°, and 60°, respectively. Applying trigonometry to the figure

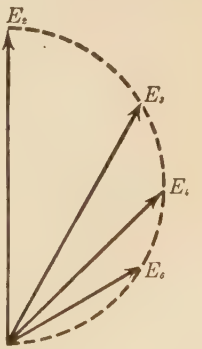


FIG. 267. Diagram for voltage relations in synchronous converters

we have, for the effective values,

$$E_2 = 0.707\ E_d,$$
$$E_3 = 0.866\ E_2 = 0.612\ E_d,$$
$$E_4 = 0.707\ E_2 = 0.500\ E_d,$$
$$E_6 = 0.500\ E_2 = 0.354\ E_d,$$

where E_d stands for the direct electromotive force at the commutator end.

All the preceding discussion refers to voltages induced in the armature; voltages appearing at the terminals will differ somewhat from them* on account of RI drop in the armature and other causes.

242. Current relations in synchronous converters. Since the direct-current power output must approximately equal the alternating-current power input, the current ratios may be found approximately from the corresponding voltage ratios (if we assume the converter to run at unity power factor, as it usually does in practice). Thus $nE_nI_n = E_dI_d$, where n stands for the number of rings (which equals the number of sections into which the armature winding is divided), I_n for the alternating current (effective) in any section of the armature of an n-ring converter, and I_d for the direct-current output. This gives

$$I_2 = \frac{1}{2} \cdot \frac{1}{0.707} I_d = 0.707\ I_d,$$

$$I_3 = \frac{1}{3} \cdot \frac{1}{0.612} I_d = 0.545\ I_d,$$

$$I_4 = \frac{1}{4} \cdot \frac{1}{0.500} I_d = 0.500\ I_d,$$

$$I_6 = \frac{1}{6} \cdot \frac{1}{0.354} I_d = 0.471\ I_d.$$

243. Instantaneous values of currents in armature coils. If a synchronous converter should be operated as a direct-current generator (its rings being on open circuit), the current in an armature coil would follow the curve a, Fig. 268. If the machine should be operated as a synchronous motor (delivering mechanical power but no direct current), the current in the

* In practice the ratios between terminal voltages may differ by 2 to 4 per cent from those given above.

armature coil would follow the curve *b*. If it is operated as a converter, the current in the armature coil will be the resultant of the currents shown in *a* and *b*, and will follow the curve *c*.

The phase relation between the component curves *a* and *b* will be different for different armature coils, however, depending on the location of the coil with reference to the connections to the rings. Thus in a coil near to a ring connection, the current curve will be like that shown at *d*, Fig. 268; in a coil midway between ring connections it will be like that shown at *e*.

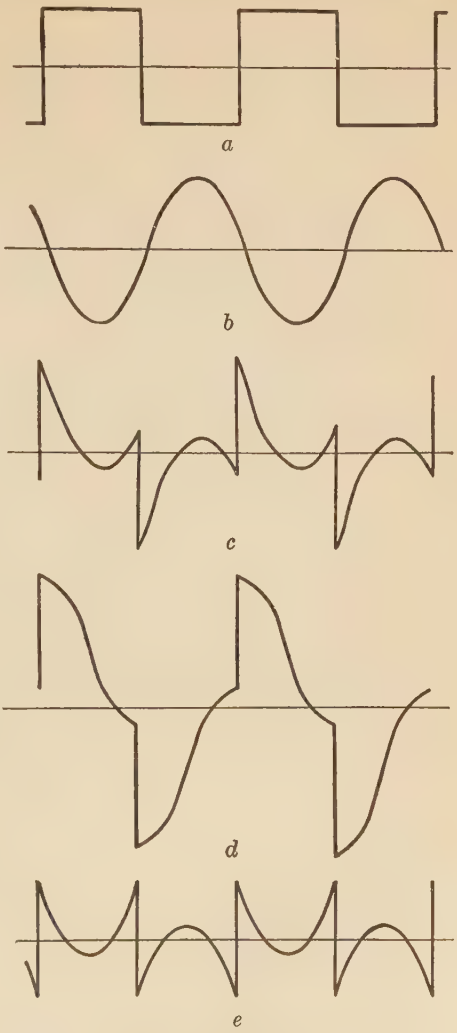


FIG. 268. Curves for instantaneous values of current in armature coils of a synchronous converter

244. Armature heating and capacity of synchronous converters. As is apparent from curves *d* and *e*, Fig. 268, the rate of production of heat (RI^2) varies considerably in the different armature conductors. However, thermal conduction tends to equalize the temperature throughout the armature, so that the capacity of the converter is determined by the average rate of production of heat throughout the whole armature winding. As would be inferred by comparing curves *a* and *c*, Fig. 268, the heating of the armature

when running as a converter differs from its heating when running as a direct-current generator giving the same output. The capacity of the machine when used as a converter is therefore different from its capacity when used as a direct-current generator. It can be shown that the ratio of converter capacity to direct-current-generator capacity for the same machine is as follows (based on armature heating, and assuming operation at unity power factor):

	Capacity
As a mechanically driven generator	1.000
As a two-ring converter	0.85
As a three-ring converter	1.32
As a four-ring converter	1.62
As a six-ring converter	1.92

This means that for a given capacity a six-ring converter is considerably smaller than a direct-current generator of the same capacity.

245. Operating characteristics of the synchronous converter. The operating characteristics of a synchronous converter are substantially the same as those of a synchronous motor driving a direct-current generator. The power taken from the converter as direct-current power affects its motor action in the same way that mechanical power taken from a synchronous motor affects its motor action, as discussed in section 209. By altering the excitation, the current intake and power factor of the converter at any load can be varied in the same manner as in a synchronous motor (see section 211).

246. Advantages and disadvantages of synchronous converters. Compared with a motor-generator set of the same capacity, a synchronous converter is in general more compact, lighter, cheaper, and more efficient; but it is inflexible in voltage ratio, and less satisfactory for parallel operation with other direct-current machines or for reversed operation.

The ratio between direct-current and alternating-current voltages in a synchronous converter is fixed by the number of rings; accordingly synchronous converters can be designed for only a very small number of different voltage ratios, whereas a motor-generator set can be designed for any voltage ratio whatsoever. Of course this difficulty can be overcome by feeding the converter from a transformer of proper ratio rather than

directly from the line, though this expedient sacrifices (to some extent, at any rate) the advantages of compactness, cheapness, and efficiency.

The voltage at the direct-current end of a synchronous converter cannot be controlled at will except by the use of more or less cumbersome auxiliary apparatus; in a motor-generator set the voltage is easily controlled, just as in any direct-current generator.

It might be thought that the direct-current voltage of a synchronous converter could be controlled within small limits by altering its field excitation, which might be expected to change the alternating electromotive force *induced between rings* (and therefore the direct electromotive force standing in fixed ratio to it), even though the electromotive force impressed upon the rings remains unchanged. It is found, however, that the changes in strength and phase of the alternating-current input which accompany a change of excitation cause such changes in the field due to armature reaction as largely to neutralize the effect of changing the main field excitation.

PROBLEMS

1. Draw a diagram like that of Fig. 266, but for a 4-ring 6-pole converter.

2. Large converters are sometimes built with twelve rings. For such a converter find the ratio between alternating-current electromotive force between rings and direct-current electromotive force between brushes; also the ratio between alternating current in one section of the armature and direct current output, at unity power factor.

3. (*a*) What will be the speed of the converter of Problem 1 when operated from 60-cycle mains? What will be (*b*) the direct-current voltage induced between the direct-current brushes and (*c*) the alternating-current voltage induced between adjacent rings? (Given: number of paths in parallel between direct-current brushes, 6; number of conductors, 596; flux per pole, 2,000,000 maxwells.)

CHAPTER XXI

POWER PLANTS

247. Location of central stations. It is the function of a central station to receive energy from fuel, water power, or other source, transform it to electrical energy, and deliver it to a transmission system for carrying it to the points where it is to be used. Evidently efficiency in the reception of energy and in its distribution is quite as important as efficiency in the dynamos that transform it into the electrical form. The location of a central station is one of the most important considerations affecting the efficiency and economy with which the energy can be received and distributed.

As regards economy in the reception of energy, when the energy is derived from fuels the station should be located near enough to a railroad or a wharf to enable fuel to be delivered by carload or by shipload close to the station's boilers. Except in very small stations, condensing engines should be used, and the station should therefore be located where a plentiful supply of water can be obtained for cooling the condensers.

An example of a station very favorably located in these particulars is the Colfax Plant of the Duquesne Light Company of Pittsburgh. This plant, designed to have an ultimate capacity of 300,000 kilowatts, is located near the mouth of a coal mine which will supply the necessary fuel for many years. The Allegheny River flows past one side of the plant, and a branch of the Pennsylvania Railroad runs past the other side, thus providing for delivery of fuel both by rail and by water whenever that from the adjacent coal mine ceases to be available. The river also supplies cooling water for the condensers.

As regards economy in transmission, the nearer the station can be placed to the points at which its output is to be used, the smaller will be the energy losses in transmission, and the smaller the cost of construction of the transmission system. Moreover, the RI drop in voltage in the lines will be less in proportion as the station can be placed near to the points of

energy consumption. When the points of consumption are scattered over an extended region, as is usually the case, it is possible to find a "center of load" within the region, and this is the most desirable location for the central station, so far as economy of distribution is concerned.

Frequently the center of load is not near to the location where a station could receive its energy most economically. If alternating currents are used for supplying a city, distribution can be effected so economically over distances up to a few miles that considerations of the center of load have little weight in determining the location of a central station; but if direct currents are used, the economy of distribution is greatly affected by the nearness of the station to the center of load, and this consideration must be taken into account as well as that of accessibility to the source of energy.

Of course the choice of a location for a central station is also affected by many other considerations as well, such as cost of real estate, possibility of good foundations, and so on.

When energy is derived from water power, the location of the station is more or less rigidly fixed by that of the water power and cannot be adapted much to suit other considerations. This is one of the reasons why water power is sometimes less valuable than it is popularly supposed to be.

248. Substations. A substation is a station which receives electrical energy from a transmission system and delivers it to a distribution system, after changing its kind — for instance, in voltage, or from alternating current to direct.

Substations are used when a direct-current distribution system must be extended beyond the limits of economical transmission of direct-current energy from the central station. The energy is then sent from the central station to the substation by alternating current, where it drives synchronous converters or motor-generator sets which deliver direct current to the distribution mains. Substations are also used in alternating-current systems when distances of transmission call for the use of higher voltages than that of the distribution system; but alternating-current substations generally operate with transformers instead of with converters or motor-generator sets.

The location of a substation should evidently be as near as possible to the center of load of the territory which it serves.

249. Load curves. The demand for electrical energy for its various applications in a given territory is one that varies

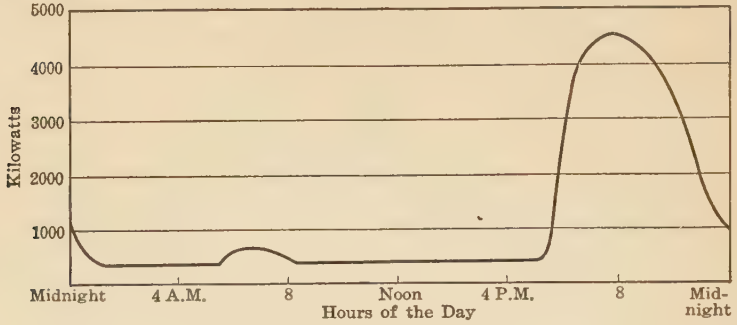


FIG. 269. Load curve for interior lighting only

widely with the hours of the day, but in somewhat the same manner on different days. In both the design and the operation of central stations it is important to know how the demand

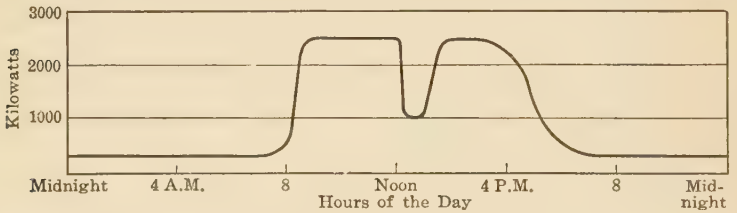


FIG. 270. Load curve for industrial power only

varies with the time of day. For this purpose "load curves," showing the output in kilowatts (as ordinates) for each instant during the twenty-four hours (as abscissas), are convenient.

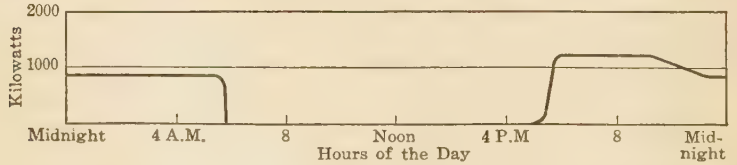


FIG. 271. Load curve for street and sign lighting only

Fig. 269 shows such a curve, where the load is for interior lighting only. Its most noteworthy feature is a very pronounced "peak," several times the average load, occurring in the early

evening. Fig. 270 shows a load curve for industrial power only, and Fig. 271 one for street and sign lighting only. Fig. 272 is the combined load curve for all three of the services of the other figures. A peak is still present, though relatively less prominent than in the other curves. The curves are intended to be typical of conditions in a city of thirty thousand inhabitants, in the spring or fall; though, of course, there is much variation among different cities, according to their industries, climate, and customs.

250. Capacity required in central stations. If we ignore, for the present, the possibility of storing electrical energy, it is

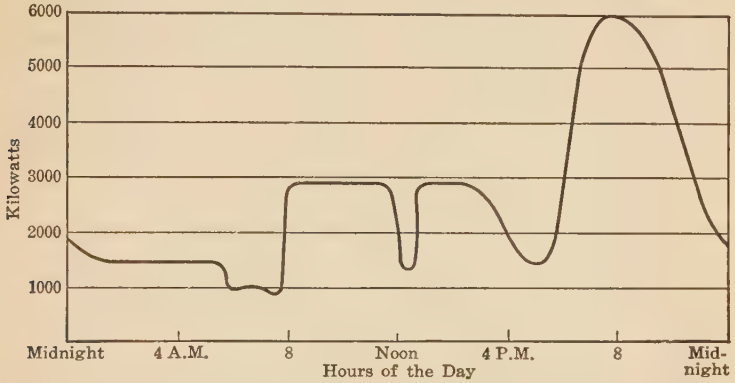


FIG. 272. Combined load curve for all the loads shown in Figs. 269, 270, and 271

evident that in any central station enough capacity of generators will have to be provided to carry the *peak* of the load.

It might be thought that this capacity could be obtained most economically by providing a single large generator of capacity sufficient to carry the peak load: the first cost would be less and the full-load efficiency greater than that of two or more smaller generators of the same total capacity. But the larger generator would have lower efficiency at half load than would a generator of half the capacity running at its full load. And with most load curves the load is much less than its maximum for a large part of the time. Consequently better operating economy is usually obtainable by installing two or more generators of combined capacity sufficient for the peak load, rather than a single one — some of the generators being cut out of service when the load is

below maximum, and the rest then running at their full load (approximately). This arrangement also has the very great advantage that any one machine can be taken out of service at some time during the day in case it needs repairs; and in case of complete breakdown of any one machine, the service need not be interrupted, since the other machines can then carry the load (probably including the peak, by temporary overload).

For instance, for the load curve of Fig. 272, three machines might be provided, of capacities 1500, 2000, and 2500 kilowatts.

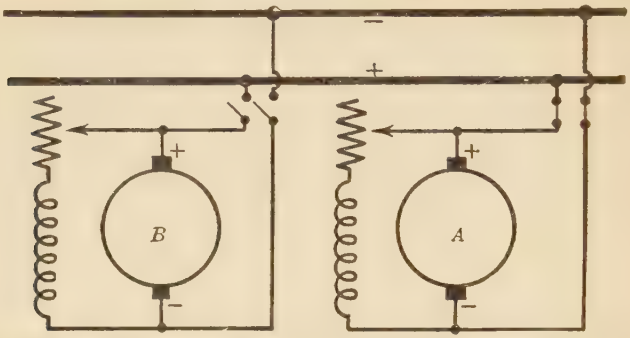


FIG. 273. Diagram of connections for operation of shunt generators in parallel

This would make possible seven different combinations: 1500, 2000, 2500, 3500, 4000, 4500, and 6000 kilowatts.*

251. Operation of generators in parallel. When a given load is to be carried by two or more generators, it is necessary to operate the generators in parallel with each other across the bus bars. The behavior of different types of generators under these conditions is very different: that of alternating-current generators was discussed in section 184: that of different types of direct-current generators is discussed below.

Shunt generators operate satisfactorily in parallel and can be made to share the total load in any desired proportion by manipulating their field rheostats. When a generator *B* (Fig. 273)

* These figures are given in order to show how the equipment may be so chosen that every machine can be run at nearly full load while it is in use. In actual operation the consideration of interchangeability of parts makes it very desirable to have the different machines alike, and it is also essential to provide some reserve capacity to use in case of disability of some one machine. For the situation assumed above, four machines, each with a capacity of 2000 kilowatts, would be suitable.

is to be connected with bus bars so as to run in parallel with a generator A and take over part of A's load, the procedure is as follows: First, B is brought up to speed (entirely disconnected from the bus bars); then, by means of its field rheostat, its voltage is adjusted to equality with the voltage across the bus bars. Then, after making sure that the polarity of B is in the right direction, its main switch is closed, connecting its terminals with the bus bars. Since the voltage of B equals that across the bus bars, B will not deliver any current, nor will any current flow backward through B against its electromotive force; a state of balance will exist; but if the field rheostat is now manipulated so as to make B generate a slightly higher voltage, E_B, then it will deliver a current equal to $\frac{E_B - V}{R_B}$, where V stands for the voltage across the bus bars, and R_B for the resistance of B's armature (including brushes and connections to bus bars). At the same time the voltage across the bus bars will be slightly increased, and the current delivered by $A \left(= \frac{E_A - V}{R_A}\right)$ will accordingly decrease. Then, by manipulating A's field rheostat so as to lower slightly the electromotive force E_A that it generates, its current output can be further decreased, and V be brought back to its original value at the same time. Thus by manipulating the rheostats of the two machines, the voltage across the bus bars can be held at some fixed value, and the load be divided between the two machines in any desired proportion.

When B is to be taken out of service, its field rheostat is first manipulated until all load is removed from B and carried by A, and then the switch is opened, disconnecting B from the bus bars, after which B can be stopped.

Series generators are not operated in parallel, but it is instructive to consider what their behavior would be if it should be attempted to operate them in parallel. Suppose two series generators to be connected across bus bars as shown in Fig. 274, and suppose that, at the moment, each machine delivers current as shown by the arrows and in such amount as to maintain the proper field excitation to make the terminal voltage of each machine equal that of the other. The generators would then operate quite satisfactorily if these conditions would *persist*.

But they cannot persist: the situation is unstable. Suppose that from some accidental cause, such as fluctuation in engine speed, the voltage of A falls momentarily a little below that across the bus bars V: the effect will be that A will deliver less current; this will lower the excitation of A; it will then generate

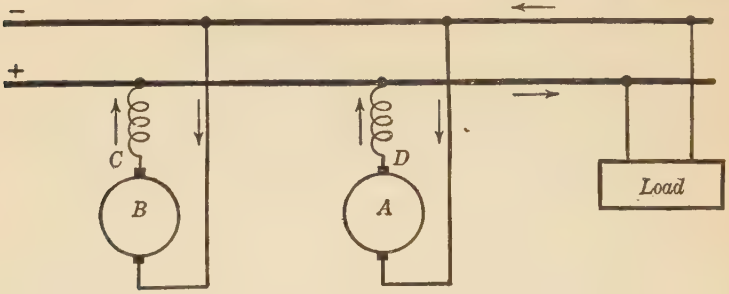


FIG. 274. Diagram showing series generators in parallel
Unstable condition of operation

less electromotive force; its current and excitation will then fall off still more, until finally its polarity is reversed, which will result in breakdown of the system.

However, series generators can be made to operate in parallel if connection is made between the points C and D (Fig. 274), as

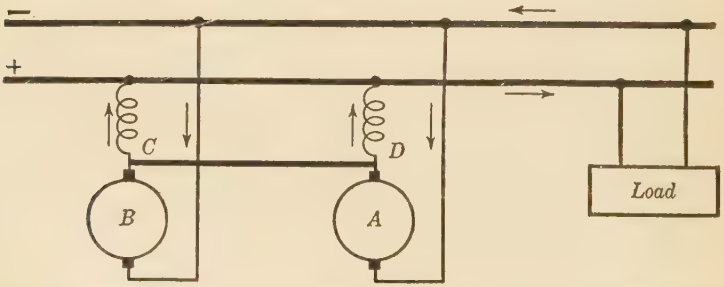


FIG. 275. Parallel operation of series generators with an equalizing bus CD
Stable condition

shown in Fig. 275. This can be accomplished by the use of an additional bus bar known as an *equalizing bus*. With this connection it is not possible for the excitation of any machine to vary independently of that of the others with which it is in parallel.

Compound generators in parallel, if under-compounded, act like shunt generators; if over-compounded, like series generators.

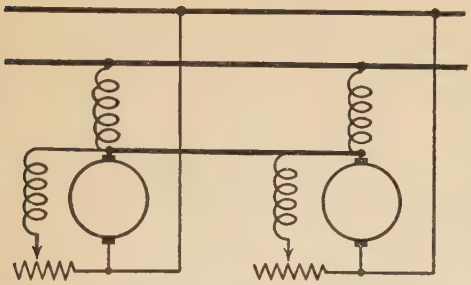


FIG. 276. Connections of compound-wound generators for parallel operation

Since the conditions of service usually require either flat-compounded or over-compounded generators, the use of an equalizing bus is generally necessary in direct-current central stations. The connections for compound generators in parallel are shown in Fig. 276. The proportion of the load taken by each machine can be adjusted at will by manipulating the shunt-field rheostats, as with shunt generators.

252. Layout of central stations. For convenience in operation and ease in enlarging a power station, the various apparatus is usually so arranged as to form a set of units, each capable of operating without the others. If increasing demand requires enlargement of the station capacity, new units are then added, without disturbance of those already installed. In a steam plant a unit would include fuel-handling appliances, a boiler with its accessories, a steam engine or turbine with condenser,

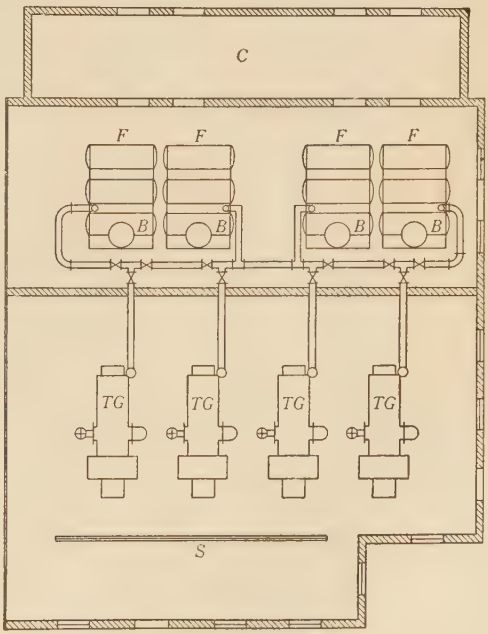


FIG. 277. Simplified diagram to show typical layout of a steam electric-power plant

C, fuel-handling appliances; *F*, furnaces; *B*, boilers; *TG*, turbine-generator sets; *S*, switchboard

and a generator with its switchboard panel. Some arrangements (such as the chimney) are not duplicated for each unit, but are used in common by all units. Although one unit can operate without the others, some interchange of facilities is possible; for example, the engine of one unit may draw its steam from the boiler of another if its own boiler is out of commission.

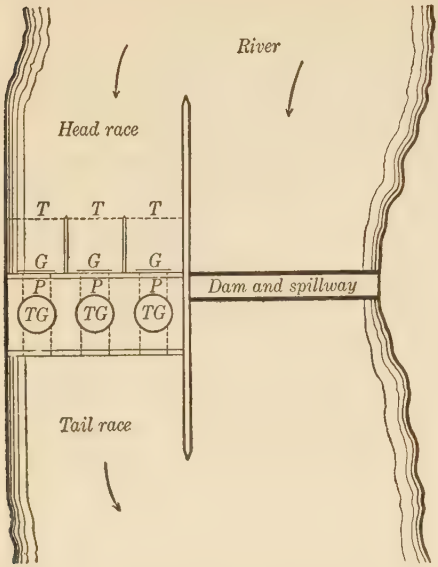


FIG. 278. Simplified diagram to show typical layout of a hydroelectric power plant

T, trash racks; *G*, gates; *P*, penstocks; *TG*, vertical turbine-generator units

Fig. 277 shows a typical layout of a steam-power station, and Fig. 278 the layout of a water-power station.

Within a unit the arrangement is governed by considerations of ease and efficiency in the flow of energy or materials through the station. The furnaces should be as near to the supply of fuel, the engines as near to the boilers, and the switchboard as near to the generators as other conditions permit.

Except sometimes in very small plants, the generator is always direct-connected to its engine or turbine, rather than belt-driven; that is to say, the armature is placed on the engine or turbine shaft. Other features of the generator and its prime mover are then so arranged that a single foundation serves for both, and they are, to a considerable extent, merged into a single machine. This arrangement has a disadvantage in that generator and prime mover must run at the same speed, but this is far outweighed by the advantages in efficiency and compactness.

253. Interconnection of central stations. Just as two or more generators within a station may be connected with the bus bars and share the load among them, so two or more distinct power stations may be connected with a transmission system and

share in contributing power to it. Such an arrangement obviously improves the reliability of the whole system. The more important transmission systems are nearly always fed from more than one generating station. An illustration is given in Fig. 279.

Often two existing transmission systems expand so as to approach each other's territory, and then it is advantageous to

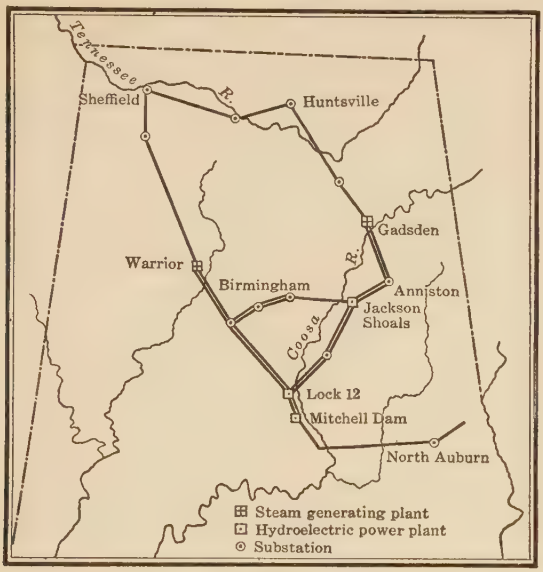


FIG. 279. The 110,000-volt transmission system of the Alabama Power Company, as of April, 1925

For convenience in drawing, the lines are shown as straight lines between the principal points of the system, although in reality the lines contain a good many curves. Note the duplication of lines in the central part of the system. In addition to the lines shown above, the company operates an extensive 44,000-volt system covering a much larger territory than that shown above

connect them together.* This will increase the effective capacity of both systems, without additional equipment, if their peak loads occur at different hours, since energy can then flow from either system to the other as needed.

* This can be done in alternating-current systems even if the voltages of the two systems differ, by the use of transformers. It can also be done between direct-current systems of different voltages, between alternating-current systems of different frequencies, or between an alternating-current and a direct-current system, by the use of suitable motor-generator sets or synchronous converters.

The course of development has been toward interconnection of transmission systems over increasingly large territories, and

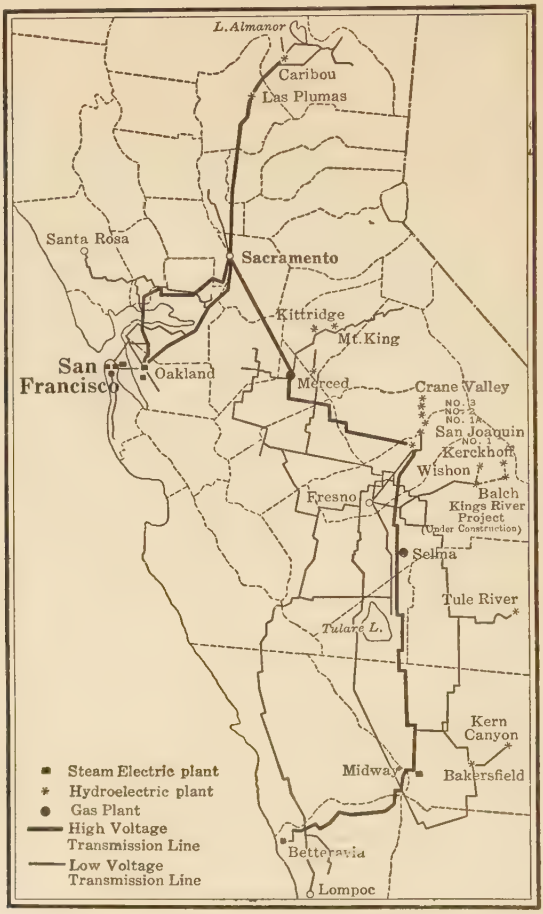


FIG. 280. Super-power system formed by interconnecting the power systems of California companies controlled by the North American Company

The territory served has an extreme length of about 450 miles and an extreme breadth of about 150 miles. The total installed generating capacity of the power plants of the system is about 350,000 kilowatts (in 1927). The total ultimate generating capacity of the water powers under development is about 1,250,000 kilowatts

doubtless this will continue until all the systems in the country are connected together and fed by an adequate number of strategically located generating stations. As steps in this direction,

a number of proposed "super-power systems" are undergoing development or receiving consideration for possible realization in the near future. The general nature of one of these projects may be seen from the map of Fig. 280.

254. Isolated power plants. In the earlier days of electrical development it often happened that a large individual consumer of electrical energy, such as a hotel, a theater, or an office building, found it economical to install a plant of its own for generating electricity; but with progress in central-station practice, the economies of large-scale production have become so great that it is no longer possible, in general, for such a consumer to generate electricity as cheaply as to purchase it from a central station. Isolated power plants for generating electricity are therefore probably a thing of the past within territory served by a central station.

The need for isolated plants still exists in situations beyond the reach of central-station distribution systems, as in country homes or institutions, and on boats, trains, and vehicles.

In all such cases the region to be served is of small extent; accordingly direct-current systems are used rather than alternating. Furthermore, the shortness of the distances of transmission makes possible the use of lower voltage than would be economical where energy has to be transmitted farther.

255. Domestic lighting outfits. The electric generating outfits for use in country homes usually consist of a low-voltage (generally 32-volt) direct-current generator and a storage battery.* The generator may be driven by any available prime mover: the gasoline engine is most often used. The usual plan of operation is to run the generator about once a week (or oftener, if necessary) to charge the battery; the battery remains connected with the line all the time and supplies energy when the generator is not running.

Fig. 281 shows the general appearance of a domestic lighting outfit manufactured by the Delco-Light Company. The power plant consists of a generator direct-connected to a gasoline engine.

The electrical energy produced by domestic lighting outfits is inherently more expensive (if the labor of running the plant and the fixed charges on the investment are not overlooked) than that

* But some outfits operate without the storage battery, the generator starting automatically when any light is turned on.

produced in central stations. But the safety, convenience, and cleanliness of electric lighting often amply justify the installation of such systems in places that cannot be served by central stations, especially where expense is not a controlling consideration.

256. Train-lighting systems. It is not practicable to supply electricity to a whole train from a single generating plant carried, for instance, in the baggage car, because during switching some cars would be left without light. Each car has to carry its own complete

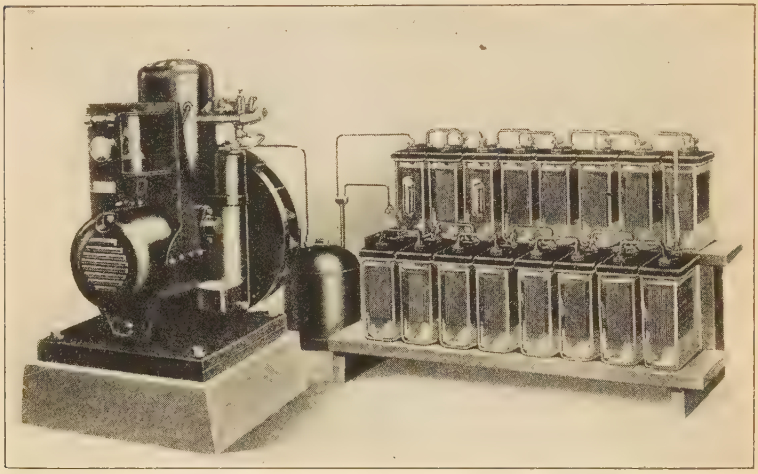


FIG. 281. Farm electric plant

Capacity, 750 watts. Power, one-cylinder air-cooled gas engine: two-bearing crank shaft; overhung armature mounted on front end of crank shaft; flywheel, part of cooling system on rear end

Courtesy of the Delco-Light Company

lighting plant, which consists of a generator and a battery. The comparatively small power required for the generator is taken from the axle of the car while it is running. The generator is of a special type that gives approximately the same voltage at all speeds above a certain lower limit. When the speed is above this limit, the generator supplies current to the lamps and at the same time charges the battery; when the speed is below this limit, the generator is automatically disconnected (thus preventing the escape of the battery current through it), and the battery alone then supplies current to the lamps. When the speed again increases, the generator is automatically reconnected. The generator is so constructed that it builds up with either direction of running, and always with the same

polarity. In Pullman cars the usual capacity of the electric plant is 1600 watts at 30 volts.

Other types of isolated plants, such as automobile-lighting systems and marine power plants, cannot be discussed within the limits of this book.

ECONOMICS OF CENTRAL-STATION OPERATION

257. Operating expenses and fixed charges. The costs of production of electrical energy can be divided into two classes. Those belonging to one class depend on the amount of electrical energy produced (being more or less perfectly proportional to it) and are called *operating expenses*. Those of the other class depend on the first cost of the equipment installed (being more or less perfectly proportional to it) and are called *fixed charges*; they continue even if the station is entirely shut down. Among the first class, or operating expenses, are the cost of fuel (in steam-driven plants), of lubricants and supplies, and of repairs and replacements. Among the other class, or fixed charges, are the cost of the use of capital (interest on bonds and dividends on stock), insurance, depreciation, and obsolescence. The cost of labor is usually classed with operating expenses. Even though employees receive fixed wages not contingent on the output of the station, nevertheless the number of employees required is somewhat dependent on the output, and the labor cost would cease entirely if the station should be shut down. Taxes belong with fixed charges in this country, where they are levied in proportion to the value of the station; though there is no inherent reason why, if our taxation system permitted, they should not be levied upon output rather than upon investment, and thus belong with operating expenses.

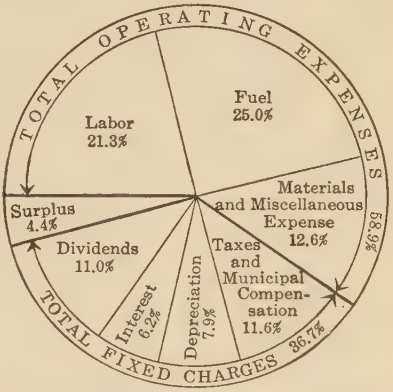


FIG. 282. Distribution of earnings of Commonwealth Edison Company, 1923*

Fig. 282 will give some idea of the relative importance of the different costs, although, of course, their proportions vary considerably for different stations and for different years. The figure applies to a total income of about $1,700,000.

*From Samuel Insull's "Public Utilities in Modern Life." Used by permission.

258. Relation of costs to peak load and average load. The fixed charges depend on the *peak load*, since the capacity of equipment required for a central station is determined by the maximum power output it must be capable of delivering. The operating expenses depend on the total energy produced, and therefore on the *average load*. This has no necessary connection with the peak load, since in most cases there are considerable periods in which the load is much less than the peak. During these periods some of the equipment is either idle or running at light load, but its fixed charges continue just the same.

259. Load factor. The total cost of producing a kilowatt-hour of electrical energy must include both fixed charges and operating expenses, and will therefore be much affected by the relation between average load and peak load. The peak load determines the fixed charges; the average load determines the kilowatt-hours of energy output. If the average load is a small fraction of the peak, there will be comparatively few kilowatt-hours among which to apportion the fixed charges; if the average load is a large fraction of the peak, then there will be a large number of kilowatt-hours to share in carrying the fixed charges. The ratio of average load to peak load is called the *load factor*.* Its greatest possible value is unity, and this would be the most favorable value for the economical production of electrical energy.

If the demand increases for electrical service (such as domestic lighting) during the peak of the load, the central station will have to install additional equipment to be able to carry a higher peak load. On the other hand, if the demand increases for service occurring at hours outside the peak (such as most industrial power), it can be met without additional equipment, merely by loading more fully the equipment already on hand. This adds nothing to the fixed charges, but only to the operating expenses. Evidently it is good business policy — as well as sound economics from the general social point of view — for a central station to try to stimulate demand for off-peak load, rather than for additional load during the peak.

260. Relative cost of water power and steam power. When electrical energy is derived from water power, the operating expenses are very low, on account of the disappearance of all expenditure for fuel. On the other hand, fixed charges may or may not be less than in fuel-operated stations of the same capacity; in many cases they are very much greater, owing to the necessity for constructing expensive dams and hydraulic works.

The popular idea that production of electrical energy from water power is always and necessarily cheaper than from fuels is erroneous.

* Of course the particular period of time concerned needs to be stated; for instance, the load factor of a plant for a Sunday would be different from that for a workday.

There are many water powers which, if developed, would produce electrical energy under such heavy fixed charges as to outweigh entirely the economy in operating expenses. Furthermore, many otherwise valuable water powers are subject to interruption at times of drought, or are situated remote from bases of supplies and from markets for the electrical energy they might deliver.

261. Large-scale production versus small-scale production. Electrical energy is similar to most commodities in that it can be produced more cheaply, per unit quantity, on a large scale than on a small scale. Some of the reasons for this are the following:

Better engineering and other expert service can be brought to bear on both the construction and the operation of large central stations than is economically possible in small stations.

The first cost of generators and their accessories is less, per unit of capacity, in the larger sizes than in the smaller ones.

The efficiency of large generators and of their prime movers is greater than that of small ones.

Better organization and subdivision of labor is possible in large central stations than in small stations. For instance, in the very small central station of a country village the same man may have to act as dynamo tender, switchboard operator, stationary engineman, and even fireman while the station is in operation in the evening; and as lineman and electrician during the daytime. In a large station all these functions are performed by different men, each with special skill in his line and with opportunity to give undivided attention to one kind of duty.

262. Switchboard cost of electrical energy. The cost of production of electrical energy per kilowatt-hour as delivered at the station switchboard varies considerably with the conditions under which the station operates. For example, the average for various local groups of stations in the northeastern United States in 1919 ranged between 1.82 cents and 6.70 cents per kilowatt-hour for steam plants and between 0.66 cent and 1.94 cents per kilowatt-hour for hydroelectric plants. Of course some individual stations have production costs lower than the lowest of the average figures just given; others have production costs higher than the highest of these figures.

263. Cost of distribution of electrical energy. The cost of electrical energy as delivered to a consumer exceeds the cost of production at the station switchboard by the cost of transmission to the consumer. This includes fixed charges on the transmission system up to and including the consumer's meter and the cost of the energy lost in the mains, meter, and transformers (if any), together with the commercial items of meter-reading, billing, and accounting. Evidently the cost of distribution increases with increasing distance of a consumer from the station. Furthermore, the average cost of transmitting a

kilowatt-hour from station to consumer will be greater when the consumers are scattered in all directions from the station than when all are grouped close together, even though at the same average distance from the station. For these and other reasons the average cost of transmitting a kilowatt-hour from station to consumer varies greatly in different stations, as well as the cost of producing a kilowatt-hour at the station switchboard.

264. Rates. The income of a central station is derived from the sale of the electrical energy it distributes. The rates charged to consumers must necessarily be such that the total income of the station will at least equal the total cost of production and distribution of its output — otherwise bankruptcy will result in course of time, though not necessarily very promptly, since the actual payment of some of the fixed charges can often be deferred.*

In most places the charges to a consumer are based on the kilowatt-hours of energy delivered to him. The cost of the various kilowatt-hours delivered differs widely with the location of the consumer and with the time at which he uses electricity — whether during the peak of the load or not. Ideal equity would doubtless require charging each consumer the exact total cost of the service that he, individually, receives; but this would require such elaborate metering and accounting as to be wholly impracticable. Nevertheless, it is easy enough to make some difference in rates according to the difference in cost of different kinds of service, and this is usually done.

The greater part of the demand for domestic lighting occurs during the peak of the load, and accordingly should be charged with fixed charges on the *whole* equipment of the station and the distribution system. The greater part of the motor load occurs during the daytime (off from the peak), and accordingly should be charged with fixed charges only on that part of the equipment in operation during the daytime. It is therefore common practice to make considerably lower rates for motor loads than for domestic lighting, separate meters being provided to measure the energy used by the motors.

Although wide differences exist in the cost of transmitting energy to different consumers, it is not customary to take any account of such differences within the territory normally served by an established distribution system; but beyond that territory electrical energy is delivered only if the consumer assumes the full cost of constructing and maintaining an extension of the mains beyond the limits.

* Especially depreciation. The gradual wearing out of apparatus goes on just the same, whether allowance is made for it or not, though actual replacement may not be necessary for many years. But when replacement finally does become necessary, if provision has not been made for it, complete failure of the station is bound to result.

A central station is put to a certain amount of expense by the mere readiness to serve a consumer, even though the consumer does not use any electricity. This expense includes fixed charges on the meter and its connections, and the labor of meter-reading and accounting. It is common to take account of this, either by making a "readiness to serve" charge in addition to the charge for energy used; or by having a minimum charge per month, which must be paid, whether or not the energy used in the month is worth as much as the minimum charge.

265. **Variations in rates between different places.** The wide variation in the actual cost of producing and distributing electrical energy in different places, which necessarily requires wide differences in rates, has been an unfortunate feature of the central-station industry. The higher rates which have had to be charged in the less fortunately located communities have often led to hostile criticism and agitation on the part of the public; in specially favorable locations, on the other hand, central stations have occasionally made unreasonably high profits on rates which were low in comparison with those of other places, but not as low as they should have been in fairness to the public. In an effort to secure fairness both to the public and to the owners of central stations, most of the states in this country have established public-utility commissions and empowered them (among other things) to fix what rates may be charged for electrical energy in the various communities of the state, and also — what is of greater real importance, though often less prominent in the popular mind — to fix and enforce standards of service.

PROBLEMS

1. It is proposed to install a central station in a country village to supply prospective customers as follows (all being on the same street):

Consumer	Distance East of West End of Street	Expected Current
	Feet	*Amperes*
No. 1	100	30
No. 2	200	10
No. 3	400	10
No. 4	700	10
No. 5	1200	40

Suppose that distribution is effected by two mains, each of resistance 0.1 ohm per 1000 feet, and the volts at the station are kept at 115. If the station is located at the west end of the street, find (when all consumers are taking full load at the same time) the volts across mains at each consumer and the total loss of power in the mains.

2. With everything the same as in Problem 1, except that the station is now 600 feet east of the west end of the street, find the volts across mains at each consumer and the total loss of power in the mains.

3. Suppose that a central station is required to supply a load of 500 kw. for four hours a day every day and only 100 kw. for the remaining twenty hours of each day. Suppose that this load is supplied by a 500-kilowatt generator running continuously (except when relieved by a duplicate machine, installed to provide reserve capacity). How much annual saving, if any, would be effected by installing a 100-kilowatt generator with a 100-kilowatt prime mover and using it to carry the load during the twenty hours of the day when the load is only 100 kw.?

Given:		
	Efficiency of 500-kilowatt generator at full load	92 per cent
	Efficiency of 500-kilowatt generator at 20 per cent load	50 per cent
	Efficiency of 500-kilowatt prime mover at full load	24 per cent
	Efficiency of 500-kilowatt prime mover at 20 per cent load	18 per cent
	Efficiency of 100-kilowatt generator at full load	90 per cent
	Efficiency of 100-kilowatt prime mover at full load	22 per cent
	Cost of energy supplied to the prime movers, per kilowatt-hour	0.5 cent
	Cost of 100-kilowatt generator and its prime mover	$8000
	Annual fixed charges on this cost	15 per cent

4. In a certain 500-kilowatt central station the annual fixed charges on the station and generating equipment amount to $15,000. The operating expenses amount to 2 cents per kilowatt-hour. Find the switchboard cost of the energy delivered per kilowatt-hour if the load factor is (*a*) 0.15, (*b*) 0.70, (*c*) 1.00.

CHAPTER XXII

STORAGE BATTERIES

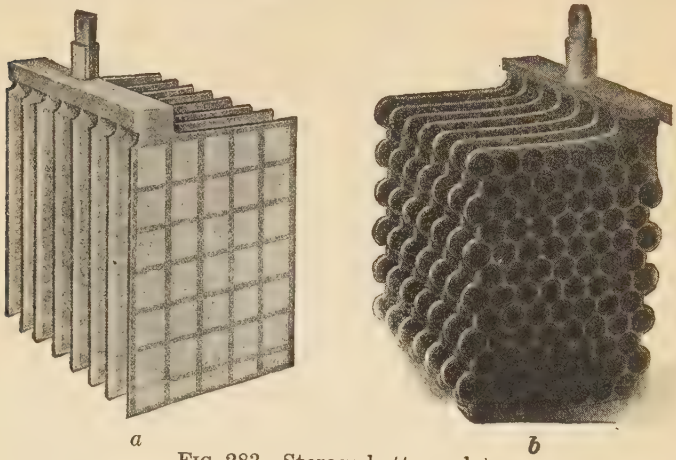
266. Storage of electrical energy. In dealing with many commodities storage is hardly less important than production and transportation. So too with electrical energy, it is often highly desirable to be able to store it so that it can be drawn upon as needed. Unfortunately there is not, in our present state of advancement, any way to store electrical energy as satisfactorily as we can store water or gas; there is only one way that is at all practicable — by the use of the storage battery. (Strictly speaking, storage batteries do not store electrical energy, but chemical energy in a form that turns to electrical energy as required.)

The problem of the storage of electrical energy is much farther from a satisfactory solution than that of its production. Notwithstanding its great usefulness, the storage battery is, in comparison with the dynamo, relatively expensive, inefficient, cumbersome, short-lived, and easily injured. In spite of all this it is, because of the great importance of storage, a very widely used and very useful appliance. But it does not offer a commercially practical means for large-scale storage over long periods of time: nearly all the electrical energy that is used has to be generated at the instant that it is wanted.

267. Kinds of storage cells. Out of the very large number of different kinds of cells only two types have been found practically useful for storage batteries: the lead cell and the alkaline cell. Of these, the lead cell is much more widely used.

In principle it should be possible to use any cell as a storage cell, since, if current from some outside source is passed through it in opposition to its electromotive force, the chemical actions that accompany discharge should take place in reverse direction, thus restoring the cell to its condition before discharge. But in most cells some of the products of the chemical reaction during discharge escape from the cell as gases, or react with other substances present, and thus are not available at the electrodes if the attempt is made to charge the cell from an outside source of electrical energy.

268. The lead storage cell. The active substances in the lead storage cell are lead peroxide (PbO_2) for the positive electrode and pure lead (in a spongy form) for the negative electrode, with



a *b*

FIG. 283. Storage-battery plates

a, negative group; *b*, positive group. (Courtesy of the Electric Storage Battery Company)

sulfuric acid (diluted with about twice its weight of water) for the electrolyte, and lead sulfate as product of the discharge reaction.

While the cell is discharging, the following chemical reactions* take place:

$$\text{At the positive electrode, } PbO_2 + H_2SO_4 + \left\{ \begin{matrix} H_2 \\ SO_4 \end{matrix} \right\} = PbSO_4 + 2\,H_2O$$
$$\text{At the negative electrode, } Pb + \left\{ \begin{matrix} \\ SO_4 \end{matrix} \right\} = PbSO_4$$

While being charged, chemical reactions take place as follows:

$$\text{At the positive electrode, } PbSO_4 + 2\,H_2O + \left\{ \begin{matrix} SO_4 \\ \\ H_2 \end{matrix} \right\} = PbO_2 + 2\,H_2SO_4$$
$$\text{At the negative electrode, } PbSO_4 + \left\{ \begin{matrix} \\ \\ H_2 \end{matrix} \right\} = Pb + H_2SO_4$$

At both electrodes the active material is attached to thin plates, or "grids," of lead alloy, which support the active materials without themselves taking part in the chemical action. Usually several plates are combined together as shown in Fig. 283; so that, except in the end plates, both sides of each plate can take part in the action, each positive plate lying between two negative ones, and each negative plate between two positive ones.

*Together with other more complex reactions, which may be neglected here.

The positive plates are easily distinguished from the negative by their appearance, the former having a reddish color, and the latter a dull gray.

269. Voltage of the lead storage cell. The voltage of a lead cell, on open circuit, is approximately 2.0 volts, but varies from about 2.2 volts down to about 1.8 volts, according to the

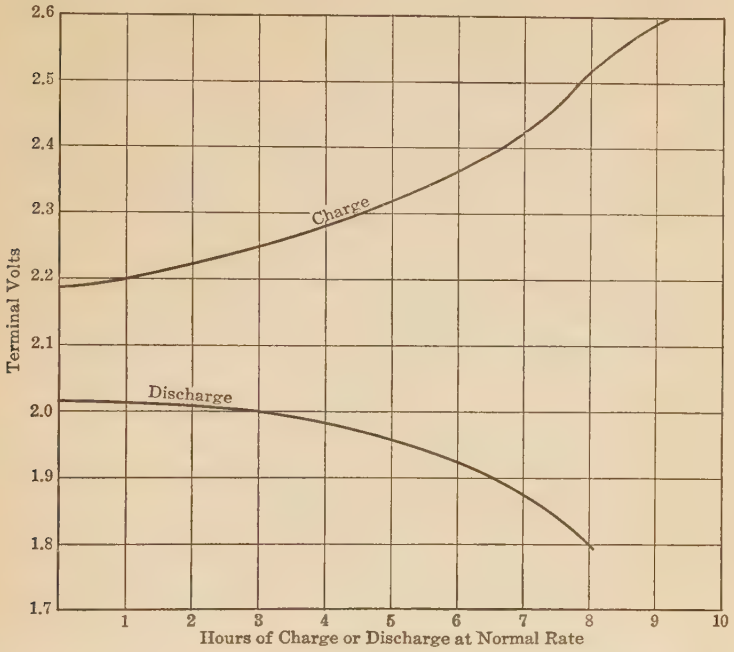


FIG. 284. Typical charge and discharge curves of lead storage cell

concentration of the electrolyte, the type of plate, the state of charge or discharge of the cell, and various other conditions.

The curves of Fig. 284 show how the terminal voltage of a lead cell varies with the state of charge while being charged, and while being discharged, at the normal rate.

The terminal voltage is higher while charging than while discharging, for two reasons: (1) The RI drop due to internal resistance of the cell increases the terminal voltage in the former case and decreases it in the latter. (2) While charging, the concentration of the sulfuric acid is greater close to the plates than elsewhere (see the chemical equations of the preceding section),

and this causes an increase in the electromotive force of the cell; while discharging, the concentration is less close to the plates, which causes a decrease of the electromotive force. These differences of concentration do not last long after the current stops, since convection currents and diffusion tend to equalize the concentration throughout the cell.

270. Rates of charge and discharge. For any storage cell it is customary to specify a normal charging rate or current. This normal current is determined by the type of the plates and by their area. Per unit area of plate it is of the order of 0.1 ampere per square inch; but its value is not sharply defined, and varies among plates of different manufacture.

When the cell is being charged, the current cannot much exceed the normal value without causing "gassing" or decomposition of the electrolyte, as discussed in the next section. This wastes energy; and the heating due to excessive current may be injurious to the cell.

When the cell is being discharged, the current may greatly exceed the normal value without injury to the cell. The rather prevalent idea that a high discharge rate, as such, is injurious to a lead storage battery is erroneous. However, a very high discharge rate is objectionable, because the efficiency of the cell is less when discharged at a high rate, and because the danger of discharging the cell too far is increased.

271. Limits of charge and discharge. The limit to which a cell can be charged is evidently reached when all the active material has been converted to PbO_2 at the positive plate and to Pb at the negative. If a charging current is continued after this condition is reached, it merely wastes energy in electrolyzing the water in the electrolyte, with the formation of bubbles of hydrogen and oxygen. This does no particular harm to the cell, although the action is inconvenient, since the rising bubbles cause the electrolyte to spray and fill the neighboring air with vapor and fine drops of sulfuric acid, which may corrode objects in the neighborhood; and the hydrogen, when mixed with air, is highly explosive, so that care should be taken not to permit open flames in the neighborhood of storage batteries while charging.

The limit to which a lead cell can be discharged is reached when about one fifth to one third of the active material is converted to $PbSO_4$. Unfortunately it is not possible to carry the

discharge far enough to utilize all the active material. If it is attempted to carry the discharge beyond the point at which the voltage has dropped to about 1.8, the voltage drops off very rapidly to a value near zero, and the cell is likely to be seriously and permanently injured.

The injury is due largely to the action of the lead sulfate ($PbSO_4$), a substance of high resistivity, which is formed at both electrodes. It is formed not only at the surface but all through the mass of the porous active material. If the proportion of it within the active material becomes too great, the material becomes more or less perfectly nonconducting, and it is then difficult to make a charging current flow into it rather than into the inactive frame of the plate. Furthermore, if there is some spot at which more sulfate has formed than elsewhere, the charging current will meet more resistance there than elsewhere, thus producing the least chemical action at the place where it is most needed. Accordingly a "sulfated" spot tends to remain permanently out of use, thus throwing an overload on to the remaining surface.

The lead sulfate which is formed during discharge occupies considerably larger volume than the active materials which enter into its production. Accordingly the plates swell during discharge; and if the discharge is carried too far, they may buckle, or pieces of active material may be forced loose and drop off the plates.

272. Capacity of storage batteries. By the capacity of a storage battery is meant the number of ampere-hours it can deliver* when discharging at the normal rate, starting from a fully charged condition and ending at the limit of safe discharge. The normal rate of discharge in stationary batteries is taken as that which will complete the discharge in eight hours.

From the laws of electrochemical action one would be led to think that the number of ampere-hours a battery can deliver would be independent of the *rate* of discharge, but in reality this number is less for a rapid discharge rate than for a slow one.†

* Strictly, the term *ampere-hour capacity*, rather than simply *capacity*, should be employed for the quantity here defined.

† The temperature also affects the capacity of a storage battery. According to the rules of the American Institute of Electrical Engineers, capacity should be measured under the conditions that the temperature of the electrolyte is 25° C. at the beginning of the discharge and the ambient temperature between 17° and 20° C. during the whole period of discharge.

The reason is that the acid within the pores of the active material is used up during discharge (see the chemical equations in section 268); and when the discharge is rapid, fresh acid cannot penetrate beneath the surface of the active material rapidly enough for the interior portions to take much part in the chemical action.

The quantity of energy available from a storage battery, in watt-hours, can be found (very approximately) by multiplying the capacity in ampere-hours by the average terminal voltage during discharge. This is called the *watt-hour capacity*.

273. Behavior of the lead storage battery when idle. There is great risk of injury to a lead storage cell if it is left standing discharged for a long period. It is very difficult to assure complete absence of chemical action while the cell is out of use. Through imperfect insulation slight leakage of current may take place and effect a gradual discharge of the cell. Such leakage may take place through accumulation of moisture and acid on the supports of the plates, or it may take place within the cell if a scrap of loose material falls between the plates and short-circuits them. Local action also takes place within the cell if impurities exist in the plates.

Before being left out of use, a lead storage battery should always be fully charged, and it is good practice to charge it again from time to time (say once a month) if possible, even if no current has been taken from it. If it must be left out of use for a long period (say six months), then the electrolyte should be removed and the plates be left dry.

274. Delicacy of the lead storage battery. Evidently the lead storage battery is a rather delicate appliance, being subject to injury (among other ways) by charging it too rapidly, by discharging it too far, or by leaving it too long without recharging. The electrolyte has to be replenished from time to time with distilled water and chemically pure acid to make up for evaporation. Altogether, the lead storage battery is better adapted for use under circumstances where regular expert attendance can be counted on than where such attendance is lacking.

With such attendance the positive plates of stationary batteries often last for ten years before needing to be replaced, and the negative ones even longer. When a battery has not received regular care (which is often lacking in the case of batteries for

domestic lighting outfits, automobile starters, and similar applications) its life is naturally very much less.

275. Methods of judging the state of charge of lead storage batteries. The condition of a lead cell as to its state of charge or discharge can be judged either by its voltage or by the concentration of its electrolyte. Judging by voltage, one may consider the cell completely charged when its terminal voltage reaches a certain value* while normal charging current is passing through the cell. In discharging, 1.8 volts is the minimum below which the terminal voltage must not be permitted to fall.

The concentration of the electrolyte is directly related to its density, which can easily be observed by a hydrometer. (There is no absolute connection between the concentration of the electrolyte and the state of charge of the cell, since it is possible to start the cell with acid of greater or less concentration; but for any given cell, there is a definite relation as long as the same electrolyte remains in use.) In practice the density at full charge usually ranges from 1.21 to 1.24; at the limit of safe discharge, from 1.185 to 1.195. In using the hydrometer one should stir the electrolyte before taking a reading.

276. Energy losses in storage cells. During both charge and discharge of storage cells energy is lost in driving the current through the internal resistance of the cell, at the rate of RI^2 watts, where R stands for the effective internal resistance and I for the current. This energy takes the form of heat within the cell. To make R small the plates are placed as near together as is practicable, and the electrolyte is used at a concentration near that for maximum conductivity.†

During charge the concentration of the electrolyte is greater close to the plates than elsewhere; during discharge it is less. This causes the electromotive force to be greater during charge than during discharge, and means that some of the energy put into the cell during charge fails to be delivered during discharge. The energy lost in this way also appears as heat within the cell, as the more concentrated parts of the electrolyte mix with the less concentrated parts.

* From 2.3 to 2.6 volts, according to the concentration of the electrolyte and the type of plates.

† The conductivity of a mixture of sulfuric acid and water is a function of the percentage of acid present, having its maximum at 31 per cent.

Loss of energy also takes place from leakage currents and from local action, when these occur.

277. Efficiency of the lead storage cell.* The efficiency of a storage cell varies greatly with the manner in which it is used: it is higher if the discharge begins promptly after completion of the charge than if an interval of time intervenes before the discharge begins, and, other things being equal, it is higher at a slow rate of discharge than at a rapid one.

The most favorable condition for efficiency is that in which a battery is alternately charged and discharged for very short intervals of time, since under this condition there is not time for the electrolyte next to the plates to become much different in concentration from that between the plates. Under such conditions lead storage cells have shown an efficiency as high as 94 per cent.

If a lead storage battery is charged at its normal rate for the normal period (about eight hours) and is then discharged at the same rate till the limit of safe discharge is reached, the efficiency is about 80 per cent.

If the battery is discharged at a more rapid rate, or if an interval of time elapses before the discharge is begun, the efficiency will have some lower value, ranging all the way down to zero, according to conditions.

278. Alkaline storage cells; the Edison cell. The most widely used type of alkaline storage cell is the Edison cell. In this cell the active materials are nickel peroxide (NiO_2) for the positive plate and pure iron for the negative plate, with a solution of caustic potash (KOH) for the electrolyte. The active materials are supported by grids of nickel-plated steel.

The open-circuit voltage of the Edison storage cell is about 1.2, varying considerably with the state of charge. The terminal voltage during charge and discharge is shown in Fig. 285.

The Edison storage cell is not injured by heavy currents; it can be short-circuited and allowed to discharge completely without damage; it does not suffer by being left for long periods without charging. On the whole, it is a rather rugged appliance and

* The term *efficiency* refers throughout this section to the ratio of the energy obtained from the battery to the energy previously put into it (the "watt-hour efficiency"), and not to the so-called "ampere-hour efficiency," or ratio of the quantity of electricity obtained from the battery to the quantity previously put into it.

can stand misuse and neglect in a way that the lead cell cannot. It weighs about half as much as a lead cell of the same capacity.*

On the other hand, the Edison storage battery is considerably more expensive in first cost than a lead storage battery of the same capacity, its efficiency is lower, and the variation of its voltage with the state of charge is greater. For these reasons it is not used in situations which call for large storage capacity or in which adequate care can be counted on; but its ruggedness and "fool-proof" qualities give it an advantage over the lead

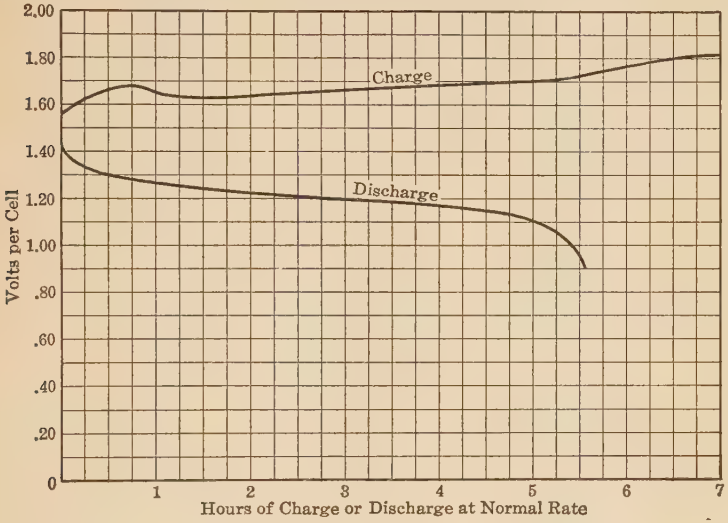


FIG. 285. Charge and discharge curves of Edison storage battery

battery under conditions where proper care is difficult to provide, and where small batteries are concerned so that questions of efficiency and cost are of minor importance. It also has the advantage where lightness is of great importance.

279. Applications of storage batteries. The principal uses of storage batteries may be classified as follows:

1. To furnish a portable supply of electrical energy to be used independently of any generator or transmission system. Thus storage batteries supply the energy for driving electric automobiles and boats, and battery locomotives and cars.

* Roughly 0.1 pound per ampere-hour of capacity, compared with (roughly) 0.2 pound per ampere-hour for lead cells.

2. To act as a substitute for a generator by supplying, while the generator is not running, electrical energy which could be taken from the generator if it were running. This may be done according to a regular routine, as in the case of domestic lighting outfits or small central stations, where the generator is operated a part of the time (charging the battery while in operation) and the battery furnishes current at other times if needed. Or it may be done irregularly as the exigencies of the situation require, as in the axle-generator system of car lighting or the lighting systems of gasoline automobiles, in which the battery supplies energy at such times as the car or the automobile is at rest; or in submarine vessels, which are driven by oil engine when at the surface, but by storage battery when submerged. Or it may be done only in case of disability of the generating apparatus; some central stations keep a "stand-by" battery ready at all times to take over the load of the station in case of temporary breakdown of the generating apparatus, thus assuring continuity of service.

3. To act in parallel with generators for the purpose of steadying a fluctuating load, the battery aiding the generators at times of heavy load, but drawing current from the generators for charging at other times. By this arrangement smaller generator capacity is needed than would otherwise be required, and the generators installed can be operated at constant load all the time — the most favorable condition for efficiency.

280. Voltage control of storage batteries. The terminal voltage of a storage battery falls off with increasing current output, and it also falls off with progress of the discharge of the cell. When storage batteries are used for driving motors, as in electric vehicles, the variation in voltage is usually unimportant; but when they are used for incandescent lamps or for supplying energy to the mains of a constant-voltage distribution system, some means must be provided for obtaining approximately constant voltage from the battery.

One method of doing this is by connecting a rheostat in series with the battery, as shown in Fig. 286. The battery must have enough cells to give the desired voltage when delivering full-load current and discharged to the safe limit (for 115 volts, this would require $\frac{115}{1.8}$, or 64, lead cells). When it is delivering less

than full-load current, or when not discharged to the safe limit, some resistance is inserted in the circuit by the rheostat; as the current increases, or as the discharge progresses, resistance must be cut out. This method, of course, wastes energy, and is therefore not adapted for use with large batteries.

Another method, free from this objection and much more widely used, is the end-cell method, shown in Fig. 287. In this method the number of cells in use is varied with the current and the state of discharge. For instance, with the battery fully

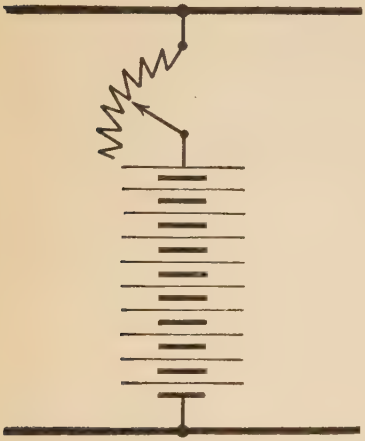


FIG. 286. Control of battery voltage by rheostat

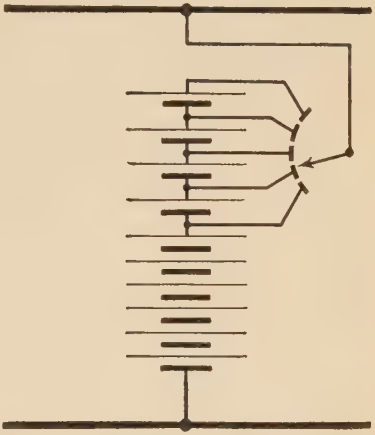


FIG. 287. End-cell method for control of battery voltage

charged (giving nearly 2.1 volts per cell) and at light load, the number of cells required for 115 volts would be $\frac{115}{2.1}$, or 55; and this number of cells would be connected with the mains. As discharge progresses or as current increases, one after another of the end cells would be placed in circuit along with the active part of the battery, until at the limit of safe discharge (1.8 volts per cell) 64 cells would be in use. In this method the voltage can be changed at will by steps of about 2 volts (the voltage of one cell), which provides sufficiently close adjustment for most purposes.

In end-cell control a special type of switch is necessary to make it possible to connect or disconnect the end cells without interruption of the current to the mains and at the same time

without momentarily short-circuiting the cell being connected or disconnected. This is accomplished by dividing the contact surface of the switch into two parts and placing resistance between them, as shown in Fig. 288.

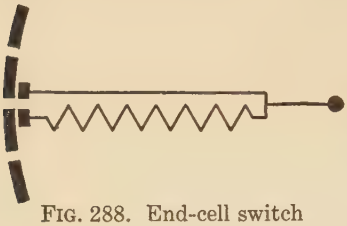


FIG. 288. End-cell switch

In recharging the battery the end cells become fully charged before the rest of the battery, and are cut out, one by one, as soon as each is fully charged.

281. Batteries in parallel with generators. When a battery is operated in parallel with generators, the intention is that at times when the demand for current is heavy the battery shall discharge and help carry the load, and at times when the demand for current is light it shall absorb current and recharge. It will not do this of itself if it is simply connected across the bus bars, since the voltage required by a battery for charging is considerably higher than the voltage it gives during discharge. To overcome this difficulty, various arrangements have been devised of which only two will be mentioned here: the floating battery and the shunt booster.

The floating battery. If a storage battery of the proper number of cells is connected across mains at a distant point of a line supplied from a central station, as shown in Fig. 289, the battery will automatically receive current at times of light load

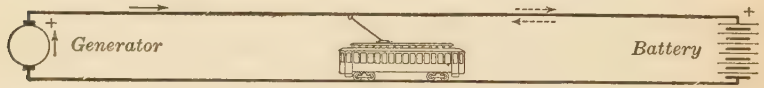


FIG. 289. Connections for floating battery

and deliver current at times of heavy load. When the load is light, the RI drop in the line from the generator is small, and the voltage then available at the terminals of the battery is high enough to drive charging current through it. When the load is heavy, the RI drop is large, and the voltage across mains at the battery is then low enough to permit it to discharge in parallel with the current from the central station. This method has the advantage of being perfectly automatic and of immediately responding to sudden as well as to gradual changes

of load; it has the disadvantage of not being readily applicable to a system of branching circuits and of requiring the battery to be placed at a distance from the station.

The shunt booster. A *booster* is a subsidiary generator connected in series with a circuit so that its voltage adds to (or subtracts from) that imparted to the circuit by another generator or by a battery. A method for operating a storage battery in parallel with a generator, by the use of a booster, is shown in Fig. 290. This method is known as the *shunt-booster* method, although in reality the booster is separately excited. The excitation of the booster is controlled by manipulation of the

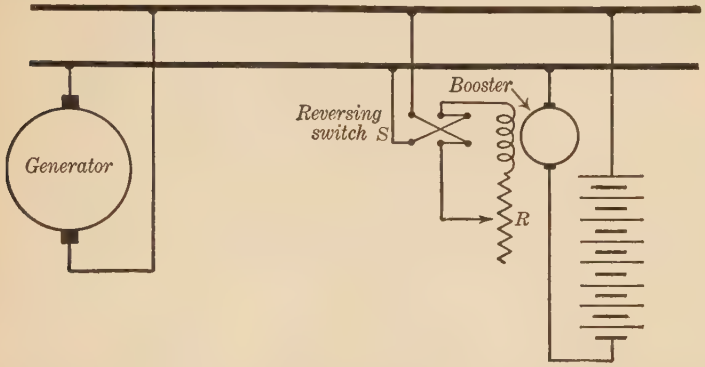


FIG. 290. Connections for operating a battery in parallel with a generator by the shunt-booster method

rheostat *R*, so that it can be made to generate more or less voltage, and its polarity can be reversed by reversing the direction of the current through its field by means of the switch *S*. The number of cells provided in the battery is such as will make its voltage approximately the same as that to be maintained across the bus bars. If the booster is wholly unexcited, the battery will behave the same as if connected directly across the bus bars; that is, its voltage being approximately equal to that maintained across the bus bars, approximately no current will flow through it, either of charging or of discharging. If, however, the booster is excited with the polarity in which its voltage opposes that of the battery, current will then flow through the battery in the direction for charging, and the amount of this current can be made more or less by making

the booster voltage more or less. If the booster is excited in the reverse direction, current will flow through the battery in the direction for discharging, and the discharge current can be made more or less by making the voltage of the booster more or less. Thus by manipulating the booster rheostat the station attendant can keep the current output of the generator nearly constant, drawing on the battery to help the generator when the demand for current is heavy, and absorbing energy in charging the battery at times when the demand for current is light. This method is not automatic; it is otherwise satisfactory when the changes in demand for current take place gradually, but is not adapted to taking care of sudden fluctuations in current.

PROBLEMS

1. The (lead) battery of a certain automobile was fully charged at the beginning of a four-hour trip, during which the battery received charging current of an average value of 8 amperes.* How much water is lost from the electrolyte in consequence of chemical action, in addition to any that may be lost by evaporation or spraying? The electrochemical equivalent of hydrogen is 0.0000104 gram per coulomb.

2. The battery in Problem 1 was then left unused for 30 days, and was found to be completely discharged. How much was the leakage current, assuming that it lasted the full 30 days, the capacity of the battery for slow discharge being 24 ampere-hours? Could this current have been detected by the ammeter?

3. To study the action of the "floating battery," consider a simplified example in which a generator maintains a constant voltage of 600 at the beginning of a street-car line 3 miles long, having a battery of 240 cells connected at its far end, and operating a single car which takes 40 amperes while running and 200 amperes while starting. The resistance of the line being 2 ohms per mile, and the battery being in such state of charge as to give 2 volts per cell on open circuit, find under which of the following conditions the battery will be charging, and under which it will be discharging and helping to drive the car: (*a*) car 1 mile from station, starting; (*b*) car 1 mile from station, running; (*c*) car 2 miles from station, starting; (*d*) car 2 miles from station, running; (*e*) car anywhere, at rest.

* In the electric system of an automobile, the motor which operates the self-starter acts as a generator while the engine is running and then supplies current for charging the battery.

4. The following data apply to a certain Exide-Ironclad lead storage cell: ampere-hour capacity, 102; average terminal voltage during discharge, 1.97; weight of cell, complete, 23.75 pounds; outside dimensions of cell in inches, $2\frac{7}{8} \times 6\frac{3}{16} \times 15$. Find the kilowatt-hours of available energy stored in the cell when fully charged, and compare this with the energy stored in a tank of gasoline (*a*) of the same dimensions as the cell and (*b*) of the same weight as the cell (neglect the weight and thickness of the tank itself). Take the heat of combustion of gasoline as 11,000 calories per gram, and its specific gravity as 0.68.

5. A certain domestic lighting outfit is to be supplied by a battery which is to be charged once a week. The average load to be carried consists of five 50-watt 32-volt lamps for four hours every night. What should be the ampere-hour capacity of the battery?

CHAPTER XXIII

SAFETY

It is the task of engineering not only to accomplish assigned objects but also to do so with maximum economy and safety.

In electrical engineering the consideration of safety is especially important, because the dangers from electricity are less easily seen than those from most other sources. For example, a dangerous bridge or structure can often be recognized as such by casual inspection and by persons without technical knowledge; but an entirely harmless-looking wire in an electric circuit may be fatal to touch, and even an expert cannot tell from its appearance alone whether it is "alive" or not. Again, in operating most nonelectrical appliances the operator has the whole field of action within his view, whereas the operator of electrical appliances often has to manipulate switches that control circuits running far beyond the range of his view (for example, in central stations, where various circuits controlled from the switchboard reach all parts of a city).

282. Safety codes. Damage from electricity may occur either to person or to property. With special reference to safety against accident to persons, an elaborate set of rules governing electrical construction has been prepared by the Bureau of Standards, under the name of "National Electrical Safety Code."* With special reference to safety of property, rules were prepared many years ago by the National Board of Fire Underwriters, and have been revised at frequent intervals. These rules, known as the "National Electrical Code," have been followed in most of the electrical installation work of this country.

These two codes furnish the electrical engineer with full specifications for a great many details of electrical construction or installation. We shall not have occasion here to consider the

* Circular No. 54 of the Bureau of Standards, 1916. Supplemented by Circular No. 72, 1918.

details of either of these codes, but shall undertake some discussion of the general principles involved.

283. Causes of electrical injuries. Damage from electricity, whether to person or property, is practically always due to one of two causes: (1) escape or leakage of current from the path it is intended to follow, so that it follows some other path, such as the human body; or (2) too heavy current in the path the current is intended to follow.

High voltage applied to a possible electrical path outside of the path which the current is intended to follow is not harmful in itself, but may be dangerous as causing an injurious current to flow. Therefore the common custom of looking upon high voltages as dangerous is well founded, since the current which would flow through a person's body in case of accidental contact with an exposed wire would be proportional to the voltage, other things being equal. The "shock" will be worse as the voltage is higher.

The mere maintenance of any object (including the human body) at a high electrical potential with reference to the earth, without flow of current through it, is harmless.

284. Effects of currents outside of their intended paths. When currents escape from the conductors intended for them and flow through other paths, damage may be caused in the following ways: (1) if the current traverses a part of the human body, it may cause serious injury or death; (2) a stray current may generate sufficient heat to cause damage directly or to start a conflagration; (3) stray currents may cause electrolysis, resulting in corrosion of metals, such as water pipes or even structural members of buildings.

Aside from considerations of safety it is obviously objectionable to permit electric currents to escape from their intended paths, since such leakage of current represents a waste of power, and, if sufficiently great, will more or less completely prevent the accomplishment of the useful purpose for which the current is wanted. Accordingly the insulation of electric circuits is of fundamental importance.

285. Insulation of electric circuits. Doubtless it would be desirable, ideally, to inclose every part of all electric circuits within a coating of solid insulation sufficient to make impossible the escape of any current under any conditions; but in

practice this cannot be done. It is often necessary that portions of circuits have no other insulation than the air,* thus permitting the possibility of accidental contact with live conductors. And even when a wire is inclosed within a covering of solid insulation, its insulation may be rendered ineffective at some point through such accidents as mechanical abrasion, overheating, or becoming water-soaked.

Every insulation permits a certain minute amount of leakage, which for all practical purposes is quite negligible.† Electrical accidents are not due to this minute escape of current, but to such escape as takes place when a conducting path comes accidentally into contact with an uninsulated portion of the circuit or with a portion where the insulation is defective. The construction of electrical circuits should obviously be so planned that the probability of accidental contact with any part of the circuit — whether provided with an insulating covering or not — will be the least possible.

286. Effect of an electric current on the human body. When an electric current traverses the human body, it causes the muscles to contract involuntarily, the effect being similar to cramps. If the current is sufficiently strong, the contraction becomes very severe. The muscles of the hand may be contracted so that one is unable to let go of a live wire that he has touched. Breathing and heart action may be brought to a stop through the rigid contraction of the muscles that carry them on, if these are traversed by the current. Upon stoppage of the current, the respiratory and cardiac muscles may fail to resume their action.

More or less severe burns may occur at the skin where the current enters or leaves the body, but its most serious effects arise from its property of contracting the muscles through which it flows.

* For instance, the trolley wires of electric-car systems, or the commutators or slip-rings of dynamos. On high-voltage transmission lines bare wires are used. In house-wiring, all fixtures are connected by winding the bare ends of wires under binding screws, though of course these uninsulated points are usually so incased within the fixtures as to prevent accidental contact with them.

† For instance, the insulation resistance of a 500-volt 100-kilowatt generator in good condition should not be less than 0.45 megohm, according to the standards of the American Institute of Electrical Engineers. The current that would leak through such insulation, under 500 volts, would accordingly be 0.0011 ampere; or one two-hundred thousandth of the current output of the machine.

287. Treatment of cases of electrical shock. When a person is overcome by electrical shock, the first thing to do is to disconnect him from the source of current if he is still in contact with it. This should be done by opening a switch in the line, if it is possible to do so without too much loss of time. If it must be done by lifting or pulling the person loose, great caution must be observed, since making contact with him may cause the current to pass also through the person who touches him. It may be possible to handle him with rubber gloves or through a rubber raincoat or by means of thoroughly dry parts of his own clothing, or to pry him loose by using a dry board. If no means of safely touching him are at hand, sometimes it is possible to stop the current by short-circuiting or grounding the line.

If the person is not breathing after disconnection from the line, steps should *immediately* be taken to restore respiration. Shaking, thumping, or other mechanical shock will sometimes cause respiration to start. If this fails, artificial respiration must be resorted to at once. The method (which applies also to cases of drowning or asphyxiation from any cause) can be learned best by demonstration and practice, but it will be described briefly here:

1. Lay the victim upon his stomach, face turned to one side, so that mouth and nose do not touch the ground; see that the mouth is free from possible obstructions to breathing (cigar, false teeth out of place, etc.).

2. The operator kneels, straddling the patient's hips, facing the patient's head (see Fig. 291).

3. The operator places his spread hands on the lower ribs of the patient and throws his body and shoulders forward so as to bring his weight heavily upon the lower ribs of the patient (see Fig. 292). This forces the air out of the patient's lungs. The operator's downward pressure should occupy about three seconds; then his hands are suddenly released (preferably removed entirely) (see Fig. 293). This gives opportunity for the natural elasticity of the victim's body to restore his ribs to their original position, thus drawing air into his lungs.

This act should be repeated an indefinite number of times, at the same rate as natural breathing (from twelve to fifteen times per minute); the operator can easily judge the proper rate by his own breathing. There is danger that in the excitement of the

occasion the rate will be too rapid. The operator must guard himself especially against this, since it may prevent the success of the method.

The artificial breathing should be continued, if necessary, for three or four hours. When the patient regains consciousness, he should be kept perfectly still; for many hours he should not be allowed to try to stand up. A number of cases are on record in

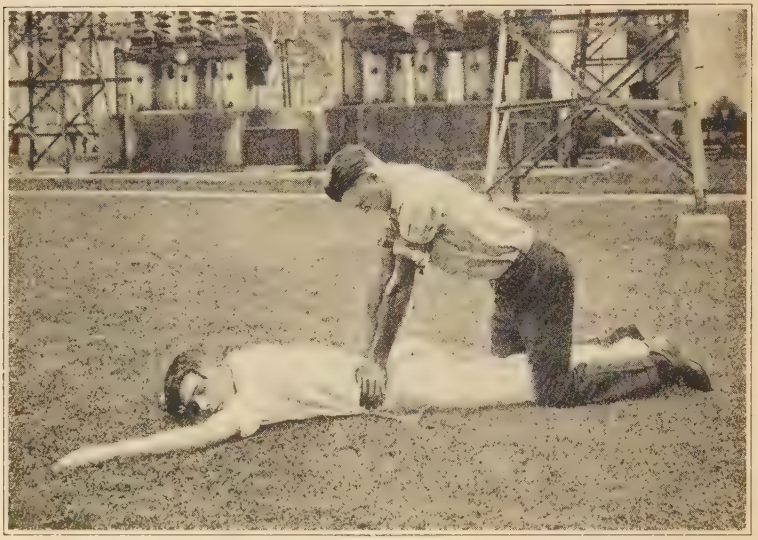


FIG. 291. Resuscitation from electrical shock

which men have been brought back to consciousness, but upon rising to their feet have again succumbed and have not responded to treatment the second time.

288. Conditions affecting injury from electrical shock. The conditions under which injuries from electrical shock occur are influenced by so many factors as to make the results seem entirely capricious. Fatal accidents sometimes occur from 110-volt circuits, yet persons sometimes make contact with circuits at much higher voltages without injury. The most important factors influencing such differences are discussed below.

The effect of electric current upon the body varies greatly with different individuals; therefore it is impossible to state very accurately how strong a current can be passed through the

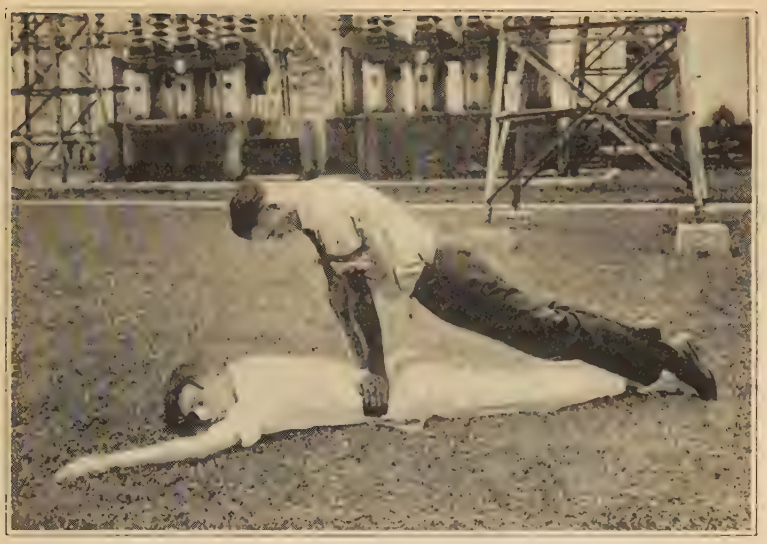


FIG. 292. Resuscitation from electrical shock

FIG. 293. Resuscitation from electrical shock

body without injury. The maximum is of the order of magnitude of rather less than 0.1 ampere, if maintained continuously; 0.5 ampere is certainly capable of producing fatal effects, even if continued only a very brief time. In general the seriousness of the effects produced increases with the strength of the current through the body and with its duration.

The effects, of course, differ also according to the parts of the body traversed by the current. The path of the current through the body is determined by the manner in which contacts have been made. The commonest case is that in which a person touches some uninsulated (or insufficiently insulated) portion of an electrical circuit with one hand, the current passing through the hand and arm and down through the body and feet into the ground. More rarely a person may touch one side of a circuit with one hand, and the other side with the other hand; this is likely to produce the worst effects. Still more rarely, one may touch both sides with the same hand or arm, in which case any injuries will be confined to the hand or arm concerned.

In any case the strength of current passing through the body will be determined by the resistance of the body and the difference of potential across it. The resistance of the body is located almost entirely at the skin, and varies greatly with the condition of the skin, being much less when the skin is wet than when it is dry.* Obviously the resistance at the contact will vary with the kind of contact made; for instance, firmly grasping a conductor with the whole hand will make a contact of less resistance than lightly touching the conductor with one finger. When a person touches a circuit with one hand, the resistance of the path to the ground varies also according to what he stands on. Dry leather shoes are fairly good insulators, and rubber shoes still better ones. A dry wooden floor is a good insulator, and a water-soaked concrete floor is a very poor one.

To illustrate how differences of the kinds just discussed may affect the results in case of accidental contact with a live wire, consider two possible cases of contact with domestic wiring. Suppose, first, that a man with thoroughly dry hands and shoes, standing on a dry wooden

* Measurements on the resistance of the body between hands when firmly grasping a conductor in each give results varying between 5000 and 140,000 ohms, according to the dryness of the skin. With hands dipping into jars of salt water, a resistance as low as 1000 ohms has been observed.

floor, grasps a bare wire of a 110-volt circuit, the opposite wire being well grounded. The resistance of the path to the ground through his body might be about as follows: body, 100,000 ohms; shoes, 300,000 ohms; floor, 1,000,000 ohms. Dividing 110 volts by the total resistance (1,400,000 ohms), we find for the current through his body 0.00008 ampere — probably too small to feel. Suppose, now, that he grasps the same wire with a wet hand while bathing in a metal bathtub. Under these conditions the resistance through his body might be about 1000 ohms; that of the rest of the path to the ground would be negligible. The current through his body would then be 0.11 ampere, which might very possibly be fatal if continued.

If he had touched the opposite wire, which we assumed to be grounded already, he would have received no shock (or possibly a slight one, since the potential of the wire at the point touched might differ slightly from that of the point where it was grounded, if a current was flowing in the wire between these points). If there were no grounds on the system at all, he would have received no shock from either wire (or possibly a slight one, due to capacitance of his body).

289. Methods of securing safety against shock from accidental contact. For protection against the occurrence of electrical shock, three general lines of action may be followed: (1) to make all portions of electrical circuits as inaccessible as practicable; (2) to insulate all portions as far as practicable; and (3) to keep all portions at as low a potential as other conditions permit, so that in case of accidental contact the current passing through the body may be small by reason of the smallness of the difference of potential driving it. The first two methods need no further discussion here.

290. What is the highest safe potential? While a conductor at very low potential (say 0.1 volt relative to the ground) will certainly be harmless to touch, and one at very high potential (say 50,000 volts) certainly dangerous, it is not possible to specify any definite value of potential at which danger begins, since so much depends on the sort of contact established. The highest potential with respect to the ground that is in common use within ordinary dwellings is about 125 volts.* The principal reason why higher potentials are not often used in dwellings is because of the increased danger of accident (either to person or to property). As determined by custom, then, 125 volts may be considered as the highest potential permissible where there is

* In this country; 220 volts is often used in Europe.

any probability that persons may make accidental contact with the circuits. It should be borne in mind, however, that potentials below 125 volts are not inherently safe, but may cause fatal accidents under special conditions of very perfect contact, which experience has shown do not often occur.

291. Conditions affecting the potential of conductors. To bring out the principles affecting the potential of conductors in electrical distribution systems it will be sufficient to limit the present discussion to the type of circuit with which the reader is probably most familiar — the two-wire constant-voltage system.

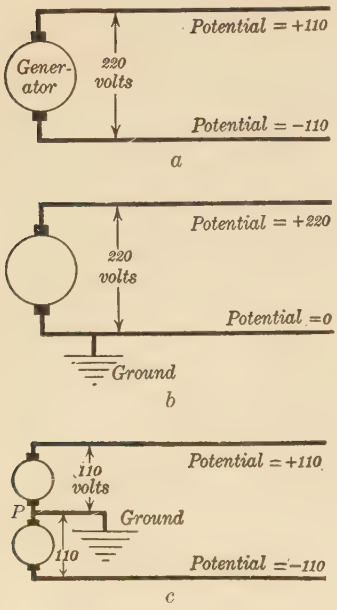


FIG. 294

The potential of any point of such a circuit is determined by the electromotive force applied to the circuit and by connections between the circuit and the ground (the distribution of potential is also slightly affected by the current flowing, which for present purposes may be neglected). If no connection exists between the circuit and the ground its potential at any point might theoretically have any value, but practically there is a tendency for the potential of one side to be as much above the ground as that of the other side is below it,* as illustrated in Fig. 294, *a*. This would also be the most desirable arrangement from the point of view of safety.

In practice, however, accidental ground contacts often develop, and may result in bringing one wire to the potential of the ground (zero), and the other to a potential equal to the full voltage between wires, as shown in Fig. 294, *b*. The increase of potential of a wire due to this cause can always be prevented by intentionally grounding a neutral point, as shown in Fig. 294, *c*.

* This would follow if the circuit were originally at zero potential throughout before the electromotive force was applied. It would also follow, regardless of the original potential, if the insulation leakage were uniformly distributed.

Thus in the three-wire distribution system (see section 77) the neutral wire may be grounded; in a circuit supplied by a transformer the middle of the secondary winding may be grounded.*

292. Protection by grounding. Many of the accidents due to contact with supposedly low-voltage wires are not due to the potential at which those wires are intended to operate, but to some higher potential improperly present by accidental contact between some part of the circuit and some high-voltage wire.

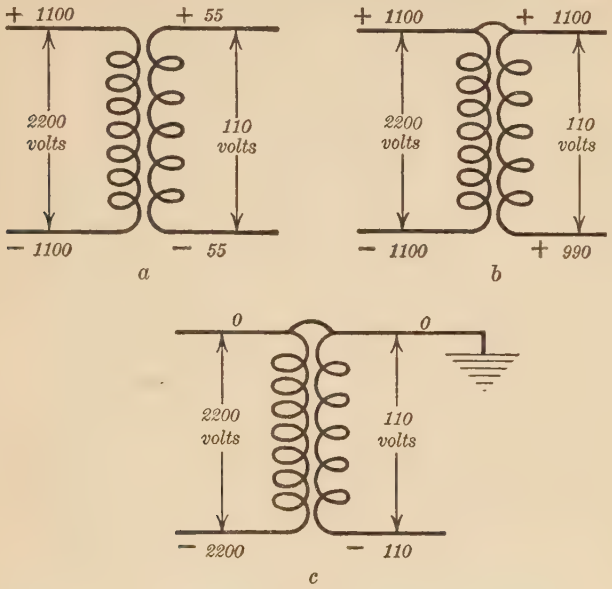


FIG. 295

The numerals next to the mains refer to potential with respect to the ground

In such cases ground connections offer protection. Consider, for example, a transformer supplying a dwelling from its secondary winding at 110 volts, its primary operating at 2200 volts. If its secondary circuit is nowhere grounded, the distribution of potential should be as in Fig. 295, *a*.† But sometimes

* If under these conditions an additional ground contact occurs accidentally on one of the wires, then, of course, there is established through the two grounds a leakage path through which more or less serious waste of current may take place. This should draw attention to the accidental ground, and lead to its removal.

† Of course the potential in this case alternates; for simplicity the figures are drawn to correspond to one of the instantaneous values (the effective value).

lightning punctures the insulation of the primary, leaving primary and secondary windings in contact and giving some such distribution of potential as that shown in Fig. 295, *b*. These dangerous potentials will not occur, however, if the secondary circuit is grounded at some point, as shown in Fig. 295, *c*. The grounding of secondaries is accordingly recommended by the "National Electrical Safety Code," but has not yet become the universal practice in this country.

All metallic objects near to high-voltage conductors, such as frames, casings, or guy-wires, should be grounded. Then, if the high-voltage conductor accidentally makes contact with them, they remain at ground potential, and no current will flow from them to a person touching them.

For a protective ground connection to accomplish its purpose, it is necessary that its resistance be low. Low resistance is not always easily attained, since the resistivity of many rocks and soils is very high, thus requiring a large area of contact to make a low-resistance connection. It is usually possible to obtain a satisfactory ground by connecting with water-piping. More detailed specifications, which would be out of place here, can be found in the "National Electrical Code."

293. Heating and electrolysis from leakage currents. Currents leaking out of the paths intended for them may cause injury to property by their heating effect or by electrolysis. Protection against these injuries, just as against injury to persons, is sought by insulating circuits as well as is practicable and by placing them as inaccessibly as is practicable, thus lessening the probability of accidental contact with objects of any kind. The grounding of circuits, however, is not always so advantageous from the point of view of protection of property, since the presence of a low-resistance ground connection, intentionally made, assures a complete leakage path if another ground occurs by accident elsewhere in the circuit. This is especially objectionable where electrolysis may occur, since electrolytic effects are cumulative, and a very small leakage current, such as is likely to remain unnoticed, may do great harm if continued for a long time. For this reason protective ground connections are frequently inadvisable on direct-current systems, under conditions where they would be otherwise desirable. Fortunately this reason does not limit their use in alternating-current systems.

294. Effects of too heavy currents. When no leakage or escape of current from its intended path takes place, damage may be done if the current in that path becomes too heavy. The injury results from the heat set free, or from mechanical force action between different parts of the circuit. The heating effect is usually more important.

295. Heating effects from excessive currents. When a current flows through any conductor, heat appears in it at the rate of $0.24\ RI^2$ calories per second (where R is the resistance of the conductor in ohms, and I is the current in amperes). This heat causes the temperature of the conductor to rise. As the conductor becomes warmer than its surroundings, it begins to dissipate heat to them at a rate which increases as the temperature of the conductor increases above that of its surroundings. Other conditions remaining constant, the temperature of the conductor will approach a value at which heat is dissipated as fast as it is generated. The greater the current, the higher the temperature the conductor will reach. With sufficiently high current the conductor will become red-hot and may set fire to inflammable material in its neighborhood; or it will melt and may scatter white-hot drops of molten metal upon inflammable materials or upon persons. When the conductor melts, the flow of current is not immediately stopped, but a spark or arc persists across the gap (always momentarily, and sometimes longer). The arc behaves much like a flame as regards ability to set fire to inflammable objects within its reach or to burn persons.

The foregoing discussion applies to cases in which the current continues long enough to set free a good deal of heat. If a current, however large, lasted for a very brief interval of time, then evidently it might not set free enough heat to produce more than a slight rise of temperature.

296. Safe current-carrying capacities. While the risk of accident increases with increase in the current carried by any given conductor, the limit of safety is not clearly defined. The current alone does not determine the temperature an assigned conductor will assume, but the characteristics of the surroundings affect it as well.* The probability of accident depends not only

* For example, an outdoor wire in a winter wind will not assume the same temperature as will the same wire carrying the same current when placed in a poorly ventilated boiler room.

on the temperature reached but also on the surroundings.* A certain amount of arbitrary judgment has to be used in deciding just what conditions should be regarded as safe or unsafe in any particular case.

In practice, empirical rules or tables are used for the purpose of deciding how large a current a given conductor may safely carry (or how large a conductor should be provided to carry a given current). Thus, for the particular case of the interior wiring of buildings, the table on page 327 gives the allowable current-carrying capacity of wires of different sizes, according to the rules of the National Board of Fire Underwriters.

This table (which is universally followed in interior wiring in this country) is intended to determine safe and conservative practice in regard to the permissible current in copper wires of different sizes within buildings. The values given in the table accordingly include a fairly large "factor of safety": injury is not likely to occur if the permissible current is slightly exceeded, and *probably* will not occur unless it is very greatly exceeded; but, of course, this does not justify taking chances by passing heavier currents through wires than the table specifies, since this would defeat the whole purpose of the conservativeness with which the values have been assigned.

297. Ways in which excessive currents occur. A wire which has been installed in the correct size for the current it is expected to carry may be accidentally subjected to heavier currents. The commonest ways in which this may occur are the following:

1. More load (lamps, motors, etc.) may be connected with the circuit than it was originally intended to supply.

2. The current taken by a motor increases with the mechanical load on the motor. Accordingly, if a motor is overloaded mechanically, the current in the wires that supply it will become greater than normal. If the overload should be sufficient to stall the motor, the current may become many times as great as normal.

3. A "short circuit" — that is, a low-resistance path between conductors intended to be at considerable difference of potential — may take place and permit excessive current to flow. For example, the two opposite wires of an electric circuit may accidentally come into contact; or some conductor, such as a tool

* For example, the probability of starting a fire from an overheated wire is much greater if inflammable objects are near it than otherwise.

ALLOWABLE CURRENT-CARRYING CAPACITIES OF WIRES IN INTERIOR WIRING ACCORDING TO THE RULES OF THE NATIONAL BOARD OF FIRE UNDERWRITERS

B. & S. GAGE NUMBER *	CROSS-SECTIONAL AREA IN CIRCULAR MILS †	AMPERES FOR COPPER WIRES WITH		
		A, Rubber Insulation	*B*, Varnished-Cloth Insulation	*C*, Other Insulation
18	1,624	3		5
16	2,583	6		10
14	4,107	15	18	20
12	6,530	20	25	25
10	10,380	25	30	30
8	16,510	35	40	50
6	26,250	50	60	70
5	33,100	55	65	80
4	41,740	70	85	90
3	52,630	80	95	100
2	66,370	90	110	125
1	83,690	100	120	150
0	105,500	125	150	200
00	133,100	150	180	225
000	167,800	175	210	275
0000	211,600	225	270	325
	500,000	400	480	600
	1,000,000	650	780	1,000
	2,000,000	1,050	1,260	1,670

slipping from the hand of a workman, may touch both wires at the same time; or some electrical appliance may burn out in such a way as to provide a low-resistance path between the mains with which it is connected.

* In the United States, sizes of wire are specified in terms of arbitrary gage numbers. Unfortunately several different wire gages are in use, the same number designating one size in one gage and another size in another gage. However, the only gage used for electrical wires is the Brown & Sharpe (or "B. & S.") gage, often called also the American Wire Gage (A. W. G.). In this gage No. 0000 is defined as exactly 0.4600 inch in diameter, and No. 36 as 0.0050 inch in diameter; the diameters for the intermediate gage numbers follow a geometrical progression. Since there are thirty-nine steps in passing from No. 0000 to No. 36, the ratio of the diameter of any size to that of the size having the next larger gage number is $\sqrt[39]{\frac{0.4600}{0.0050}} = \sqrt[39]{92} = 1.122932\cdots$. It happens that the third power of the square of this ratio is almost exactly 2; accordingly the cross-sectional area of any wire is twice as large as that of the wire whose gage number is larger by 3 — a relation which is often convenient to keep in mind in using the Brown & Sharpe gage.

† The circular mil is a unit in common use in this country for expressing the cross-sectional areas of electrical conductors. It is equal to the area of a circle the diameter of which is 0.001 inch. Obviously the cross-sectional area of a wire, in circular mils, is equal to the square of its diameter in thousandths of an inch (assuming circular cross section).

4. Most objects which come into accidental contact with electrical wires supply a path — although often of very high resistance — from the wire to the ground. If such contact, or "ground," occurs on a wire whose potential is intended to be different from that of the ground, and the resistance of the path to the ground is sufficiently low, a heavy current may flow.

To prevent the accidental occurrence of dangerously heavy currents, automatic protective devices are used. These are usually either fuses or circuit-breakers.

298. Fuses. A fuse is a piece of easily fusible metal included in an electric circuit, and is designed to melt and thus open the circuit when the current in it exceeds some definite value. Fuses are usually made of alloys containing two or more of the metals lead, tin, bismuth, zinc, and cadmium, the composition varying with different manufacturers. Aluminium also is used as a fuse material.

The simplest form of fuse is that of wire, of which a short piece can be cut off and connected in the circuit requiring protection. The action of such a fuse is rather uncertain, since the rate of dissipation of heat from it is greatly influenced by its surroundings (especially by conduction of heat into the connections at its ends), so that fuse wire of the same size and material may "blow" at somewhat different currents under different conditions. Exposed fuse wire also has the disadvantages that drops of hot metal may be scattered when it melts under current, and that whatever arc occurs has access to the immediate surroundings.

To obtain safer and more certain action, *inclosed fuses* are used. In an inclosed fuse the fusible element is soldered to brass or copper terminals and surrounded by walls of noninflammable material. The whole assembly is arranged for insertion in a *fuse block*, to which the circuit requiring protection is connected. If the fuse blows, the whole assembly is removed from the fuse block and replaced by a new one.

The commonest types of inclosed fuses are cartridge fuses and plug fuses. In the cartridge fuse (see Fig. 296) the fusible element is placed within a cylindrical tube of fiber having brass caps, or ferrules,* at the ends, to which the ends of the fusible element are soldered. The space between the fusible element

* Or, in some cases, other forms of terminal.

and the walls is generally filled with some inert powder (silicic acid is often used) which cools and quenches the arc when the fuse melts. The fuse block consists of a block of noninflammable insulating material (usually slate) supporting at each end a spring clip provided with means for connection to the circuit

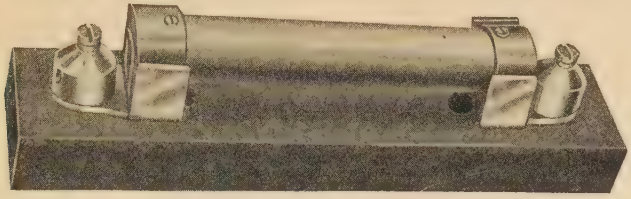


FIG. 296. Cartridge fuse

requiring protection; the spring clips engage with the terminals of the fuse when it is in place, and permit its easy removal for replacement when necessary.

The plug type of inclosed fuse (see Fig. 297) has the same shape as the Edison base of an incandescent lamp, and is connected in the circuit requiring protection by screwing it into its fuse block (often called a *cut-out*) in the same way that an incandescent lamp is screwed into its socket. Its side walls are made of porcelain, but the top is covered with mica to permit easy inspection of the fusible element inside. It is not customary to surround the fusible element with powder, since fuse plugs are used in circuits of comparatively low current and voltage, in which there is little danger of a bad arc when the fuse melts.

FIG. 297. Plug fuse

The current rating of a fuse, which is marked on each inclosed fuse and on spools of fuse wire, refers to the permissible current in the circuit to be protected by the fuse. It is customary to give a fuse a current rating somewhat less than the current at which it is expected to melt, so that the current in the circuit under protection will have to *exceed*, and not simply reach, its maximum permissible value before the fuse blows. The standards of the American Institute of Electrical Engineers require that "fuses shall be so constructed that they will carry continuously 110 per cent of their rated current." This is equivalent to

permitting a continuous overload of 10 per cent in a circuit protected by a fuse. For a brief period of time a greater overload may take place, since the fuse does not melt at the instant the current reaches the fuse's maximum carrying capacity, but only after such a current has lasted long enough to heat the fuse to its melting point. It is no disadvantage that a fuse fails to respond to a brief temporary overload; because, if the current does not have time to liberate sufficient heat to melt the fuse, it will also not have time to liberate heat in dangerous quantity in the rest of the circuit.

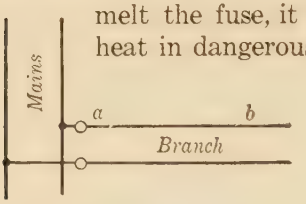


FIG. 298. Diagram to show location of fuses and a branch circuit

Inclosed fuses are rated for voltage as well as for current, fuses to be used with higher voltages making greater requirements for means to extinguish the arc than those to be used with lower voltages.

It is standard practice to use fuses as the protective devices against excessive currents in house-wiring, and they are largely used in all circuits which are not intended for extremely heavy currents. They are made for currents up to several hundred amperes.

The point at which a fuse (or other protective device) is placed in the circuit it is meant to protect is not a matter of indifference. Evidently the proper place should be at the beginning of the circuit, where it branches off from the mains that supply it, as shown in Fig. 298, *a*. If the fuse were placed at a distance from the point of branching, as at *b*, then it would fail to act if short circuit should occur between *a* and *b*. A fuse should be placed in *each* wire of a circuit requiring protection, since a low-resistance ground might occur on one wire only and cause heavy current to pass through it (if a ground existed elsewhere on the system) without affecting the current in the other wire.

299. Circuit-breakers. The circuit-breaker is a protective device against excessive current, which depends upon the magnetic action of the current to operate a mechanism for opening the circuit when the current reaches some predetermined value. This is usually accomplished by making the current pass through the winding of an electromagnet provided with a

movable armature of soft iron, which, when the current is strong enough, is set into motion and strikes a trigger; a strong spring then opens a switch which had been held closed by a catch connected with the trigger. To close the circuit again

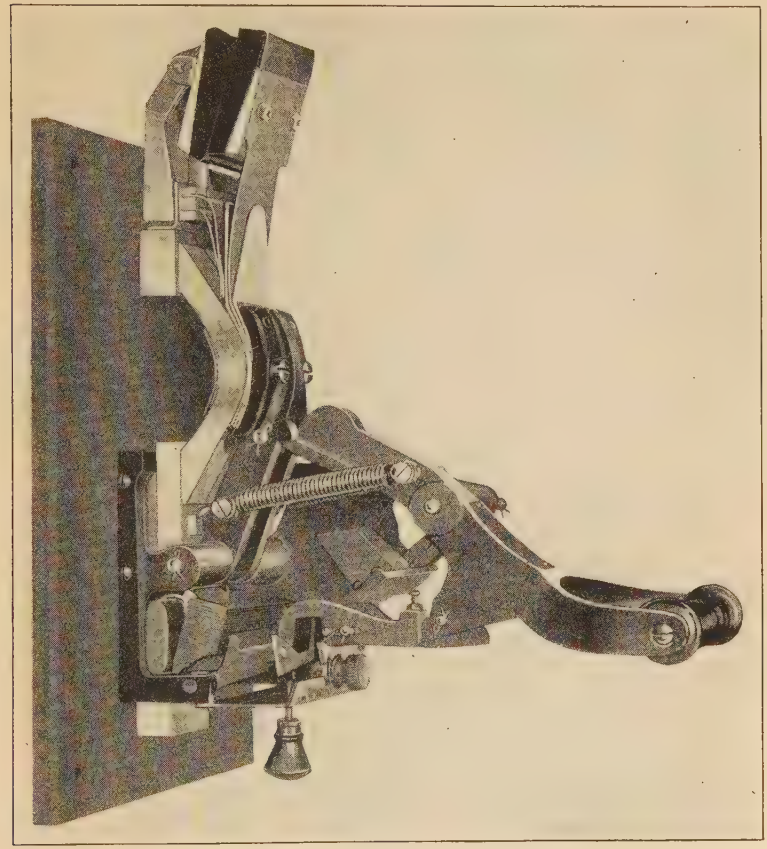


FIG. 299. Carbon circuit-breaker
Courtesy of the Cutter Company

after the circuit-breaker has acted, it is merely necessary to close the switch; the breaker is then in condition to act again.

It is necessary to provide against destructive effects from the powerful arc formed when a heavy current is broken. In the "oil circuit-breaker" this is done by keeping the contacts immersed in oil, which quenches the arc and absorbs much of its

heat when the current is broken. In the "carbon circuit-breaker" the switch is provided with auxiliary contacts of carbon in addition to the main contacts of copper. In the act of opening, the copper contacts separate first, leaving the current to continue for a brief interval of time (against increased resistance) through the carbon contacts; the final break occurs between them, and whatever burning of the contacts is caused by the arc takes place in the carbon contacts, which are cheap and replaceable.

Circuit-breakers are especially suited for heavy currents. They are made for the heaviest currents used (10,000 amperes, and more). They are not often used for currents smaller than 10 amperes.

Compared with fuses, circuit-breakers possess the following advantages:

1. *Greater certainty of action.* The action is uninfluenced by the temperature of the surroundings, as that of a fuse would be.

2. *Cheaper operating cost.* When a fuse blows, it has to be replaced by a new fuse; when a circuit-breaker opens, the circuit is restored by merely closing its contacts.

3. *Quicker action.* The circuit-breaker opens almost instantaneously when the current reaches the critical value; the fuse requires time for heat to accumulate.

4. *Less probability of misuse.* When a fuse blows, and another one of proper size is not at hand, there is great temptation to close the circuit by use of a larger fuse or a piece of copper wire, which, of course, leaves the circuit without protection.

On the other hand, the first cost of a circuit-breaker is much greater than that of a fuse of the same rating.

300. Rules for personal safety. To conclude this chapter, attention will be called to a few of the most important rules for personal safety in working on electrical installations.

Never work on a live circuit, if this can be avoided; open a switch and disconnect the circuit before beginning to work.

If work must be done on a live circuit, as far as possible use only one hand, keeping the other hand behind the back; this lessens the probability of accidentally making contacts with the two hands on opposite sides of the circuit.

Never stand on wet floor or ground while working on electrical circuits; if necessary, place dry boards (or other insulating supports) to stand on.

Do not permit any loosely held objects (dangling ends of wire, tools projecting from pockets, etc.) to be near live wires; they may cause short circuit, with consequent injury from burning or flash.

If it is necessary to approach high-voltage apparatus, remember that contact may be made through the air near the conductors without actually touching a conductor itself.

More important than any formulated rules is the "safety spirit," or general attitude of cautiousness and of alertness in recognizing danger. This spirit should be cultivated especially in regard to electricity. Remember always that electricity is a dangerous agency. Never let yourself drift into an attitude of recklessness toward it. Do not take chances with it or permit others to do so.

PROBLEMS

Leakage Currents

1. The insulation resistance between mains on a certain 2200-volt distribution system amounts to 10 megohms for each mile of line under average weather conditions. There are 12 miles of line in the whole system. Find the insulation resistance between mains for the whole system, the total leakage current, the total power lost in it, and the annual cost of the loss, allowing 3¢ per kilowatt-hour.

2. One ampere of current strays from the rails of an electric railway through the ground into an iron pipe line, out of which it passes again into wet ground, causing electrolysis. How much iron will be affected (corroded or dissolved) in a year? (The electrochemical equivalent of iron is 0.000289 or 0.000193 per coulomb, according as the iron reacts as ferrous or ferric iron.)

3. The ground connection in Fig. 294, *c*, has a resistance of 1 ohm (this is too high for a good ground connection). Suppose an accidental ground having a resistance of 54 ohms now occurs on the main whose potential is intended to be + 110 volts. Find the leakage current, and the potential of each of the mains and of the point *P* (neglect the resistance of the mains, and assume that the electromotive force of the generators is unchanged).

Short Circuits and Overloads

4. A transformer supplies a building with electricity at 220 volts through two mains of No. 8 copper wire (resistance 0.4 ohm per 1000 feet). If a short circuit occurs across these mains 100 ft. from the transformer, find what current would flow in the absence of

protective devices. The internal resistance of the transformer is 0.02 ohm; the resistance of the short circuit is negligible. Assume that the transformer maintains an electromotive force of 220 volts, whatever the current.

5. The motors of a certain street car are protected by a circuit-breaker, set to open the circuit at 200 amperes. This turns out to be insufficient to start the car up a steep grade; in order to get through, the motorman fastens the circuit-breaker shut, thus causing the motors to take 400 amperes (for a short time). Compare the rate of production of heat in the motors under this condition with that when taking 200 amperes.

6. What size of wire should be used for a circuit in a house, intended for three 100-watt 110-volt lamps? Suppose that one of these lamps is replaced by an electric iron, taking 5 amperes, while the other two lamps are burning; find how much the rate of production of heat in the wires is increased.

BIBLIOGRAPHY

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For special apparatus, a valuable source of information is offered by the bulletins issued by the manufacturers.

TABLES FOR USE WITH THE PROBLEMS

Factors for Translation of Units

1 foot = 30.5 centimeters
1 inch = 2.54 centimeters
1 pound (mass) = 454 grams
1 pound (force) = 445,000 dynes
1 foot-pound = 1.36 joules
= 3.78×10^{-7} kilowatt-hours
1 horse power = 746 watts

Weight and Resistance of Copper Wire

Gage Number, Brown & Sharpe Gage	Diameter in Inches	Weight per 1000 Feet in Pounds	Resistance per 1000 Feet at 20° C. or 68° F. in Ohms
0000	.460	640	.0490
000	.410	508	.0618
00	.365	403	.0779
0	.325	320	.0983
1	.289	253	.124
2	.258	201	.156
3	.229	159	.197
4	.204	126	.248
5	.182	100	.310
6	.162	79.5	.395
7	.144	63.0	.498
8	.128	50.0	.628
9	.114	39.8	.794
10	.102	31.4	.999
11	.0908	25.0	1.26
12	.0808	19.8	1.59
13	.0721	15.7	2.00
14	.0641	12.4	2.52
15	.0571	9.86	3.18
16	.0508	7.82	4.02
17	.0453	6.20	5.06
18	.0403	4.92	6.38
19	.0359	3.90	8.05
20	.0320	3.09	10.15
22	.0254	1.94	16.14
24	.0201	1.22	25.67

ANSWERS TO PROBLEMS

NOTE FOR THE STUDENT. The answers given here are intended merely for your convenience in checking the results of your computations. They should not be looked upon as having any quality of infallibility, or as in any way relieving you of the responsibility of independent thought in working the problems. The answers are intended to be correct, and are correct in most (if not all) instances; but their correctness is not guaranteed. This is better for you.

CHAPTER I

1. 2 gausses, eastward; 2 gausses, eastward; 2 gausses, westward. **2.** 184 dynes, to the left. **3.** 1.45 cm. **4.** Floor and ceiling, 540,000 maxwells; east and west walls, none; north and south walls, 100,000 maxwells. **5.** .077 lb., southward. **6.** 24.7 lb. **7.** $H = \frac{2I}{D}$. **8.** $F = \frac{2 I_1 I_2}{D}$; the force will be an attraction if both currents travel in the same direction, and a repulsion if they travel in opposite directions; 100 dynes per cm. **9.** 25 ohms. **10.** 0.20 gauss; 0.636 amp. **11.** 31.4 gausses. **12.** 0.0754 volt; 0.754 amp. **13.** 0.00034 volt, upward. **14.** 0.0015 volt. **15.** 0.001 sec. **16.** 20 amp.; + 10, 0, and − 10 amp. **17.** 0.0178 henry. **19.** 0.790, 1.19, 2.00 amp.; 2/3, 2/3, 2/3 amp. **20.** 12.57 gausses; 15,500 gausses; 1233. **21.** 1257 gilberts; 0.01257 oersted. **22.** 3.58 amp. **23.** 76,000 maxwells. **24.** 0.0002 coulomb; 0.2 sec. **25.** 0.0528 amp. **26.** 12×10^9; 480,000,000 times as much. **28.** 20, 11.9, 0.000045, and 0 amp.

CHAPTER II

1. 5.66 volts; 19.4, that is, 20 conductors. **2.** 0.539 lb. **3.** 2 kw.; 3.16 h.p. **4.** 66.3 + 2, or 68.3 lb. **5.** 2250 pound-inches. **6.** 2.9%.

CHAPTER III

1. 79.8 amp. **2.** 204.8 volts. **3.** 2.97 lb. **4.** 0.8 volt; 189,000 amp.; 152 kw. **5.** 15.1 volts. **6.** $93\frac{1}{3}$ times.

CHAPTER IV

1. 0.00138 sec.; 13,000 amp. per sec. **2.** 80 ohms. **3.** 130 volts. **4.** 163 pound-inches. **5.** 162 amp.; 793 pound-inches. **6.** 2.27 amp.; 97.0 ohms; 2200 turns; 3300 feet; a little smaller than No. 24.

CHAPTER V

1. 100, 70, and 20 amp.; 109, 107.6, and 106.8 volts; 714 watts. 2. 0, 30, 50, 20, 20, and 50 amp.; +111.5, +109.7, +0.3, 0, −109.0, and −0.9 volts; 612 watts. 3. 109.4 volts; 109.2 volts; 111 volts. 4. No. 000.

CHAPTER VI

1. 109.8, 108.4, and 107.6 volts. 2. 2 amp.; 22 amp.; 261 : 251. 3. 9 volts. 4. 115 volts.

CHAPTER VII

1. 1.726 ohm; 1.659 ohm; 1.596 ohm. 2. 0–1, ∞; 1–2, 0.05 ohm; 2–3, 0.0167 ohm. 3. 0.05 ohm; 112.5 volts.

CHAPTER VIII

1. 0.3 volt; 30 watts. 2. 0.04 amp.; 8.8 watts. 3. 1147.6 watts. 4. 110.74 volts. 5. 0.00075 ohm. 6. 630 amp. 7. (*a*) 110, 110; (*b*) 220, 0; (*c*) 146.7, 73.3.

CHAPTER IX

1. 20.4 rev. per sec.; 126.1 pound-inches. 2. 1.71 amp.; 22.0 rev. per sec. 3. (*a*) 220 amp.; 1540 pound-inches; (*b*) 100 amp. 4. (*a*) 2.20 rev. per sec., 8460 pound-inches; (*b*) 4.50 rev. per sec., 8460 pound-inches. 5. 222 rev. per sec., or 13,320 r.p.m. 6. (*a*) 180 amp.; (*b*) 40.4 rev. per sec. (neglecting armature reaction).

CHAPTER X

1. 75.5%. 2. 74.3%. 3. 65.7%. 4. Motor *B*; $73. 5. Yes; it would save $1200 annually.

CHAPTER XI

1. 4.52×10^7 cal. per hour; 5660 g. or 12.45 lb. of coal per hour. 2. 48,000 cu. in. 3. *B* would cost half as much as *A*. 4. 558,000 cal.; 516,000 cal.

CHAPTER XII

1. $8\frac{4}{7}$ rev. per sec.; 30 rev. per sec. 2. 0, 1.2, 4.2, 5.4, 6.0, 6.6, 6.0, 5.4, 4.2, 1.2, and 0 volts; 4.53 volts. 3. 52.8 kw.; $2197; $878,800. 4. 334.

CHAPTER XIV

1. 60°; 1908 volts, leading the voltage of *B* by 27°. 2. 90°. 3. 18.3 ohms; 18.3 ohms; 0.0485 henry. 4. 155.7 volts. 5. 0.909 amp.; 0.868 amp. 6. 105 volts; 32.7 volts; 91.2 watts; 0 watts. 7. 0.5; 1/6 cycle. 8. 9.27 amp.; 1640 volts.

CHAPTER XV

1. 78.7 amp.; 136.3 amp., or 3 times as much. 2. 1114 watts; 1485 watts, or 4/3 times as much. 3. Main *A*, 90.1 amp.; main *B*, 66.2 amp.; main *C*, 109 amp.; 1462 watts.

CHAPTER XVI

1. 943, 1244, and 12,000 volts. 2. 1089, 1436, and 12,690 volts; 1089, 1436, and 11,380 volts.

CHAPTER XVII

1. 1017. 2. 107.0 amp.; 185.3 amp. 3. 2571 volts; 3020 volts; 2012 volts. 4. 96.5%.

CHAPTER XVIII

1. 50 amp.; 1100 volts. 2. 4.55, 90.9 amp. 3. 97.7%. 4. 88.6%. 6. 250,000 maxwells; 94,300,000 maxwells per sec.; 0.943 volt; 0.666 volt; 3450.

CHAPTER XIX

1. 590 or 610 r. p. m. 2. 208 volts, leading the supply voltage by 56.5°; 166.5 amp., leading the supply voltage by 3.4°. 4. 1800 r.p.m.; 1 cycle per sec. 5. Ratio of currents, 13.0 : 22.2; ratio of RI^2 losses, 34.5%. 6. 18.27, 12.57 volts. 7. 166.6°; 161.9 kw. 8. 168.5 h. p.; 1790 r. p. m.; 127.2 kw.

CHAPTER XX

2. 0.183, 0.455. 3. 1200 r.p.m.; 238.4 volts; 119.2 volts.

CHAPTER XXI

1. 113.0, 111.6, 109.2, 106.2, and 102.2 volts; 752 watts. 2. 110.8, 111.4, 113.0, 114.0, and 110.0 volts; 392 watts. 3. $21,000. 4. 4.28 cents; 2.49 cents; 2.34 cents.

CHAPTER XXII

1. 10.8 g. 2. 1/30 amp. 3. (*a*), (*c*), (*d*), discharging; (*b*), (*e*), charging. 4. 0.201 kilowatt-hour; 38 kilowatt-hours, or 190 times as much; 138 kilowatt-hours, or 690 times as much. 5. 220 ampere-hours.

CHAPTER XXIII

1. 0.833 megohm; 0.00264 amp.; 5.81 watts; 50.9 kilowatt-hours; $1.53. 2. 9114 g. or 6076 g. 3. 2 amp.; + 108, − 112, and − 2 volts. 4. 2200 amp. 5. 4 times. 6. No. 18; 6.25 times.

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